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MACHINE DESIGN.

PART II.

FORM, STRENGTH, AND PROPORTIONS
OF PARTS.

BY

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THIRD EDITION, REVISED.

FIRST THOUSAND.

PREFACE.

THE matter presented on the following pages is confined to such subjects as the designer must deal with daily. Equations and formulas are put into such a form as to afford a ready means of application to problems under consideration. Numerical examples and data from practice illustrating principles are introduced wherever it seems that a clear understanding can be brought about in this way. The data thus presented have been gathered from numerous sources during the last fifteen years. That coming from modern practice is always given the preference, however, except where the older matter is undoubtedly the most valuable. Whenever possible, the results of practice or experiments as presented by some engineer or experimenter which fairly well represent the general experience along their lines are given in preference to abstract statements. This is believed to be the most satisfactory method, since it affords a means of studying all the facts incidental to the particular case, in addition to furnishing information fully as well as abstract statements.

The publications of engineering societies and engineering journals have been freely consulted. Much courtesy has been shown by the representatives of engineering and manufacturing concerns, who have invariably been kind in answering inquiries, and many of them have furnished valuable data. Whenever possible, credit for such information is given in connection with the information itself, but much of it was of such a nature as to make it impossible to separate it from general statements.

The author desires to express his thanks to all those who have so kindly treated his petitions for information. Of those not men-

tioned in the body of the book Mr. Edw. L. Bateman of Fraser
& Chalmers and Mr. Peter Conner of the Gisholt Machine Co.
have been especially kind, the former in giving information, and
the latter in giving both information and his time in making
sketches and drawings of many of the figures.

The printer's proof of most of the matter closely related to the
mechanics of engineering has been kindly read by Mr. E. R.
Maurer, Professor of Applied Mechanics in the University of Wis-
consin, who pointed out where several improvements might be
made and ambiguities eliminated. By following his suggestions it
is believed that the work has been materially improved.

<div align="right">FORREST R. JONES.</div>

MADISON, WIS., January, 1899.

PREFACE TO THIRD EDITION.

SEVERAL new subjects have been included and a considerable
number of examples introduced to illustrate the matter that appeared
in the earlier editions. New data from leading engineering con-
cerns have also been added. In all, the book is increased in size
by about eighty pages of new matter.

<div align="right">F. R. J.</div>

ITHACA, N. Y., November. 1908.

NOTE.

In the second impression of the Third Edition, Tables of
Dimensions and Capacities of Ball Bearings are added, and some
changes made in the text.

<div align="right">F. R. J.</div>

ITHACA, N. Y., March, 1904.

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FORM, STRENGTH, AND PROPOR-
TIONS OF MACHINE PARTS.

CHAPTER I.

BEARINGS AND LUBRICATION.

1. Introductory.—In machinery the name **bearing**, or **bearing surface**, is applied to a part which presses or bears against another and moves over it at the same time.

If the surfaces are dry, or as left by the tool used to finish them, they have a strong tendency to abrade, or " cut," each other when the materials commonly used for machine members are rubbed together. To prevent this cutting, a lubricating substance, such as oil, grease, or graphite, is introduced between them.

The necessity of thorough lubrication is often of such vital importance that the bearing must be designed with especial regard to securing a continuous application of the lubricant.

BEARINGS FOR RECTILINEAR MOTION.

2. Planer and lathe "ways" or "V's."—The table or platen of a metal-working planer reciprocates on bearings which must give it an accurate rectilinear motion in order that the cutting tool may form a plane surface, or a curved surface whose elements are right lines. The V form of bearing, such as is shown in Fig. 1 at *A* and *B*, is most commonly used. In this style of bearing the weight of the table, together with the work upon it, is relied upon to hold

the table in place.   The cutting tool frequently exerts a side
pressure tending to force the table off its bearings.   The ability of
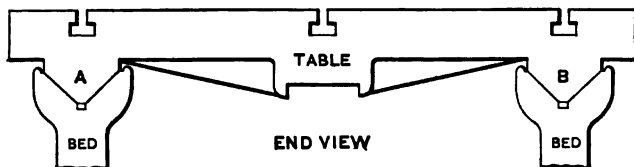
FIG. 1.

the table to resist this pressure depends on the angle θ, Fig. 2, be-
tween the sides of the V.   It is plain that the smaller this angle the
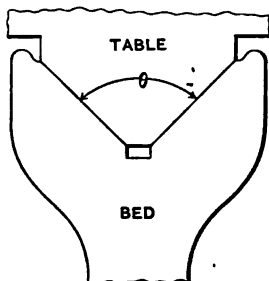
FIG. 2.

greater will be the resistance of the table to being displaced.   The
smaller the angle, however, the greater the total pressure between
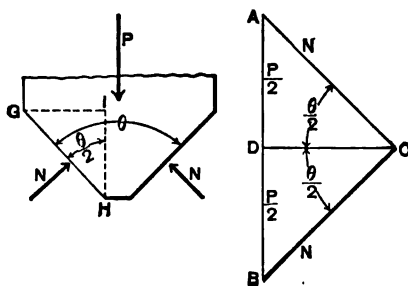
FIG. 3.

the bearing surfaces.   This can be seen by the aid of Fig. 3, which
represents the way on one side of the table when it is removed from

the bed. The load which this V must support may be called P. The pressure against each side of the V is normal to the bearing surfaces, friction being neglected. These normal pressures are represented by N and N in the figure. The amount of N and N is found by resolving P into two forces normal to the bearing surfaces. This is done by taking AB parallel to P, and of such a length as will represent the magnitude of P according to any convenient scale of pounds per inch, or other units, and then drawing AC and BC parallel to N and N. Then $AC = BC = N$ according to the scale selected for AB. In the figure it can be seen that

$$N = \frac{P}{2} \div \sin \frac{\theta}{2} = \frac{P}{2} \csc \frac{\theta}{2}.$$

The equation shows that reducing the angle between the sides of the V increases the total pressure upon the bearing surfaces. The angle should therefore be kept as large as possible without getting it so great that the table may be thrown from the bed by the side pressure of the tool.

The V form of bearing is self-adjusting for wear, since it naturally settles down as the surfaces wear away.

For ordinary service the V's for a small planer should be made with a smaller angle θ than for a large machine. This is due to the fact that, as the size of the planer increases, the weight of the table increases more rapidly than the side pressure of the tool.

Varying the angle θ does not change the pressure per unit area on the bearing surfaces as long as the load P and the horizontal projection of the surfaces remain unchanged, for the bearing surface is increased in the same proportion as the total pressure. The power necessary to move the table increases as θ is decreased, however, and the wear on the ways is increased, so that the table will settle down more rapidly. There is also more liability to abrasion with the smaller angle, since localization of the pressure, due to unevenness of the bearing surfaces, causes a greater pressure on the high parts.

That the pressure per unit area on the bearing surface is unchanged as long as the horizontal projection of the surface and the load remain the same, whatever the value of θ, can be shown

in the following manner: In Fig. 3 the horizontal width of the bearing surface GH is GI; therefore

$$GH = GI \csc \frac{\theta}{2}.$$

The total pressure upon GH is N, whose value is given in the preceding equation. The pressure per unit area, as found by dividing the total pressure N normal to the bearing surface by the total area GH, is

$$\frac{N}{GH} = \frac{P}{2} \csc \frac{\theta}{2} \div GI \csc \frac{\theta}{2} = \frac{P \div 2}{GI}.$$

This shows that the pressure per unit area is always equal to the quotient obtained by dividing the load supported upon the bearing surface by the horizontal projection of the surface, and is therefore not affected by the angle θ. The other half of the load P is supported by the opposite side of the V.

The angle θ is generally 90° or slightly less in planers for general use up to 24 inches between housings. It grows larger as the machines increase in size, reaching as much as 120° for 72 in. to 144 in. between housings. The width or slant height of one side of the V in planers built by the Niles Tool Works branch of the Niles-Bement-Pond-Co. is given below. The width of bearing surface is given for only one side of one V, and corresponds to the distance GH in Fig. 3.

<div align="center">WIDTH OF PLANER V'S.</div>

Distance between housings, inches.	26	32	36	38	42	44	48	54	60	72	84	96	120	144
Width of bearing surface = GH, Fig. 3. Inches.	2	$2\frac{1}{4}$	$2\frac{1}{2}$	$2\frac{1}{2}$	8	$8\frac{1}{4}$	$8\frac{1}{2}$	$4\frac{1}{4}$	$4\frac{1}{2}$	$5\frac{1}{4}$	6	$6\frac{1}{4}$	$8\frac{1}{2}$	9

The Bement-Miles branch of the same company has, for a planer 72 inches between housings, the following values referring to Fig. 3:

$$\theta = 120°; \; GH = 4\frac{22}{32} \text{ inches}; \; GI = 4\frac{1}{4} \text{ inches}.$$

The G. A. Gray Company planers have, referring to Fig. 3, $\theta = 90°$ and $GI = 1$ inch in a planer 22 inches between housings; in one 72 inches between housings $\theta = 110°$ and $GI = 4$ inches. Between these sizes the angle θ and width of way are proportional to the distance between housings.*

Planers for special work, where the piece to be machined is comparatively light, and the side pressure of the tool heavy, should have the angle θ smaller than 90°, and the width of the bearing surfaces greater than for the larger angle more commonly used. The greater width of the bearing surfaces is necessary to keep down the pressure per unit area on the bearing.
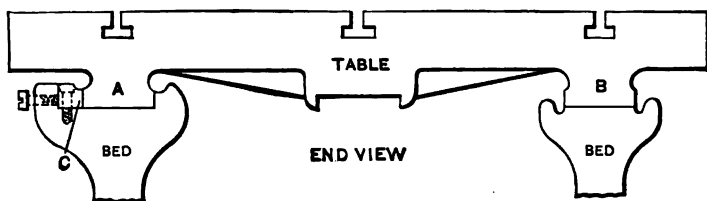
FIG. 4.

Fig. 4 shows a pair of less commonly used bearings for a planer or other reciprocating table which is partly held in place by its own weight. The cutting tool cannot displace a table having this form of ways, except when a heavy side pressure is applied against a piece of work at a considerable distance above the table. The pressure on the horizontal part of the bearings is equal to the load, instead of being greater than the load, as in the case of the V bearings. This is an advantage, since it reduces the liability to abrasion.

The strip C is adjustable horizontally to allow for wear on the vertical surfaces. If attached to the bed as shown, it can be set so as to give the table a snug running fit from end to end of the bed, even though the wear near the centre of the length of the bed is greater than at the ends, as is generally the case.

Fig. 5 shows a form of bearing used upon one side of the table of a planer taking work 120 inches square.† The flat angle of 150°

* The proportions of planer bearings given above were courteously furnished by the officials of the establishments named.

† Used by the Wm. Sellers Co. Sketch kindly furnished by them.

is sufficient to hold the table in position when making the return stroke, or when the tool is taking a light cut. The sides of the way, which make an angle of 8° with a vertical line, or 16° with each other, prevent displacement of the table by a heavy side pressure of the cutting tool. On account of their angularity, no adjusting device is necessary to take up side wear, for the settling of the table in the ways makes this adjustment.

V-form bearings can be continuously lubricated by placing oil-pockets, of the form shown in Fig. 6, along them at intervals. This pocket is simply a recess cast into the bed. A double-cone
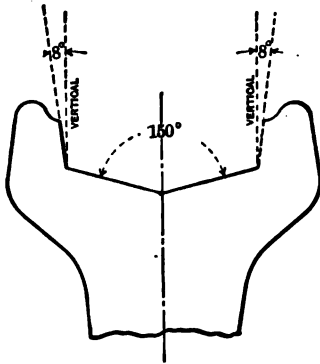
FIG. 5.
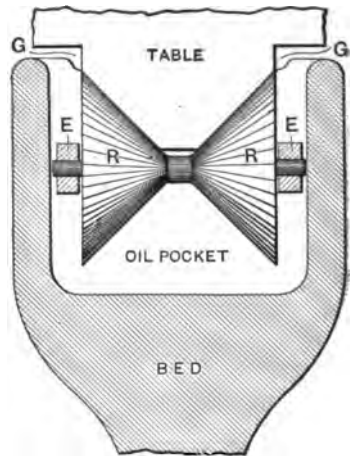
FIG. 6.

roller RR is placed in the pocket so as to be partly submerged in oil. The angles of the cones are such that they will fit the table V's. The roller is supported at the ends by bearings EE which are held up by some means, as springs or counterweights, so that the roller will press lightly against the V as the table passes over the roller. The slight frictional resistance between the surfaces turns the roller, so that the oil adhering to it is carried up to the V. Suitable means should be provided to prevent the roller from being raised too high when the table passes from over it. After the oil is thus placed on the V, some of it is carried between the bearing surfaces by the motion of the table. The pressure between the

surfaces generally squeezes the oil out, most of it going to the groove at the bottom of the V, and then flowing back to the oil-pocket; a smaller portion is forced out at the top into the grooves GG, and either flows along these grooves back to the pockets or runs down the inclined surfaces of the bearings when the table passes from over them. The groove at the bottom of the V not only serves for an oil-channel, but affords a space into which particles of foreign substances may be scraped from the exposed bearing surfaces of the bed by the end of the table V as it passes over them. The supporting pins EE on which the cones revolve should be as small as is consistent with good design, and turn freely in their bearings: otherwise there is danger that the cones will stop turning and a flat place will be worn on them by the rubbing of the table.
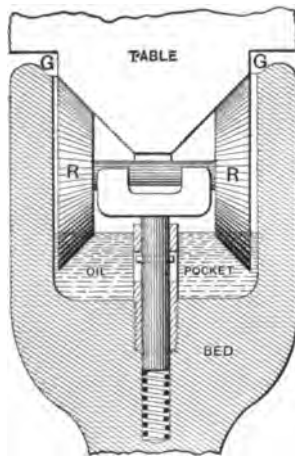
FIG. 6.1.

Fig. 6.1 is a more compact form of the same device. The cone frusta are made short by cutting away more of the part near the apexes. The support is between the two parts that bear against the table, instead of outside. There is consequently less widening, or none, of the bed at the pocket. The cones may be pressed up against the bed by a coiled compression spring, as shown, or a

counterweight may be made to act on the small pin by extending its ends out through the slots in the quill that supports the shank of the yoke bearing. As shown, this pin is a stop to prevent the cones from rising too high when the table clears them.
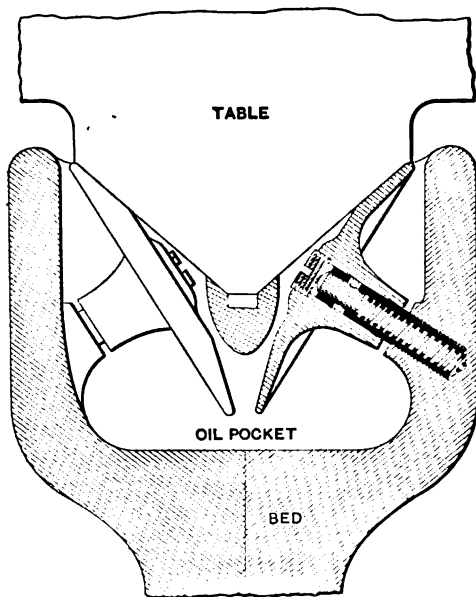
FIG. 6.2.

Fig. 6.2 has a feature of some importance that does not appear in the preceding devices. By the use of two separately supported mushroom-shaped oilers the material of the bed can be carried over the oil-pocket. This secures a continuous groove at the bottom of the V which may be utilized as a drain to carry the oil to wells or buckets at the ends of the ways. The dirt and particles of iron from the ways are not carried into the oil-pocket as in the other designs. Clean oil can therefore be constantly supplied to the bearings.

Flat bearings can be lubricated in a similar manner. In this case a pair of flanged rollers, as *RR*, Fig. 7, mounted on the

same spindle and forced lightly toward each other with a spring, so as to bear against the vertical part of the bearing, give good service. The faces of the flanges on the inner sides at F and F should be
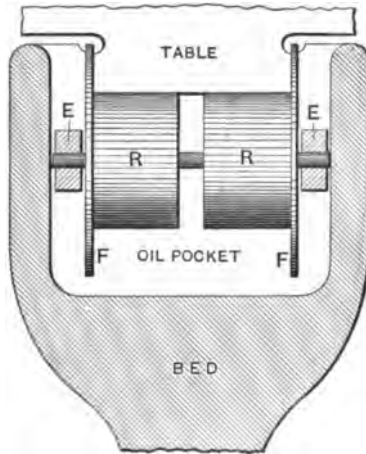
FIG. 7.

corrugated or indented, or the flanges perforated with small holes, to facilitate the carrying of the oil up to the ways.

The ways or V's forming the bearings between the carriage and bed of an ordinary engine-lathe commonly have the form of an inverted V, as shown in Fig. 8. The pressure of the cutting tool
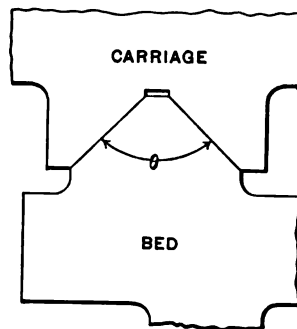
FIG. 8.

against the work often has a tendency to throw the carriage from the bed. Since the carriage is comparatively light, clamps are

necessary to hold it in place. The V's can therefore have such an angle as will allow the carriage to rest on them without pressure against the clamps when the tool is taking a light cut, but the clamps may come into service when a heavy cut is taken. The tops of the ways should be slightly crowned to prevent marring of the rubbing surfaces by tools, etc., laid across them.

The angle θ is commonly made 90° or less on engine-lathes for ordinary service. The width of the bearing surface on each side of the V is about ¾ of an inch to 1¼ inches for a lathe which will swing a piece 24 inches in diameter.

While it is not common practice to provide any method for lubricating the ways of a lathe other than by an oil-can, a machine is sometimes found in which an oil-pad or bunch of waste rubs against and lubricates the bearing surface. The carriage has a pocket over the ways, into which the pad fits. Extensions of the carriage are sometimes carried out over the ways to shield them from chips and turnings.

These devices are found in smaller tools rather than larger ones. The very frequent cut and grooved appearance of the ways of lathes indicates very clearly that such appurtenances are a desirable addition to all such machines.

3. The bearings for the ram or tool-carrier of a shaper for metal-working must guide the ram positively, so that it cannot be thrown out of position by a heavy pressure against the cutting tool in any direction. Fig. 9 is an end view of a common form of bear-
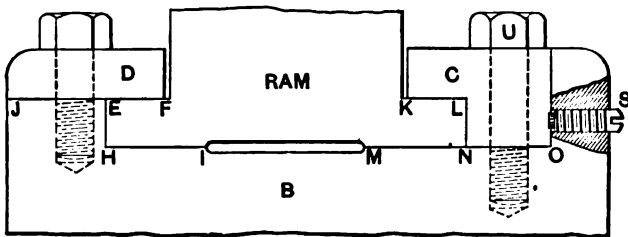
FIG. 9.

ing for this purpose. The ram reciprocates in the part B, which may be either the frame of the machine or a carrier for the ram, according to the design of the shaper.

In order to take up wear on the faces EF and HI, the clamp D is removed from the machine and a light cut taken off the surface

EJ, and the clamp is then replaced. Another method is to plane *B* a little lower at *JE* than *EF* and place " shims " or " liners " of thin sheet metal or paper between *D* and *B* on the surface *JE*; as wear occurs some of the shims are removed and the clamp drawn down tight again. It is essential that the clamp be firmly held against the part *B*. Wear along the surfaces *KL* and *MN* is taken up in the same manner by cutting away the surface *NO*, or by removing liners which had been placed between the clamp and part *B*. Wear along *EH* and *LN* is taken up by the set-screw *S*, which is used to force the clamp *C* against the ram, the cap-screws *U* being loosened for this purpose.

Another form of shaper-ram bearings that has been much used, but which seems to be giving way to that just described, is shown in Fig. 10. The bearing surfaces are *EF* and *HI* on one side, and
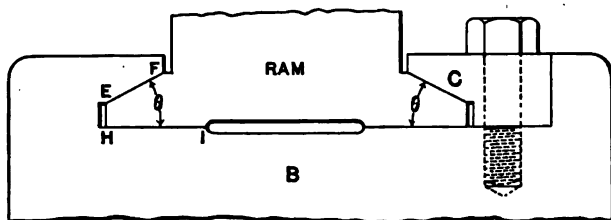
Fig. 10.

the similar surfaces on the other. The objectionable feature of this bearing is that, on account of the wedge-like form of the bearing surfaces, which make the angle *θ* with each other, the pressure between them is apt to be so great locally that cutting will occur. Otherwise this bearing presents the excellent feature that it can be adjusted for wear in all directions by simply lowering the clamp *C* slightly, or by forcing it towards the ram with a row of set-screws placed as the one at *S* in Fig. 9.

The angle *θ*, Fig. 10, is commonly from 30° to 45° in practice.

The width of the bearing surface *EF* is about 1 inch on a shaper having *θ* = 30° and a maximum stroke of 18 inches.

Another method of taking up the side wear in a ram having bearing surfaces at right angles is shown in Fig. 11, where the bearing strip *E* is held against the ram by a line of set-screws *S*, each provided with a lock-nut. Some method of preventing end

motion of the strip must be supplied.   This may be done by having
the end of one or more of the screws made cylindrical, so as to fit
into a small hole in the strip.   Screws with conical points, fitting
into conical countersinks in the strip, will hold it in place, but the
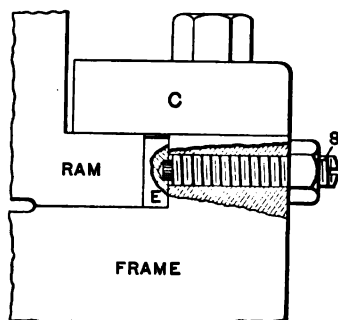
FIG. 11.

strip has a tendency to slip off the screws, thus causing unnecessary
pressure between the rubbing surfaces.   A cylindrical pin, driven
into the frame on the line of the set-screws with its end projecting
into a cylindrical hole in the bearing strip, will prevent end motion
of the latter.   Flat- or round-point adjusting screws can then be
used against the flat surface of the bearing strip.

On account of the thinness of the strip, the pressure is some-
what localized at the set-screws.   The consequent wearing away of
the strip more rapidly at these places than between them, makes it
necessary to spring the strip by setting up the screws, in order to
hold the ram rigidly when taking a heavy cut.   This causes a con-
stant pressure of the strip against the ram, even when on the
return stroke.   The consequent wear and loss of power are dis-
advantages.

Another method of adjusting the bearing strip, which almost
entirely prevents its springing so as to press constantly against the
ram, is to tap the screws into it and allow them to pass through
smooth holes in the frame; liners can then be placed back of the
strip, and the screws tightened to hold it back firmly against them.

4. For planer-saddles, lathe tool-rests, and similar parts, where
one of the sliding parts is much longer than the other, the " gib "
shown at *G* in Fig. 12 can be conveniently used.   In Fig. 12, *B*

may be taken to represent a portion of a planer cross-rail with the upper portion of the tool-carrying saddle resting upon it. The gib is tapered on two sides, from end to end ; the other sides are
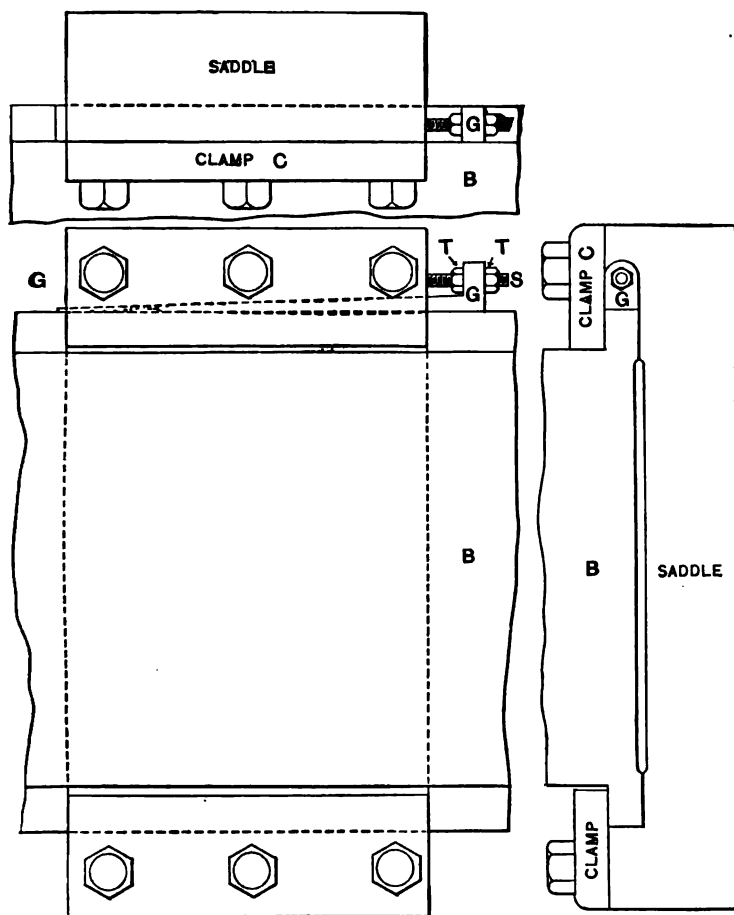
Fig. 12.

parallel. One of the sides forming the taper rests against the corresponding sloping surface of the saddle, and the other against the rail. A stud S, tightly screwed into the saddle, and having

two lock-nuts T and T, is used for adjusting the gib so as to secure the desired fit between the moving parts. Wear on the vertical faces is taken up by the clamp C at the top, and the corresponding clamp at the bottom. The dead weight of the saddle and the parts appertaining to it is carried on the horizontal surface under the gib. The taper of the gib is generally between $\frac{1}{4}$ and $\frac{3}{8}$ of an inch per foot.

The tool-slide which moves on the carriage of a lathe may be made with a gib G, as in Fig. 13. Here the gib bears against the

FIG. 13.
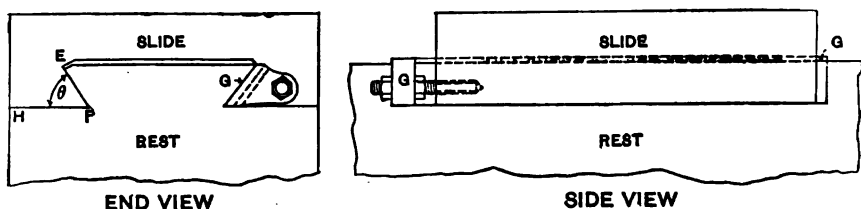
inclined surfaces so that, by adjusting it alone, wear in all directions is taken up. The taper of the gib may be the same as for rectangular bearings, viz., $\frac{1}{4}$ to $\frac{3}{8}$ of an inch per foot.

The angle θ generally lies between 45° and 60°. For a lathe which will swing 24 inches in diameter, EF may be 1 inch and FH $1\frac{1}{2}$ inches; for one swinging 12 inches in diameter, the corresponding dimensions are $\frac{5}{8}$ inch and 1 inch.

By chamfering off the corner of the rest and filling in the re-
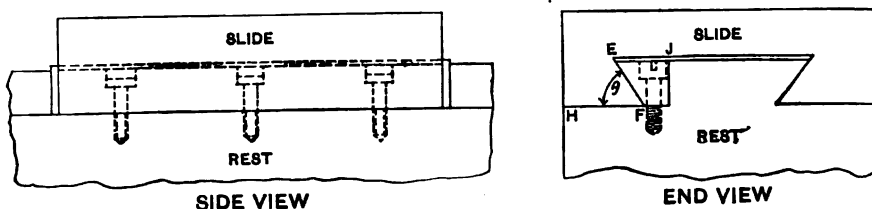
FIG. 13.1.

entrant angle at E as shown, the slide is made stronger and the sharp corner of the rest is less liable to injury.

Fig. 13.1 shows another device that is used for the same purpose

as the preceding. In this case the heavy beveled clamp that is used instead of a gib does not taper from end to end, but is prismatic. It is held down rigidly against the rest by the screws as shown. In order to take up wear some of the material is planed off the bottom of the clamp where it is drawn against the rest by the screws. A somewhat less satisfactory method is to place shims or liners under the gib, and remove them to compensate for wear.

SIDE VIEW   END VIEW
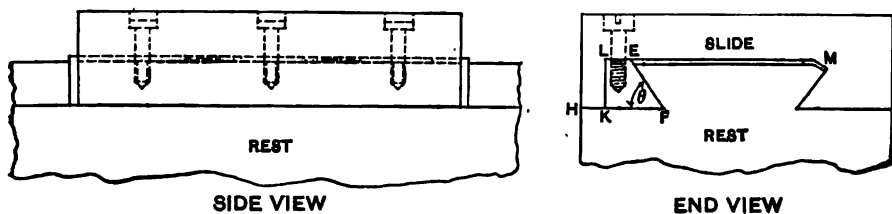
FIG. 13.2.

Fig. 13.2 is very much the same as the preceding one. In this case the clamp is attached to the slide instead of the rest. The adoption of either of these devices must naturally depend upon the judgment of the designer in accordance with the conditions to be met.

Another device that is often used for rectangular bearing surfaces is that of Fig. 14, where the right-angled piece G, resembling
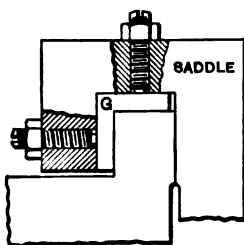
FIG. 14.

a box-square, is adjustable with both a vertical and a horizontal row of set-screws, thus compensating for wear in all directions.

The thin strip E, Fig. 11, can be used on angular bearings such as shown in Fig. 13. If the set-screws are kept at right angles to

the outer vertical side of the slide, their points should be either conical, so as to have line contact with the side of the strip, or else should be fit into a hole drilled into the strip. The object in both these cases is to get as much contact surface between the points of the screws and the strip as possible, in order to prevent crushing the metal at the place of contact.

Another method of adjustment is shown by the dotted lines at the left side of Fig. 13. This is by using an angular strip *EFKL* which bears against the surface *LK* of the slide, and is drawn upward by screws passing through smooth holes in the slide and into the threaded holes of the strip. Or, leaving the slide solid, as indicated by the full lines, the same device can be used on the rest, the strip being *EFIJ* in this case. The screws must be put in from the opposite direction for this construction; they are not shown in the figure. The gib should be placed, when possible, where it does not receive the pressure of the cutting tool.

5. Cross-head guides for engines, pumps, and similar machines having a crank and connecting-rod, are frequently made of the
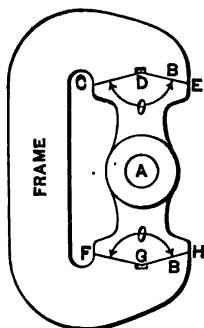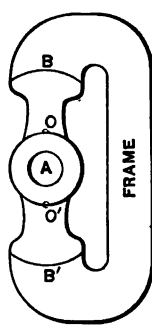
FIG. 15.   FIG. 16.

form shown in Fig. 15, which is an end view of the guides *BB* and piston-rod *A*. The bearing surfaces are at *CDE* and *FGH*. They must take the pressure due to the angularity of the connecting-rod with the piston-rod, this pressure being in either direction along a line *DG*, not drawn. Since there is very little or no side pressure at right angles to *DG*, the angles θ can be made large. Adjustment for wear can be made either by moving apart the bearing surface of the cross-head or bringing those of the frame *B* together.

To do this, the cross-head must either be made of several pieces, or the guides made of separate pieces and attached to the frame.

Fig. 16 shows another form of cross-head guides. The bearing surfaces here are parts of cylindrical surfaces, which are represented as arcs of circles in the figure. The centres of these arcs are at O and O' on a line through B and B', the radial distances OB and $O'B'$ being less than half the distance BB' between the bearing surfaces. It can be readily seen that this construction prevents the cross-head from rotating about A.

In contrast with this construction for preventing the turning of the cross-head in the guides, some prominent engine-builders bore the guides concentric with the cylinder, in order to allow the cross-head to adjust itself to the crank-pin and connecting-rod.

Flat bearing surfaces normal to the line BB', Fig. 16, are frequently used for cross-head bearings. In order to prevent lateral motion the cross-head generally extends over the edges of the guides and has a bearing against their sides.

6. **The relative length of sliding bearings for rectilinear motion,** and their positions at the ends of the stroke, should be taken into consideration when designing bearings which must perform considerable service during the life of the machine of which they form a part.

If, in a machine of the iron planer type with its reciprocating table, held in place by gravity, the table is much shorter than the supporting bed, and always moves the same distance between the same limiting positions, then the extreme ends of the bed will not be worn at all, but shoulders will form at the limits of travel of the table. By making the table longer, or cutting off the ends of the bed, so that the table just reaches the end of the bed, the shoulders will be obviated, but still the bed will be worn more rapidly at the middle than at the ends.

On the other hand, suppose that the table and bed are of the same length and the stroke is but slightly shorter than the table and bed—only enough shorter to prevent the table from tipping off the end of the bed as it stops moving—then the wear will be greatest at the middle of the table and the ends of the bed. The bearing parts will wear to a curved form similar to that shown

in Fig. 16.1.   The table becomes concave and the bed convex on the bearing surfaces.
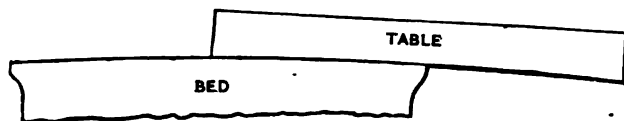
FIG. 16.1.

The remedy for the above defects is, for the kind of machine mentioned, shown in Fig. 16.2.   The bed in this figure is of such a
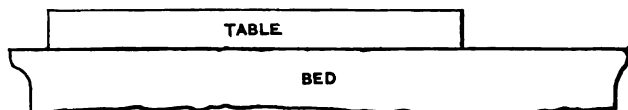
FIG. 16.2.

length that the table will overhang enough at the ends of the stroke to make the wear due to such overhanging just counterbalance the tendency to wear rapidly at the middle of the bed.

It has been demonstrated in practice that it is possible to get such a relative length of table and bed that the wearing parts will retain their straight form through many years of service.

In a high-speed steam-engine with a crosshead of the form shown in Fig. 16.3 it has been found in practice that, with the
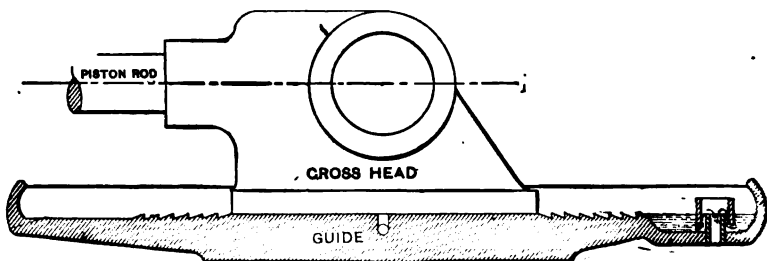
FIG. 16.3.

length of the bearing surfaces of the slide and guide equal to each other and to the length of stroke, the parts retain plane bearing

surfaces and show no appreciable wear after long service when well lubricated. This is contrary to what has been found for planer-table and bed ways. The cause of the difference is due to the fact that the centre of gravity of the crosshead is not in line with, but lies below, the piston-rod. When the latter pulls upon the cross-head and rapidly accelerates it from a position of rest at the end of a stroke there is a tendency for the crosshead to rotate about a horizontal axis perpendicular to the direction of motion. This brings a heavy pressure against the forward end of the bearing surface of the slide which causes more rapid wear at the middle of the guide and end of the slide than in any other part when the crosshead is still near one end of the stroke. When the stroke is nearly finished, the retarding action of the piston-rod upon the crosshead causes the heel of the crosshead to press hard against the guide, for a reason the reverse of that which causes the for-ward end to press hard when the speed of the crosshead is accel-erated. At the middle of the stroke, when the crosshead is moving uniformly, the wear is distributed quite equally over the bearing surfaces. The result of the whole is that, as already stated, the parts retain their form in an engine which "runs over" so that the crank-pin moves away from the cylinder when above the centre line of the piston-rod.

6.1 **The method of lubricating the crosshead and guide** bearing surfaces that has been adopted by the Straight Line Engine Company for bearing surfaces of equal length is of interest. It is shown in Fig. 16.3. On account of the great over-reach of the crosshead shoe at the ends of the stroke there would be undesirable splashing of the oil, if any were to collect, unless some special device for preventing the splashing were used. As shown in the figure, a "sawtooth" form of corrugated surface has been adopted. The edges of the ribs just clear the slide. Oil is kept at a height just above the level of the bearing surfaces by the device shown at the right of the ways. It is a trap consisting of a tube standing as high as the bearing surfaces. About it is a ring-like hood some-what higher than, and reaching some distance down below, the top of the tube. When oil and water accumulate in the shallow recess, the water flows up inside the hood and down through the tube, leaving the oil in position to lubricate the guide.

6.2. The material used for V's and other rectilinear bearings in machine tools is almost universally cast iron. The durability of the rubbing surfaces is a question of some importance. There is a considerable difference in the amount of wear that takes place in the different grades of cast iron. A number of experiments made by the Alfred Herbert Co. of Coventry, England, upon cast-iron sliding surfaces submerged in an oil-bath demonstrated very clearly the fact that an open-grained soft cast iron wore away very much less rapidly than a close-grained, hard material. While the tool marks were not removed from the porous material, a very perceptible wear took place in the harder iron under the same duty.

Two mating pieces were made from each grade of cast iron. The larger one, used as a stationary bed, was recessed at the top to two depths; the higher surface of the recess was planed for bearing surfaces, one strip along each side of the length of the recess. The latter was filled with oil so as to cover the bearing surface. The smaller piece, used as a slide, was reciprocated over the bed by a mechanical drive.

The oil remained clear with the soft iron, but soon became cloudy with the hard. The rubbing motion was reciprocating. Each bearing surface rubbed through a distance of about eighty-four miles over its mate under a pressure of approximately 90 pounds per square inch. As much as one-fiftieth of an inch was worn off the rubbing surface of some of the harder irons.*

Materials the same as used for journal-bearings are used for rectilinear surfaces for heavy duty in many cases.

BEARINGS FOR ROTARY MOTION.

Journals, Boxes, and Journal-bearings.

7. Definitions.—When two parts of a machine rotate with regard to each other, as a wheel and its axle, the part of the one which is enclosed by, and whose surface rubs against, the other is called the **journal**; the part which encloses the journal is called the **box**, or, less specifically, the **"bearing."** The name "bear-

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* The parts tested and results obtained were shown the writer at the works of the Alfred Herbert Co., in September, 1902.

ing " or journal-bearing is very commonly applied to the whole assembly of parts embodying both the journal and its box.

8. **The cylindrical journal-bearing** is the most common form. When the pressure upon it is always in nearly the same direction, the box may be made to extend only partly around the journal. The boxes used on the axles of railway cars furnish an example of this kind. These boxes encompass about one-third of the circumference of the journal. This is sufficient to hold the axle in place, and furnishes almost as much efficient bearing surface as if the box covered half of the circumference of the journal.

A difficulty that sometimes assumes serious proportions is liable to be met when a box, extending half-way around the journal, is used, especially if the box is well fitted to the journal before going into operation. The nature of this trouble can be described with the aid of Fig. 17, in which B is the box and A is the supporting
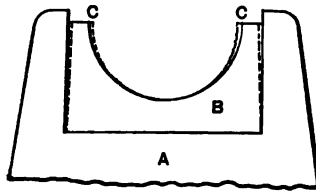
FIG. 17.

frame. Before operation the box is of the form indicated by the full lines, the semicircle CC just fitting the journal. When the journal is rotated in the box, the latter is apt to have a tendency to take the form shown by the broken lines, the points CC trying to approach each other. This action causes the sides of the box to press unduly hard against the journal. The result is increased frictional resistance and wear between the rubbing parts.

One remedy for this is to make a very loose fit along the surfaces near C and C. Another and more certain one is to provide some means for holding the box back against the frame so that the parts CC cannot move away from it; this may be done with bolts passing through the frame and into the box, or by dovetailing the sides of the frame and box together.

When the pressure acting on a bearing is reversed, as indicated by the arrow-heads in Fig. 18, the box must generally completely

surround the journal. The adjustment for wear under such conditions should be in the direction of the forces. For the vertical pressures in Fig. 18 the separation should be on a horizontal plane through the centre of the journal. The short lines at D and D show the separation on this plane. The small shoulders at the ends of these lines are to hold the cap C in the proper position. Adjust-
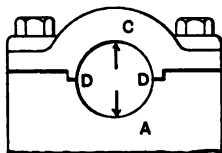
FIG. 18.

ment is made by bringing the cap down nearer the frame as wear occurs. Liners placed between the cap and frame may be removed for this purpose, or some of the cap or frame cut away. If the pressure is horizontal and reversed, the separation should be made on a vertical plane.

In many cases the wear is both vertical and horizontal. The horizontal steam-engine with its heavy fly-wheel furnishes an example of this. The downward pressure caused by the weight of the fly-wheel, as indicated by the vertical arrow in Fig. 19, wears away
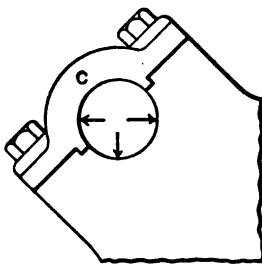
FIG. 19.

the bottom of the box. The pressure, indicated by the horizontal arrows, is due to the steam-pressure on the piston; this causes wear on the sides of the bearing. When close fitting of the running parts is not of great importance, a box divided at an angle with both lines of pressure, as shown in the figure, can be adjusted with

a fair degree of satisfaction. For more accurate adjustment the box must be divided into three or more parts.

Fig. 20 shows a box divided into four parts, and adjustable
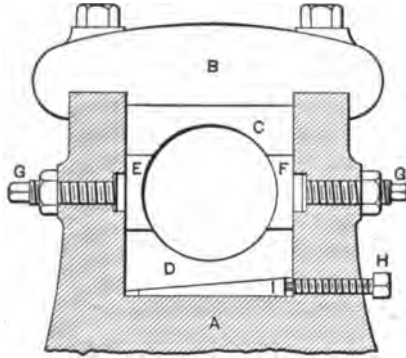
FIG. 20.

both vertically and horizontally. The four parts of the box, C, D, E, and F, are enclosed by the frame A and the cap B. The wedge I, at the bottom of the box, is used for the vertical adjustment, the top C being held down by the cap B. The side screws G and G are for adjusting the side pieces E and F horizontally; these screws are made with enlarged ends, for bearing against the sections of the box, in order to prevent crushing of the metal where they come in contact. With a box of this form the axis of the journal can always be kept in the same position, whatever the direction of the wear. This is an important consideration in some classes of machinery.

A unique and very compact adjustable journal-box is shown in Fig. 21.* It consists of an outer shell or casing A enclosing an adjustable lining B, made in two parts. Both the shell and the lining have the same eccentricity, so that when the lining is placed in the shell with the thickest part of one next the thinnest part of the other, the outer surface of the shell and inner surface of the lining are concentric. The complete box can therefore be placed in a hole bored in the frame of a machine, and turned about its own axis without displacing the axis of a journal fitting into it. The

* Used by the Straight Line Engine Co., Syracuse, N. Y.

lining does not completely encircle the journal, but the circumference is completed by a wedge *D* at the thickest part, together with
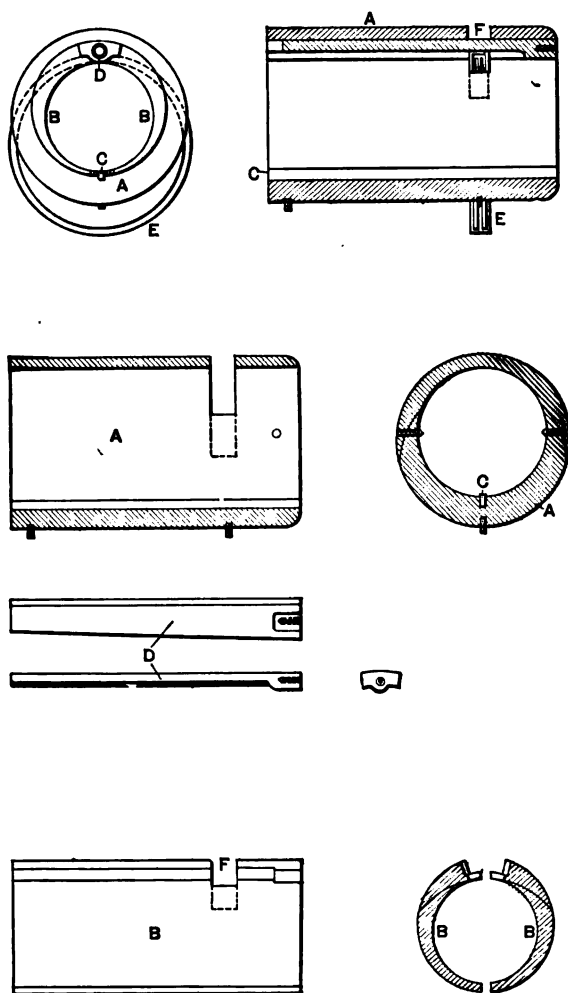
FIG. 21.

thin strips or liners *C* placed at the thinnest part, when the bearing is new. As wear occurs these strips are removed from the thin side and placed alongside the wedge, thus reducing the diameter of

the bore of the box, and at the same time moving the centre of the bore slightly nearer the wedge side of the bearing. By placing the thin side of the bearing where the greatest wear occurs, each
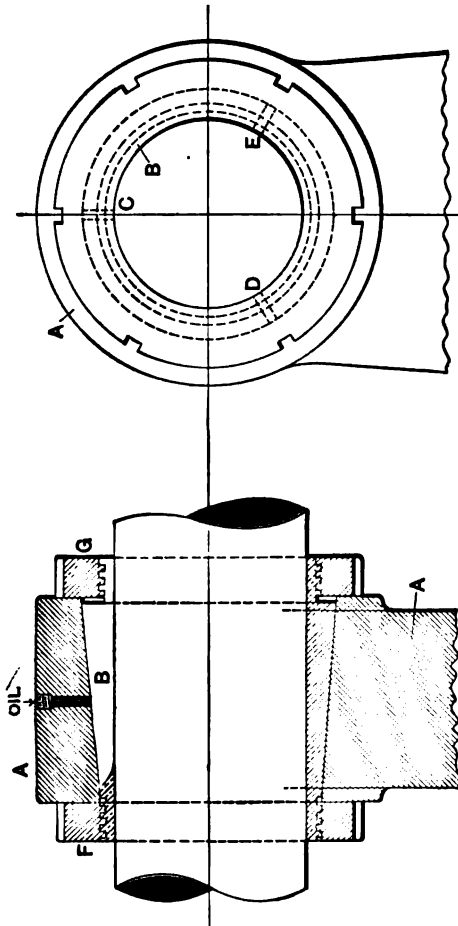
FIG. 22.

adjustment of the lining tends to bring the journal back to its original position concentric with the outer surface of the shell.

When but a slight adjustment for wear on the journal and box is required, and compactness is desired, the one-piece box of Fig. 22 is serviceable. The box *B* is made of a single piece, cylindrical

in the bore, and coned on the central part of its outer surface to fit the conical hole in the frame A of the machine; it is cut through at C, D, and E, from the large end to near the small one; the ends are threaded for nuts F and G. When new, these nuts are adjusted so that the inner surface takes the circular form and size to which it was bored when the box was solid. In order to take up wear on the rubbing surfaces, G is loosened and F tightened so as to draw the box into the frame, thus springing the sides together so as to diminish the diameter of the bore; G is then tightened to hold the box firmly in place.

Fig. 23 represents what is probably the simplest form of adjust-
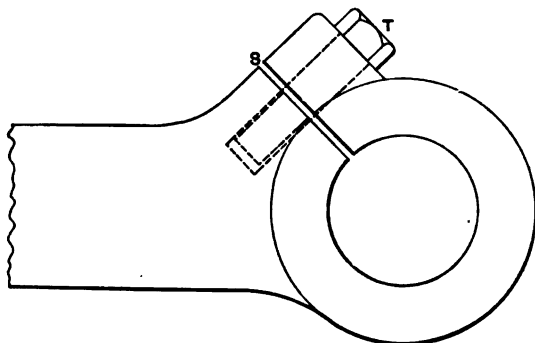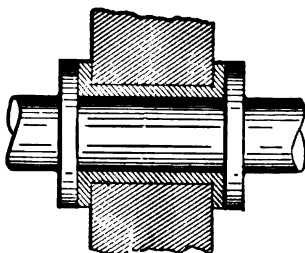
FIG. 23.   FIG. 24.

able box that can be devised. The slot S, cut along one side, allows it to be adjusted by the cap-screw T. This design is also frequently used where it is desirable to have a pin pass easily through the bore when putting the parts together, but held tightly in place during the operation of the machine.

In order to prevent excessive end motion of a journal in its box, collars are frequently placed at the end of the journal, forming a part of it, as in Fig. 24. It frequently occurs that if the collars are fitted so as to allow free running, but no end shake, of the parts, there will be excessive pressure and binding between the collars and the box when the machine is put into operation. This is probably due to the fact that the heat generated by the rubbing of the surfaces over each other raises the temperature of the thin shell more rapidly than that of the journal, consequently expanding it more rapidly, and causing it to press against the collars as stated.

The higher coefficient of expansion of the material of the box may also have a slight tendency to increase the binding with a material that is highly expansive. The remedy is plainly to make a loose fit when constructing the machine.

A bearing having a journal with a single collar is shown in Fig. 25. This is a form that is used on the spindle of a lathe where it rests on the head-stock at the end next the face-plate or live-centre. A smaller adjustable collar is placed at the other end of the spindle to keep the spindle from moving endwise, toward the right in this case.

9. Self-aligning bearings are desirable in many classes of machinery. Fig. 26 represents a common and compact form. The outer surface of the box or sleeve B is partly made up of a portion of the surface of a sphere. The supporting part A has a concave spherical surface to conform to that of the sleeve. This device allows the axis of the sleeve to adjust itself in line with the journal and maintain such adjustment, even if the shaft of which the journal forms a part is temporarily sprung, or permanently bent so as to wabble. Movement of the support A has no effect on this alignment of the box. Some means of preventing the box from rotating in the support must be provided. This can be done by a pin C fitting tightly in the sleeve and loosely in the support.

For convenience of construction and compactness of design it is best to place the centre of the sphere on the axis of the bore of the box. This is not absolutely necessary, however. When so placed, the centre line of the shaft will always pass through the same point, the centre of the sphere, in relation to the supports. This is not true when the centre of the sphere is not on the axis of the bore.

The requirements of construction can be met, in many cases, by casting the spherical recess in the frame larger than the part that fits into it, and filling the space between the sleeve and frame with Babbitt or other white metal that fuses at a low temperature. This obviates the usually expensive operation of machining the concave spherical surface.

It is not necessary to have the spherical surface extend completely around the sleeve; two diametrically opposite segments of the surface can be used, as shown in Fig. 27, where B is the sleeve having the spherical segments under the spherically concave points. of the two screws C and C, which are supported by the frame A.

OIL
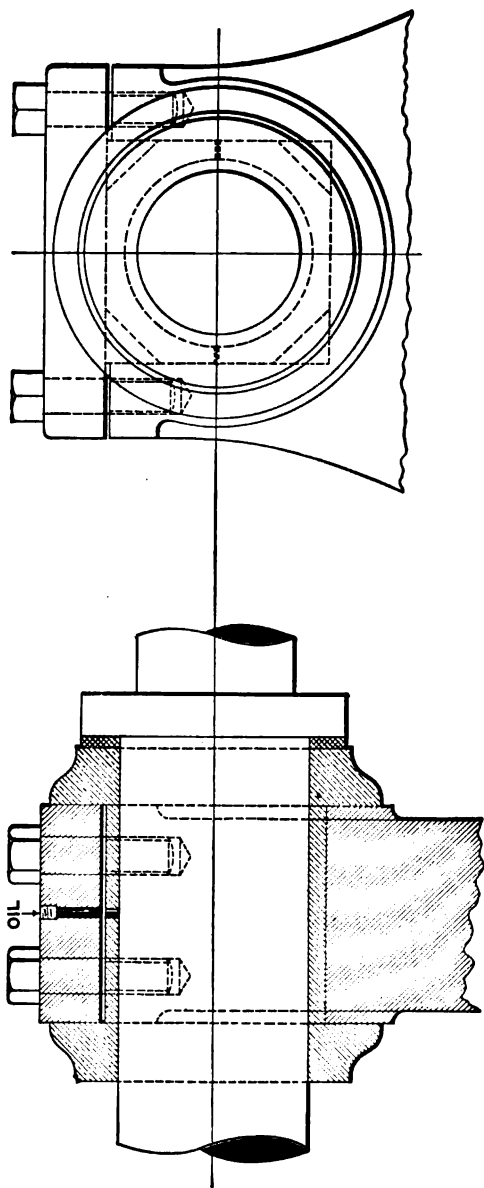
FIG. 25.

The lips D and D, extending around the segments, prevent the sleeve from rotating, by striking against the end of the screws. (See also Fig. 29.)
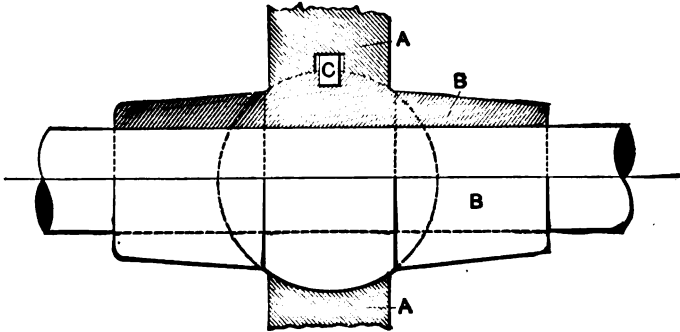
FIG. 26.

A self-aligning bearing can readily be made without the use of a spherical surface, but it is necessarily not so compact. Fig. 28 shows such a bearing, which is also adjustable both vertically and horizontally. The sleeve B is supported on the points of two
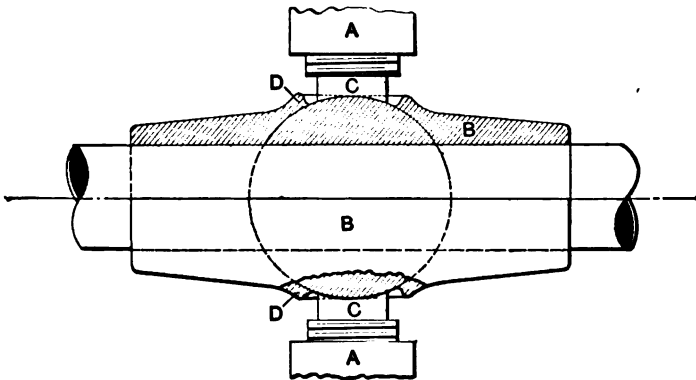
FIG. 27.

coaxial screws which pass through threaded holes in the extremities of the arms of the yoke C; the lower part of the yoke passes through a smooth hole in the supporting frame A, and has lock-nuts D and D for adjusting and locking it in place, still leaving it free to turn in the frame.

10. Lubrication of journal-bearings.—Either of the three forms of lubricants—liquid, pasty, and solid—may be applied to a journal bearing.   The liquid lubricants consist chiefly of oils, the pasty of
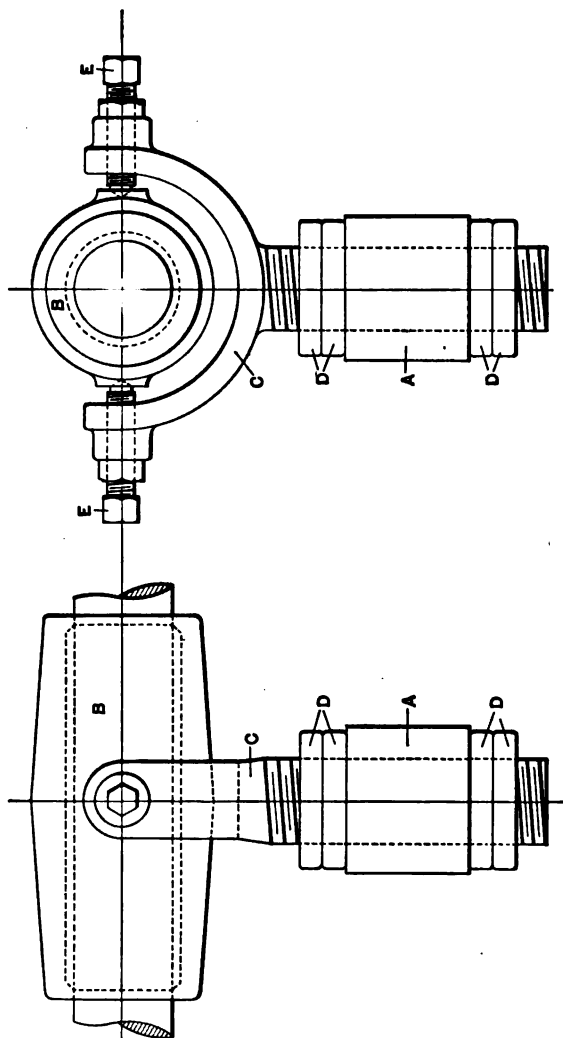
FIG. 28.

grease, and the solid of graphite.   The oils are by far the most commonly used.

11. Oil- and grease-lubrication for journal-bearings.—Both experimental investigation * and practice show that when an oil-lubricated journal is run under a uniform load, constantly applied in the same direction, a film of oil adheres to the journal and is carried between the rubbing surfaces, unless the pressure between these surfaces is so great as to squeeze or rub it off. As long as the film of oil is maintained between the rubbing surfaces, the lubrication is effective; but when the oil is so completely squeezed out, or burned by the heat due to friction, as to break the film and allow the metallic surfaces to come into contact, abrasion and consequent seizure or destruction of the surfaces follow. Seizure can occur only when the box completely or nearly surrounds the journal, and is sufficiently rigid to grip and hold it when the metals come into contact. Probably the seizure is often due to sudden heating and expansion of the material forming and lying just beneath the rubbing surfaces, when the lubricant has ceased to separate them.

The film of oil must withstand the greatest pressure that occurs at any part of the bearing. If a box presses vertically downward on its journal, the maximum pressure will usually be at or near the top and centre of its length. From this point the pressure gradually decreases towards the sides and ends.

If a hole is drilled in the middle of the box at the top, so as to communicate with the atmosphere, the oil will be forced out of it as the journal turns. If a pressure-gauge is attached to the opening, it will show the pressure acting on the oil at that place,† which is probably about the same as would exist if the hole had not been made. Any attempt to introduce oil at this point in the usual manner, as with a drip-cup, would clearly be unsuccessful. Even if a groove were cut along the top of the box for some distance, but not reaching the ends, it would only collect the oil and force it up through the opening. The oil must therefore be applied at some other place. If an oil-cup which gradually feeds the oil on the bearing is used, it will give the most satisfactory results when the opening through the box is on the side where the surface of the

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* Tower's experiments : Proc. Inst. of Mech. Engrs., 1883, p. 632 ; also other experiments.

† Tower's experiments.

journal is approaching the top or point of highest pressure of the rubbing surfaces.   This applies especially to boxes which only partly surround the journal.   If the box completely encircles it, satisfactory lubrication can be obtained by applying the oil at any point where it will pass through the opening to the rubbing surfaces, provided suitable oil-grooves for distributing the oil over the surfaces are cut into either the box or journal, or both.   Such grooves are generally cut into the box.

When a " half-box," as one which covers only half or less of the circumference of the journal is called, rests on top of the journal, it can be lubricated very successfully by letting the lower part of the journal run in a reservoir of oil.   This is called oil-bath lubrication.   The oil adhering to the journal is carried up between the rubbing surfaces and thoroughly distributed over them.   If the half-box is placed under the journal, the oil can be most successfully applied at the top.

Oil-pad lubrication is an excellent substitute for bath lubrication when the latter cannot be advantageously applied on account of waste of oil or inability to provide a suitable reservoir for retaining it.   The oil-pad may be made of a piece of porous woven material which is saturated with or dips into oil and presses against the journal.   An excellent, and at the same time comparatively inexpensive, material which is used instead of the woven pad, is the cotton " waste " so commonly used for cleaning machinery. It is almost wholly, possibly wholly, used in this country for lubricating car-axle journals.   The waste and oil are placed in a closed box beneath the journal, filling the space so that there is good contact between the waste and journal.   The capillary action of the waste carries the oil to the journal in sufficient quantity to lubricate it.

Mr. E. Charbal states that woollen wicking was found better and more economical than cotton for railway service, the delivery of oil being from 50% to 100% better, and the renewals only 68% of those for cotton.[*]

While experimenting with a pad-lubricated journal 4 inches in diameter and 8 inches long, running against a " half-box " at 266

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[*] Min. of Proc. Inst. Civ. Eng., 1894–95, Part II., p. 412.

revolutions per minute under a total load of 15132 pounds = 756.6 pounds per square inch of projected area, Mr. John Dewrance found a vacuum of 28.4 inches of mercury, corresponding to 13.9 pounds per square inch between the rubbing surfaces, at a point where the rotating part had passed the position of maximum pressure.*

The nature of the rubbing surfaces of a bearing is of importance. In general it may be stated that the smoother they are the more satisfactory service will they give. It has been found that by burnishing the surface of a journal with a roller burnisher pressed hard against it while rotating in a lathe, so that the burnisher rolled on the bearing surface, the frictional resistance to the rotation of the journal when placed in its bearing was lowered and more satisfactory operation secured. It is the practice of the C., M. & St. P. Ry. to roll the journals of their car-axles while turning the wheel-fit in the lathe. It is found decidedly beneficial.

In contrast with the above, it is not an uncommon occurrence to find that a journal, turned and finished in the ordinary manner by polishing or grinding, which runs hot despite the best attention and care in both its manufacture and operation, can be put into satisfactory working order by rubbing it with emery-cloth so as to make the scratches of the emery parallel to its length; a file has been used satisfactorily for the same purpose by draw-filing the journal so as to make the scratches of the file parallel to the axis of the journal, as with the emery-cloth.†

There seems little reason to suppose otherwise than that the smoothest, truest surface that can be obtained is the best for a bearing, both in point of low frictional resistance and great durability. But if the surfaces fit very truly together over a considerable area which has no oil-grooves or other indentations, the oil may not be carried between them in sufficient quantities for satisfactory lubrication. It is in such a case that the scratches of the emery-cloth or file have a beneficial effect. The scratches act as little reservoirs or pockets which carry the oil in between the surfaces and distribute it over them. An increased number of oil-grooves in the bearing, properly arranged, or a few in the journal, so as to cut the

* Minutes of Proc. of Inst. Civ. Eng., vol. cxxv., p. 362 ; *Engineering*, London, Jan. 1, 1897, p. 29 ; *The Engineer*, London, Jan. 8, 1897, p. 42.

† Trans. Amer. Soc. Mech. Engrs., vol. vi., pp. 849-857.

rubbing surfaces into small areas, would doubtless have the same effect as the numerous scratches. Oil-grooves, sufficiently large not to fill with dirt and gummy oil, should be provided plentifully in a bearing having considerable area and doing heavy service. It might seem that this should be applied to railway-car boxes. There is a reason why it could hardly be successfully done, however, without objectionably great expense. The reason is as follows: Owing to the nature of the service they perform, it is scarcely possible to examine the rubbing surfaces of the bearings from the time the boxes are put in place until they are worn out. The allowable wear is so great that grooves of a practicable depth would be worn out of the box long before it had completed its life. And even if this were not true, the probability is that the grooves would be filled with dirt in a short time, and thus become useless; this is also true of grooves cut in the journal, which is a less desirable place.

An intermittent or a reversed force, causing pressure on a bearing, offers an opportunity for the lubricating oil to get between all parts of the rubbing surfaces. On account of this beneficial action greater bearing-pressure can be used than for a constant pressure in one direction. This is markedly shown in practice in the main bearing and crank-pin of the ordinary form of double-acting steam-engine.*

For very heavy, continuous bearing-pressures and slow speeds grease is better than oil for lubricating journal-bearings. The greater viscosity, or "body," of the grease prevents it from being squeezed out from between the rubbing surfaces as quickly as the thinner oil, and slow speeds do not carry the oil in between the rubbing surfaces with sufficient rapidity to overcome the squeezing-out action.

In journal-bearings, it may be taken as a rule that the faster the speed and lower the bearing-pressure, the thinner or less viscous the lubricant that will give the best service. It is frequently found that a change from a thick to a thin lubricant, or *vice versa*, will overcome difficulties of heating and abrasion of a journal-bearing.

A case has been cited where a journal which ran hot in its box when abundantly supplied with oil, cooled down and worked satisfactorily when the supply of oil was diminished.

* Trans. Amer. Soc. Mech. Engrs., vol. vi., p. 856.

12. Special devices for journal-lubrication with oil and grease.
—On account of the importance of securing a constant supply of oil
on a bearing when working hard, some of the typical appliances for
this purpose are worthy of description.   One of the simplest and
most commonly used for a bearing whose journal rotates, is a light
ring, of a greater diameter than the journal, which is hung over,
and rests on, the top of the journal, a part of the top of the box
being cut away for this purpose.   The lower part of the ring dips
into a chamber of oil below the journal.   Since the weight of the
ring rests on the top of the journal, the rotation of the latter causes
the ring to turn also, so that the oil adhering to it is carried up and
deposited on the journal.   Such a ring-oiling device is shown in
Fig. 29.*   By providing a large and deep oil-reservoir, connected
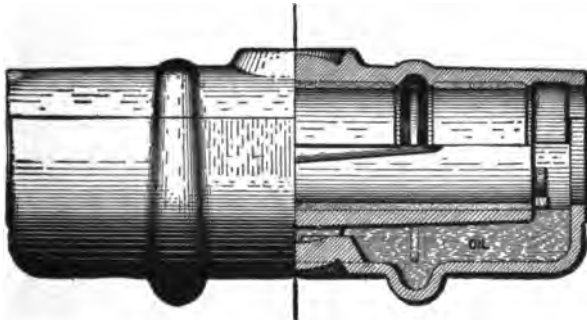
FIG. 29.

by suitable channels to both ends of the box, so that the oil will be
led back to the bottom, the particles of foreign matter gathered up
by the oil while passing over the journal have an opportunity to
settle out, leaving the oil clean for its next application.

A short, endless chain is frequently used instead of the oil-ring.
It has the advantage that, in certain forms of bearings having the
box cast in one piece with the reservoir, it can be introduced
through a smaller opening than the ring, or even without any
external opening through the side of the box.   This fact makes it
especially applicable to line-shaft boxes and similar light bearings
which are not attached to a heavy frame.   The point of superiority
sometimes advanced for the chain, that it will carry a more copious

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* Bearing manufactured by Rice Machinery Co.

supply of oil to the bearing, is hardly worthy of consideration, for the ring will furnish an ample supply.

A recent and apparently excellent substitute for the woven pad or cotton waste, mentioned in the preceding section, is a block of hard wood, concaved on one end to fit the journal, and having broad but very thin slits from this surface to the opposite end, which dips into oil. The concave end is held against the journal by a spring, and the capillary action of the slits carries the oil up to the surface of the journal.

Wicking, similar to that of candles or lamps, is used for lubricating by placing it so that the lower end is in oil and the upper touches, or hangs over, the journal; capillary action carries the oil up to the journal.

When the box of a bearing rotates around the journal, as when a pulley turns on a shaft, oil can be readily applied as shown in Fig. 30, provided the shaft does not extend far beyond the bearing on
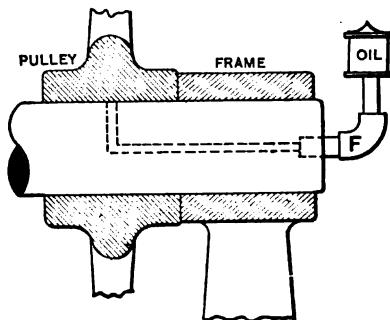
FIG. 80.

one side, at least. A hole is drilled into the end of the shaft as far as the centre of the pulley, and then another from the surface of the shaft to meet it. The oil-cup is attached by a piece of pipe which screws into the end of the shaft. This arrangement answers when the shaft is held rigidly in the supporting frame, so that it never revolves. If the shaft turns at times, a running pipe-coupling can be placed at F and the oil-cup supported by the frame.

A comparatively short box on a long shaft, when the conditions

of operation are such that one rotates continuously and the other at intervals, as a pulley on a line- or counter-shaft, presents the most difficult of all problems in oil-lubrication. Numerous appliances have been devised, but none seems to have come into general use. When the parts are not readily accessible, and are comparatively light, as line- and counter-shafts and their pulleys in most shops and factories, a bearing whose box is made of a material which is itself a lubricant, is the most satisfactory (see § 13). A grease-cup having a piston which is pressed against the contained grease by a spring, so as to gradually force the grease between the rubbing surfaces, is often used when the parts are easily accessible, or heavy; it can be used in almost every case, but the attention required makes it frequently undesirable. If the hub of a pulley can be enlarged in diameter without inconvenience, a recess can be made in it, with an opening to the journal, and an absorbent pad fitted into it so as to press against the journal; by saturating the pad with oil at intervals lubrication sufficient for light loads can be secured.

Grease-lubrication is often used for a bearing whose box does not rotate. One of the most common methods of applying it is by means of a grease-cup having an opening or oil-hole leading straight to the shaft, and a copper or brass pin which partly fills the hole. The pin stands vertically, with its lower end resting on the journal, being held in contact with it by its own weight. The grease in the cup completely surrounds the pin. As the journal becomes warm by running, the heat is carried through the pin to the grease, warming and melting it, so that it flows down to the journal. The slight heating of the pin by its own rubbing against the journal is hardly appreciable. While this device can be quite safely relied upon to keep the bearing from getting dry and cutting, it does not afford economy of power, since the journal must have enough frictional resistance to generate heat for melting the grease. The end of the pin is apt to wear a groove in the journal, which may be objectionable.

* Fig. 30.1 is a device for curing the ills of loose pulleys that seems most excellent. A sleeve fits over the shaft and forms the journal for the loose pulley. It is provided with oil holes and chan-

* _American Machinist_, April 26, 1900.

nels for carrying the lubricant to both the shaft and pulley. An arm extending out from the sleeve and attached to some stationary

Make the end of this rod fast to prevent bush from turning.

Oil Cup

Oil Holes
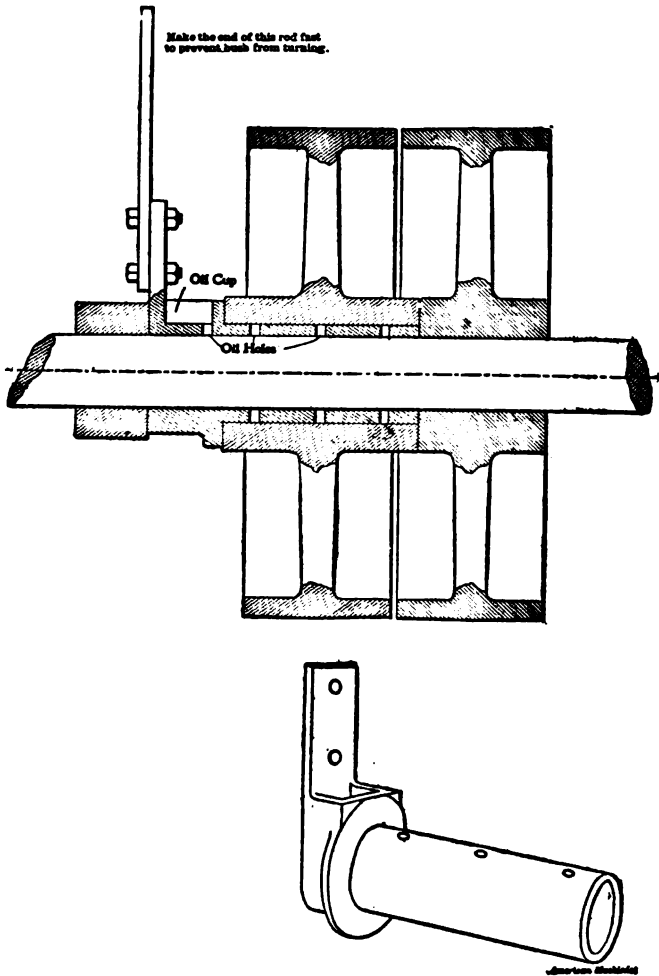
FIG. 80.1.

object, as the ceiling, wall, or floor, prevents the sleeve from rotating. This device has the advantage that it can be applied to any free portion of a counter- or jack-shaft, and does not require any

supporting shaft-hanger, floor-stand, or wall-bracket. A case would seldom occur where no stationary object could be found to which to attach the arm that prevents the sleeve from rotating.

A crank-pin which revolves about a crank-shaft in the ordinary manner can be lubricated the most successfully by a device of the general nature of that shown in Fig. 31. A hole is drilled into the end of the pin, and another, to intersect it, from the rubbing sur- face at about the middle of its length; the latter hole should start from the part most distant from the crank-shaft. A piece of tubing is screwed rigidly into the end of the pin, with its free end extend- ing towards the centre of the crank-shaft, where a hollow ball is attached, the centre of the ball being coincident with the centre line of the crank-shaft. A hole B in the side of the ball, drilled concentric with the crank-shaft, so as to remain stationary when the parts rotate, permits the introduction of oil by any suitable
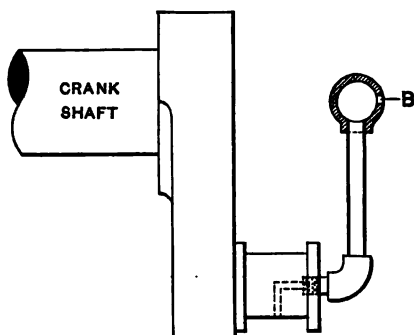
FIG. 31.

means, as an oil-cup with a drip-tube extending through the hole into the ball. After the oil is introduced it is carried out to the rubbing parts by centrifugal force.

The pin of a double crank or pair of disks can be continuously oiled by means of a slight modification of the device just de- scribed. Fig. 32 illustrates the method. A groove C is turned in the crank or disk, or a grooved collar attached to it. An oil- hole leads from the bottom of the groove to the surface of the crank-pin at E. Oil that is put into the groove will be carried to the crank-pin bearing when the parts are rotating. The drip from

the inner end of the crank-shaft bearing *B* may be thus used for the crank-pin, or oil can be fed into the groove by a tube leading from an oil-cup down to the under side of the crank-shaft.

Reciprocating journal-bearings, such as that of a cross-head pin of a horizontal steam-engine or pump, are most commonly lubricated by a method represented by the following device: A piece of wicking is either suspended from an attachment to the frame of the machine, or stretched over a curved surface, and a supply of oil is fed to the wicking so as to keep it completely saturated and almost ready to allow a drop to fall from it. A metallic scraper or "wiper" is fastened to the parts to be lubricated, which, as it is brought under the wicking at each stroke of the moving parts, scrapes or picks some of the oil from it; the oil is led from the wiper to the rubbing surfaces through suitable oil-ways.

There are numerous modifications of this device, a notable one consisting chiefly of a thin strip of metal attached to an oil-cup so
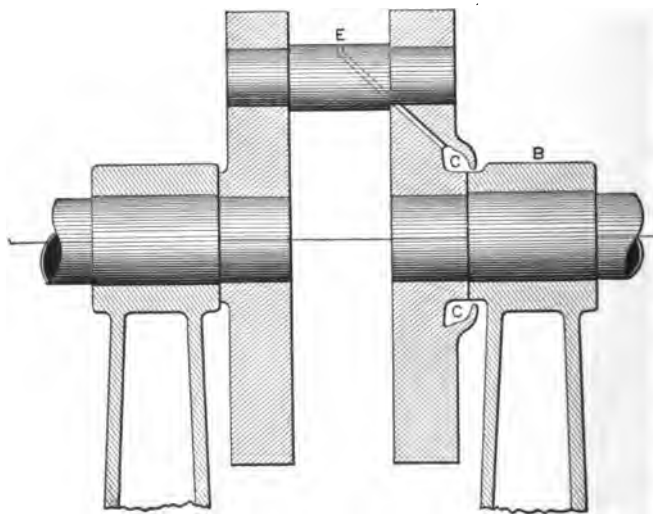
Fig. 32.

that the oil fed from the cup will gather at the lower end, which is pointed. Another thin strip of metal, bent back upon itself so as to almost come together near the ends, is fastened to the reciprocating parts so that the ends will pass on each side of the point to pick off

the drop of oil and then let it flow down to the rubbing parts. These oiling devices are generally placed so as to act at one end of the stroke, thus preventing the throwing of the oil which would occur in high-speed machinery if it were picked off near the middle of the stroke.

A less common device for oiling a reciprocating journal consists of a pair of telescoping tubes, the end of one being fastened to the centre of the reciprocating pin, and one end of the other to a standard on the frame of the machine. The end of the standard to which the tube is attached is above the line of travel of the part to be lubricated. The fastenings at the ends of the tubes are made to permit rotary motion; this, together with the telescopic action, furnishes a means of providing a continuous passageway from an oil-cup on the standard to the journal. Suitable holes in the latter complete the path for the oil to the rubbing surfaces.

When the parts to be lubricated reciprocate vertically, an open cup can be attached to one of them, and the oil dropped directly into it from a feeding-cup on the frame.

As a precaution against injury to an oil-lubricated bearing, should it run dry from neglect or failure of the oiling device, sufficient grease to lubricate it for a reasonable period is placed in a suitable recess formed in the box. As long as the lubrication is sufficient to keep the bearing cool, the grease remains; but as soon as the temperature rises the grease melts and flows to the journal.

Oil-grooves should be made with regard to the effective distribution of the oil over the rubbing surfaces. If a horizontal journal always rotates in the same direction in its box, it is not the best practice to cut the grooves longitudinally, for the oil is then aided but slightly, if at all, in its flow along the grooves by the action of the revolving surface. By cutting the grooves as right- and left-hand threads of very rapid pitch, both starting from the oil-hole in the direction that the journal turns, the oil will be drawn along the grooves by the journal, thus securing both a more rapid circulation and effective lubrication. This applies especially to ring- and chain-oiled bearings.

Forced lubrication is frequently applied to journal-bearings. An oil pressure as high as 60 pounds per square inch or even higher is sometimes used. By this means the duty that a journal

can perform is very greatly increased. It is especially useful for high speeds of rubbing. The pressure may be obtained by a pump driven by the machine itself, or by an overhead reservoir with pipe connection to the bearing.

13. Materials for journal-bearings.—As long as a suitable lubricant is continuously introduced between the rubbing surfaces of a journal and its box, almost any of the metals common to engineering practice can be used for these surfaces. Some will stand much higher bearing-pressures than others, but this depends generally upon the strength of the material. The serious injury that is almost certain to overtake many of the metals when copious lubrication fails, even for an instant, precludes them from use except for slow speeds or light bearing-pressures. Others, while running without injury under both speeds and pressures comparatively high, offer so much frictional resistance to the rubbing motion, that the power lost in overcoming it is so great as to be objectionable in some classes of machinery. A bearing that is called upon to perform heavy service continuously should, and often must, be made of a material, or materials, which will continue to run even though the lubrication is defective, and whose frictional resistance to the rubbing of the parts over each other is low. This last quality is generally, but not always, a necessity for continuous, heavy service; it is always desirable, however. If the frictional resistance is high, the material must either have good heat-conducting capacity, so that it will carry away the frictional heat, or else a special cooling device, such as a hollow box with water circulating through it, may become necessary. A crude method of cooling, which is much used in rolling-mills on the bearings of the rolls, is to let a stream of water flow over the bearing. This would clearly be unsatisfactory where flowing water is objectionable.

For heavy service, mild steel and wrought iron both run satisfactorily on brass or bronze. The journal is generally necessarily made of steel or iron on account of the required strength to endure such service. Brass and bronze exert a comparatively low frictional resistance to the rubbing of the steel or iron over them, and are both softer than iron and steel. This latter is an important property for the material of the box, for, should the lubrication fail, the iron or steel journal will continue to rub over it for a considerable time without serious injury to the bearing, this being a not infre-

quent occurrence with the journals and half-boxes in railway prac-
tice; or, if the box completely encircles the journal, it may seize
and stop it without serious injury to the rubbing parts, since the
softer metal will not cut into the surface of the steel or iron, and
so long as the latter remains comparatively smooth, it will not cut
and groove the softer metal to a serious extent. This cannot always
be taken as true for the harder grades of the alloys just mentioned

The "**white metals**" form a group of bearing *alloys* extensively
used for lining the boxes of bearings. They have low coefficients
of friction, a desirable property for bearing materials. On account
of their weakness they are used only as a thin lining to fill a recess
in the shell of the box. They fuse at low temperatures, and readily
flow into moulds when melted. They can be conveniently and
economically used. For ordinary service their surfaces are left
rough as they form when cast into place. No machining is
necessary. For high-grade, heavy service they are cast in place
and then expanded to fit the shell tightly, either by peening with
a hammer or spinning with a blunt burnishing-tool, and then
machined with a cutting-tool.

These alloys do not injure the surface of the journal when
lubrication fails, except possibly the hardest compositions. In
case of excessive heating they melt and flow out of the box. This
is a valuable property for some classes of high-speed machines in
which sudden stoppage by seizure of the journal would be disas-
trous.

The composition of "white metal" alloys is exceedingly varied.
They generally contain either tin, lead, or zinc in considerable per-
centage. Frequently much of two of these metals is present.
Antimony is added to give hardness.

Babbitt metal is a white alloy largely used for bearing surfaces.
Its composition as now made covers so wide a range that the
name cannot be taken to indicate any particular alloy.

It is common practice among the best engine builders to line
the principal bearings with a "white metal" alloy. The main
shaft and crank-pin bearings are so made.

In railway practice the "half-box" of the axle-bearing is often
coated over with a thin layer of lead when new. The lead, on

* See § 14 for compositions of bearing alloys for railway use.

account of its softness, allows the journal to adjust itself to the bearing and obviates extreme localization of pressure, and at the same time acts as a metallic lubricant.

Cast-iron boxes for steel and iron journals are quite extensively used, especially for line-shafting. The pressure between the rubbing parts must be kept low, however, and even then the parts are in great danger of serious injury if the lubricant is not constantly applied. A continuous-oiling device, such as a ring or chain dipping in a reservoir of oil, becomes a necessity when the speed and pressure are at all high. If abrasion once begins, it is almost certain to injure both surfaces considerably, the cast iron generally suffering the most. The appearance of a bearing so injured seems to indicate that a particle of the cast iron is ground off and imbedded in the journal so as to slightly groove the box, and then the loose pieces of metal continue the grinding action in connection with the grooving, which roughens the surface of the journal still more and causes the destruction to go on, cutting away the box until the machinery is stopped. The brittle and crumbly nature of cast iron makes it exceedingly liable to great injury when abrasion once begins. The more close-grained the cast iron, the less liable is it to cutting. In order to keep the pressure per unit area as low as possible, cast-iron self-aligning boxes are made very long—as long as four times the diameter of the bore, or even longer in proportion.

The comparatively low cost of cast iron, and the ease with which it can be cast into form and machined, make it a desirable material from the point of first cost; hence the reason for its somewhat extensive use for journal-boxes, as before mentioned.

Chilled cast-iron journal-boxes are used on some classes of machinery not requiring accurate fitting of the rubbing parts. The bore of the box is cast against a chill which gives it a fairly accurate form. On account of the extreme hardness of the chilled surface, it is almost impossible to finish it with tools, but it may be ground smooth. In its almost exclusive application to the less accurate classes of machinery, the chilled cast-iron boxes are used without finishing the bore other than by cleaning it carefully. The hardness of the surface eliminates the liability to the abrasion and con-

tinued cutting common to iron which has been cast in the ordinary
sand mould, and against sand or clay cores.

Cast-iron journals are used on the rolls of rolling-mill machinery.
Each roll is cast as a single piece, body and ends. The journals or
necks are cast in sand, so they have the ordinary crystalline struc-
ture and softness of sand castings. These journals run successfully
on brass, bronze, or Babbitt-lined boxes. The speeds and total
pressures are frequently quite high, but the journals are of large
diameter, so that the bearing-pressure per unit area is comparatively
low.

In some classes of metal-working machine-tools, where great
rigidity is desired, a cast-iron spindle is used, whose journals run
on boxes of the same material. When so used, the speed of rubbing
is generally low, and the bearing-pressure per unit area always so.
If well lubricated at first, the rubbing surfaces soon take on a
glazed, glass-like surface, which, once thoroughly fixed, will stand
much neglect in the way of lack of proper lubrication. The same
is true, possibly even to a greater extent, when the cast-iron journal
runs on Babbitt metal, or on cast iron and Babbitt metal in alter-
nate strips.

Hardened steel, or case-hardened iron or mild steel, journals
and boxes, ground to form, are used frequently where it is especially
desirable to maintain accuracy in the running parts. These hard
parts run well together when the rubbing surfaces are smoothly
finished. They will also run excellently on any of the materials
that can be used on steel, wrought iron, or cast iron. Their cost
is much greater than that of the unhardened materials.

A self-lubricating material for journal-boxes has been invented
in recent years. It is made of graphite held together by wood
fibres. The process of making is to mix together the pulverized
graphite and wood pulp in a bath of water, run the mixture into
moulds having the form of the piece wanted, and then subject it to
heavy pressure, forcing the water out through minute openings in
the moulds for this purpose. The compressed "fibre-graphite" is
then treated with oil and baked to give it desirable qualities. The
material thus manufactured is strong enough to work under
pressures ordinarily required for line- and counter-shaft boxes,
pulleys, crank-shafts of small engines, etc. No lubricant is re-

quired or should be used, for the graphite is itself a solid lubricant. The application of oil or grease to a fibre-graphite box, not only does no good, but is harmful.   The cause of their harmful action is as follows: The surface of a journal is always somewhat rough. When it is first revolved in a fibre-graphite box, this roughness abrades the box to some extent, and the small particles of graphite thus loosened collect in the depressions on the journal, finally filling them flush with the higher parts of the metallic surface, thus forming.the best possible rubbing surface.   If oil is now applied to the bearing, it loosens these small particles from the journal and floats them away, leaving the surface in the original condition; the continued application of oil will not allow particles of graphite to collect on the journal, and consequently the wearing away of the box continues much more rapidly than if it were left dry.

The fact that the fibre-graphite bearing requires no attention whatever when running satisfactorily, and that oil and grease are entirely absent, makes it a desirable material for places difficult of access, and when oil and grease may be harmful.

Some of the minerals are used in their natural condition for journal-boxes.   Probably these are confined to the more close-grained ones almost exclusively.   They are used without any lubricant for much the same reason as just mentioned for fibre-graphite. When an iron or steel journal first runs on a stone box, the latter grinds off the surface of the metal, which lodges in the interstices and irregular depressions of the stone, filling them and making a smooth surface which works satisfactorily.   If oil or water is applied, the particles of metal are floated out of the concavities of the stone, and the grinding action is again started.

The action of a stone box and steel journal can be readily understood by rubbing the blade of a knife over a clean oil-stone until the stone becomes glazed and stops cutting or grinding away the steel, and then applying oil, thus causing the stone to cut freely again.

It is believed that no materials except the more precious stones are used in their natural state for bearing surfaces in this country. These are used only for light machinery, such as watches, clocks, electric meters, chronographs, etc.   It is said that the spindles of large grindstones are quite commonly run on stone boxes in the cutlery-manufacturing establishments in England.

Journal-bearings that are submerged in water and cannot be conveniently enclosed so as to be lubricated with oil or grease, frequently have the boxes made of wood. Lignum-vitæ, letterwood, and camwood are the varieties chiefly adopted. The former seems to be the most largely used. It is generally fixed with the end of the grain against the journal. The water itself is a good lubricant for steel, wrought iron, or bronze, running on wood. It is therefore only necessary to provide a way for the water to get between the rubbing surfaces. This is often done by using thin strips of the wood set in a metal frame or casing so as to run parallel with the length of the journal, and extending out a short distance from the casing toward the journal, thus leaving small passageways between them for the water.

The wood has another point in its favor, which is that there is little or no electrolytic corrosion of the journal or shaft passing through it, such as is always present when the journal runs on a metallic box. This corrosion is greatest in sea-water. It affects the journal most seriously at the end of the bearing, a groove forming around it at that locality. The steel of the shaft, the bronze or brass box, together with the salt water, present all the elements of a galvanic cell, and it can only be expected that electrolytic action will occur. When the shaft and box are in direct contact, the electrical resistance through the water at their exposed surfaces is low, and consequently the electrolysis is rapid. The casing holding the strips of wood can be made of the same material as the journal, thus reducing or eliminating the corrosion. Moreover, the fact that the journal and metal of the box are not in direct contact may be beneficial in reducing corrosion, but this can be of little moment if the box is supported on metallic framing which in any way has contact with the shaft of which the journal is a part.

14. Frictional resistance of journal-bearings.—If a horizontal journal rotates in its box under a vertical load P, and the sum of all the elementary frictional forces acting tangent to its rubbing surface, to oppose rotation, is f, then $f \div P = \mu$ is called the coefficient of journal friction for the bearing.

The quantity f may also be defined as the force which must be applied to a rotating piece at a distance from the axis of the journal

equal to its radius, and normal to the axis, in order to keep it rotating uniformly.

In an oil-lubricated bearing, the frictional resistance partakes partly of the nature of the friction between two unlubricated solid bodies rubbing over each other, and partly of that of a solid moving through a considerable volume of liquid. The value of μ therefore lies between that for solid friction of the rubbing materials of the bearing, and that of the same surfaces passing through a volume of the lubricant. Any variation of the rubbing surfaces, the kind of oil, or the quantity that is applied, will affect the value of μ.

Many experiments have been made to determine the value of μ and its method of variation with change of speed and pressure. These experiments, made by different investigators, are unquestionably nearly all, probably all, thoroughly reliable. The values obtained for μ, and the method of its variation, are so greatly different in nearly all cases that it seems impossible to arrive at any definite conclusion regarding them. The following facts seem to be fairly well established for mild steel running on brass, bronze, or white-metal alloys, operating under a steadily applied load:

1st. In an oil-lubricated journal-bearing, the coefficient of friction μ at first decreases as the velocity of rubbing increases from zero, then increases with further increase of speed. The speed at which this change in the variation of μ occurs becomes higher as the bearing-pressure is increased.

2d. In an oil-lubricated journal-bearing, the coefficient of friction μ decreases, rapidly at first, then more slowly, and finally begins to increase, as the load is increased from zero up to the limit of working.

3d. In an oil-lubricated journal-bearing, for the higher speeds, the coefficient of friction μ decreases as the temperature rises, until the lubricant either becomes so thin as not to sufficiently separate the rubbing surfaces, or is disintegrated; but for low speeds μ increases as the temperature rises.

A series of tests for the coefficients of friction of the alloys given in Table I were made by Messrs. Joseph Kuhn and Robert T. Mickle.* One journal each of wrought iron, steel, and cast iron

* " Variation of the Coefficient of Friction with Different Loads and Bearing Metals." *Engineering News*, May 18 and 25, 1893, pp. 468 and 494.

TABLE I.*

COMPOSITION OF BEARING-METAL ALLOYS. PERCENTAGES.

Alloy.	Copper.	Tin.	Lead.	Zinc.	Phosphorus.	Remarks.
A	79.70	10.00	9.50	....	.80	Penn. Ry. standard phosphor-bronze.
B	74.90	9.40	9.45	5.45	.80	Same as A with 6% zinc added.
C	78.45	9.80	11.75	....	....	Baltimore & Ohio Ry. standard bearing alloy.
D	74.90	9.30	11.15	5.85	....	Same as C with 6½% zinc added.
E	87.50	12.50	....	....	....	Copper 7, tin 1.
H	85.71	14.29	....	....	....	Copper 6, tin 1.
I	83.34	16.66	....	....	....	Copper 5, tin 1.

* *Engineering News*, May 25, 1893, p. 496.

TABLE II.†

TESTS OF FRICTIONAL RESISTANCE OF ALLOY A.

Pressure, pounds per square inch.	Wrought-iron Journal.			Steel Journal.		Cast-iron Journal.	
	Velocity of Rubbing, feet per minute.	Temperature, Fahr. deg.	Coef. of Friction, μ.	Temperature, Fahr. deg.	Coef. of Friction, μ.	Temperature, Fahr. deg.	Coef. of Friction, μ.
35	246	76.0	.0176	75.5	.0151	77.8	.0159
	503	89.0	.0199	88.0	.0181	91.5	.0179
	1066	127.5	.0204	112.8	.0256	115.8	.0173
65	230	85.3	.0085	83.8	.0095	84.5	.0092
	483	102.1	.0112	96.9	.0076	103.0	.0103
	968	136.0	.0122	123.6	.0107	129.3	.0137
125	222	89.3	.0044	87.3	.0054	92.6	.0120
	451	100.9	.0074	99.1	.0065	100.8	.0078
	865	137.3	.0092	133.8	.0048	133.5	.0082
250	247	86.6	.0038	87.0	.0050	89.8	.0057
	500	105.4	.0025	107.9	.0042	104.8	.0040
400	234	91.5	.0018	94.0	.0035	92.1	.0038

† *Engineering News*, May 25, 1893, p. 495.

was used.   The dimensions of all were the same, viz., 2 feet in cir-
cumference and 1⅝ inches long.   The boxes were each 1.42 inches
long, parallel to the axis of the journal, and 2⅓⅝ inches wide circum-
ferentially.   The projected area was 4 square inches.   The journal
was moved endwise ⅜₆ of an inch 26 times a minute.   The lubri-
cant was "Extra" lard oil, as used by the Pennsylvania Railway
Company.   The results of the tests are given in Tables II and III.

TABLE III.*

COMPARATIVE TESTS OF BEARING ALLOYS UNDER PRESSURE OF
250 POUNDS PER SQUARE INCH, AT A RUBBING SPEED OF
500 FEET PER MINUTE.

Alloy.	Velocity, feet per minute.	Wrought-iron Journal.			Steel Journal.			Cast-iron Journal.		
		Temperature, Fahr. deg.	Wear, milligrams per 1000 feet travelled.	Coef. of Friction, μ.	Temperature, Fahr. deg.	Wear, milligrams per 1000 feet travelled.	Coef. of Friction, μ.	Temperature, Fahr. deg.	Wear, millimetres per 1000 feet travelled.	Coef. of Friction, μ.
A	500	105.4	33.4	.0025	107.9	41.2	.0042	104.8	22.5	.0040
B	475	117.8	51.7	.0040	125.8	47.1	.0061	121.0	35.2	.0060
C	478	110.5	29.8	.0055	113.5	29.5	.0040	122.4	54.9	.0091
D	458	113.0	43.5	.0056	117.8	68.7	.0049	127.8	35.0	.0070
E	478	113.1	49.5	.0055	113.1	31.4	.0052	116.1	9.9	.0074
H	468	111.1	7.7	.0043	112.6	25.9	.0045	116.1	16.6	.0066
I	500	115.0	25.7	.0043	117.3	32.1	.0088	120.1	13.9	.0046

* *Engineering News,* May 25, 1893, p. 495.

In Table II it can be seen that, for the wrought-iron journal,
when working under 250 pounds per square inch, μ falls from .0038
to .0025 when the velocity of rubbing increases from 247 to 500
feet per minute; evidently the speed at which μ begins to increase
has not been reached.   The same is true for the steel and cast-iron
journals for the same pressure and speeds.   At a pressure of 125
pounds per square inch, the wrought-iron journal seems to have
passed the speed for the change in the variation of μ from decrease
to increase; the cast-iron journal seems to have nearly reached the
turning-point. for μ increases from .0078 to only .0082 when the

speed changes from 451 to 865 feet per minute. In nearly all the other cases the turning-point has been passed. There is, as shown by the table, a considerable increase of temperature in all the cases of increase of speed, but the rise of temperature runs fairly uniform for similar increases of speed.

It was found, in a test of commercial-power-transmission machinery, that the coefficient of friction increased rapidly after a rubbing speed of 2.9 feet per second = 174 feet per minute was reached.*

Table II also shows a continuous decrease in μ as the pressure increases for an approximately constant speed. The pressure does not reach a value high enough to cause μ to begin to increase.

Fig. 33 † shows, for a rubbing speed of 314 feet per minute and
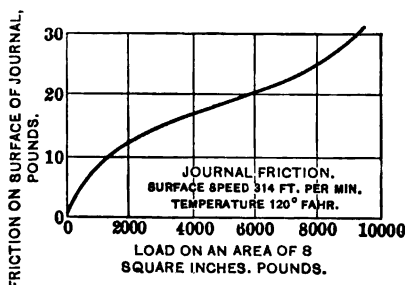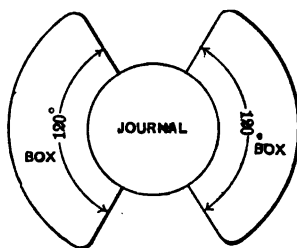
FIG. 33.   FIG. 33.1.

a temperature of 120° Fahrenheit, a decrease of μ up to a bearing-pressure of about 800 pounds per square inch, and a slight increase of μ for increasing pressure above 1000 pounds per square inch, of projected area of the journal.

Fig. 34 shows how μ decreases as the temperature rises, for a speed of 314 feet per minute and a load of 1000 pounds per square inch of the projected area of the journal.

‡ Exceedingly low values of the coefficient of journal friction

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* " Experiments upon Friction in Electric Motors and Transmission Shaft-ing," by S. Hanappe. Minutes of Proc. Inst. Civ. Eng., vol. cxxii., p. 496.

† Fig. 33 may be found in *Digest of Physical Tests*, July, 1897, p. 175; Fig. 34 on p. 174.

‡ Private communication from Professor Kingsbury, March, 1903.

have been obtained recently by Prof. Albert Kingsbury. The tests were made on a hardened, ground, and polished steel journal 1⅜ inches in diameter and 2 inches long. Each of the two "brasses" embraced 120° of the journal's circumference, as shown in Fig. 33.1. Great care was taken to obtain good rubbing surfaces and an accurate fit. The parts were submerged in a bath of the lubricant, with the axis of the journal vertical. The pressure between the bearing surfaces was secured by a spring that tended to force the two "brasses" together when they were in position on opposite sides of the journal. Table IIIA shows the minimum values obtained for the coefficient of friction at a bearing-pressure of 340 pounds per square inch of projected area of the journal and different speeds. The lubricant was "velocite," a mineral spindle-oil. Kerosene or other oils of slight viscosity gave practically the same results. The bearing-pressure was never put on until the journal was at full speed, and relieved before stopping. Otherwise the rubbing surfaces would be liable to injury at slow speeds.

TABLE IIIA.

FRICTION TESTS OF AN ACCURATELY FITTED JOURNAL-BEARING.

Journal 1⅜ inches diameter by 2 inches long, hardened, ground and polished. Two brasses each embracing 120° of circumference of the journal on opposite sides. Bearing submerged in oil. Axis vertical. No oil grooves or channels.

Rev. per minute.	Velocity of rubbing. Ft. per min.	Temperature of oil-bath. Degrees Fahr.	Minimum coefficient of friction, μ.
186	67.1	175	.000458
118	42.4	135	.000455
75	27.1	120	.000458

A striking example of the effect of the condition of the rubbing surfaces upon the coefficient of friction, μ, has been stated by Mr. A. H. Emery.* The materials rubbing together were hardened

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* Trans. Amer. Soc. Mech. Engrs., vol. vi., p. 852.

steel and non-hardened steel. By changing the hardened-steel rubbing surface, which had been ground on a fine, solid emery-wheel, to the fine finish produced by a polishing-wheel, μ was changed from 30% to 3%; the pressure per square inch and the lubricant remained the same, and the same pieces of steel were used in both cases. The pressures were exceedingly high, reaching 50000 pounds per square inch in some cases.*

When no liquid lubricant is used, but the solid surfaces of the journal and box rub directly against each other, as in the fibre-
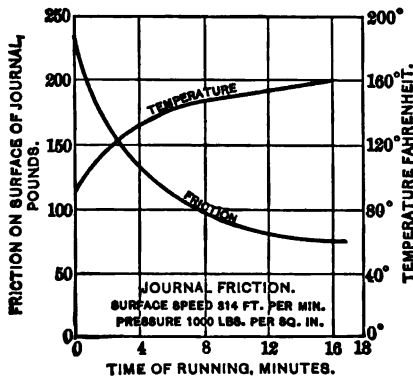
FIG. 34.

graphite bearing, it is probable that there is little, if any, change in the value of μ with change of load and speed, except such as may be caused by deformation of the box, unless there is excessive heating.

The power which is expended in overcoming the frictional resistance of a journal is transformed into an equivalent amount of heat at the rubbing surfaces. This heat must be conducted away, first by the lubricant to the rubbing surfaces, when a lubricant is used, and then through the materials of the journal and box, so that it may radiate, or be carried away by some cooling substance, as water, when special means of cooling are provided. The oil, therefore, must have, as one of the qualities of a good lubricant, considerable heat-conducting capacity when used for heavy service. It is always desirable, however, that its unctious properties shall be

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* Mr. Emery states in a private communication that this pressure was used on the wedge-shaped jaws of a testing machine holder.

such as to reduce the generation of heat to a minimum, thus reducing the power expended in overcoming friction.

15. Effect of changing the proportions of journal-bearings.—In the majority of cases where a bearing is to perform heavy service, the diameter is made as small as the requisite strength will permit. The length is then made such that the bearing-pressure per unit area will be low enough for satisfactory operation.

In order to examine the effect of changing the diameter of a journal, let it be assumed that a bearing, running with a solid lubricant, and designed to operate under a given load and speed, heats to excess. It therefore becomes necessary to change its size in order to secure cool running. First, assume that the diameter is increased, the length remaining unchanged. For convenience in pointing out the effect of this change it will be assumed that the diameter is doubled. The frictional resistance to rotation $= f$, acting tangent to the surface of the journal, is unchanged; for the coefficient of friction μ is the same in both bearings before excessive heating occurs, and the load is the same for both. The distance through which f acts, during one revolution of the larger journal, is twice that for the smaller. If $D =$ the diameter of the smaller, and $2D$ that of the larger, journal, the energy converted into heat by the frictional resistance during one revolution will be $\pi D f$ for the smaller, and $2\pi D f$ for the larger; that is, twice as much mechanical energy is wasted, and twice as much heat produced, in the larger as in the smaller journal. The larger bearing has twice as much area of material at the rubbing surface for carrying away the heat, as the smaller; therefore the quantity of heat to be conducted away per unit area of the material is the same for both. This shows that the liability to excessive heating is just the same in both bearings, and that, while increasing the diameter brings no improvement in the way of cooler running, it doubles the waste of energy in the bearing.

Again, it may be assumed that the length of the journal is doubled, the diameter remaining $D =$ that of the original journal. The frictional resisting force f is the same as before; the diameter is also the same. The mechanical energy transformed into heat is therefore the same as before, and equals $\pi D f$. The area of the material through which the heat is conducted away from the

rubbing surfaces is twice as great in the longer as in the shorter bearing; hence the heat that must pass through a unit area of the material is but one half as great in the elongated bearing as in the original. The longer journal is consequently less liable to excessive heating; the mechanical energy lost in it is the same as in the shorter. In a general way, it may be said that doubling the length, without changing the diameter, of a bearing running with a solid lubricant, reduces its liability to excessive heating by one half, without changing the power necessary to overcome its frictional resistance.

When excessive heating of an oil-lubricated bearing cannot be prevented by the best lubrication, or by putting the rubbing surfaces in their best condition, together with proper precautions for cooling, etc., it is evident that there is need of change in the dimensions. It is assumed that the journal is to run at a given rotative speed and under a given total load. If the bearing-pressure per square inch is very high, and the velocity of rubbing comparatively low, it may be remedied by increasing the diameter; or, if the bearing-pressure is low, and the velocity of rubbing high, decreasing the diameter may prove effective. The strength of the journal must be considered, of course. Lengthening the journal will reduce the liability to heating; but if, on account of too great length, the pressure becomes localized, as at one end, the trouble may be increased. On account of the numerous causes affecting the successful running of an oil-lubricated bearing, it does not seem possible to arrive at any more definite conclusions than these. Each problem must be considered with regard to the conditions affecting it, and dealt with accordingly.

16. Proportions of journal-bearings, and examples from practice.—The most recent and valuable formulas for the proportions of the bearings of engines have been given by Professor John H. Barr of Cornell University.* The formulas, or at least the constants in them, are derived from American practice in engine-construction. The engines are divided into two classes: "low-speed" for Corliss and other long-stroke engines usually making not more than 100 to 125 revolutions per minute, and "high-speed" for those having a stroke from one to one and a half times the piston

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* Trans. Amer. Soc. Mech. Eng., vol. XVIII., 1897, p. 756.

diameter, and a rotative speed of 200 to 300 revolutions per minute. The data for the formulas were obtained from about eighty separate engines classed as high-speed, and about eighty-five classed as low-speed. The high-speed engines ranged from 20 to 240 H.P., and the low-speed engines from 45 to 740 H.P. The practice of thirteen builders is represented in the high-speed, and of twelve builders in the low-speed, engines. The equations are given for the average maximum, average minimum, and mean sizes of the parts.

The notation for journal-bearings is:

A = area of piston, square inches;

B = a constant;

C = a constant;

D = diameter of piston, inches;

H.P. = rated horse-power;

K = a constant;

L = length of stroke, inches;

M = a constant;

N = revolutions per minute;

S = steam-pressure, taken at 100 pounds per square inch above exhaust, as a standard pressure;

$a = dl$ = projected area of journal, square inches;

d = diameter of journal, inches;

l = length of journal, inches.

MAIN JOURNALS:

$$d = C\sqrt[3]{\frac{\overline{\text{H.P.}}}{N}}; \qquad \cdots \cdots \quad (1)$$

$$l = Kd. \qquad \cdots \cdots \cdots \cdots \quad (2)$$

The projection of the journal equals

$$dl = MA. \qquad \cdots \cdots \cdots \cdots \quad (3)$$

The values of the constants for the main journals are:

High-speed Centre-crank Engine. For each of two journals.			Low-speed Side-crank Engine. One journal only.		
Mean.	Maximum.	Minimum.	Mean.	Maximum.	Minimum.
$C = 7.3$	8.5	6.5	6.8	8.0	6.0
$K = 2.2$	3.0	2.0	1.9	2.1	1.7
$M = $ .46	.70	.37	.56	.64	.46

CRANK-PIN:

$$l = C\frac{\text{H.P.}}{L} + B; \quad \dots \dots \quad (4)$$

$$d = \frac{KA}{l} \quad \text{or} \quad dl = KA. \quad \dots \dots \quad (5)$$

The values of the constants for the crank-pin are:

	High-speed Engines.			Low-speed Engines.		
	Mean.	Maximum.	Minimum.	Mean.	Maximum.	Minimum.
B = 2.5″		2.5″	2.5″	2.0″	2.0″	2.0″
C = .30		.46	.13	.6	.8	.4
K = .24		.44	.17	.09	.115	.065

CROSS-HEAD PIN:

$$a = dl = CA; \quad \dots \dots \quad (6)$$

$$l = Kd. \quad \dots \dots \quad (7)$$

The values of the constants for the cross-head pin are:

	High-speed Engines.			Low-speed Engines.		
	Mean.	Maximum.	Minimum.	Mean.	Maximum.	Minimum.
C = .08		.11	.06	.07	.10	.054
K = 1.25		2.0	1.0	1.3	1.5	1.0

Problem in the design of journal-bearings.—The application of the above formulas may be illustrated in the following problem:

It is required to determine the dimensions of the main bearings, crank-pin, and cross-head pin of a centre-crank engine having a stroke $L = 14$ inches, piston diameter $D = 12$ inches (corresponding to $A = 113.1$ square inches), and rated at 100 H.P. for a speed $N = 250$ revolutions per minute.

This comes under the high-speed class.

Main journals:

$$d = C\sqrt[3]{\frac{\text{H.P.}}{N}} = 7.3\sqrt[3]{\frac{100}{250}} = 7.3 \times .7368 = 5.38'';$$

$$l = Kd = 2.2 \times 5.38 = 11.84''.$$

This corresponds to a value of $M = .56$ in the equation $dl = MA$.

Crank-pin:

$$l = C\frac{\text{H. P.}}{L} + B = .3\frac{100}{14} + 2.5'' = 2.14 + 2.5 = 4.64'';$$

$$d = \frac{KA}{l} = \frac{.24 \times 113.1}{4.64} = 5.85''.$$

Cross-head pin:

$$dl = CA = .08 \times 113.1 = 9.048 \text{ sq. in.};$$
$$l = Kd = 1.25d.$$

Therefore;

$$d \times 1.25d = 1.25d^2 = 9.048;$$
$$d^2 = 9.048 \div 1.25 = 7.238;$$
$$d = 2.69'';$$
$$l = 1.25d = 1.25 \times 2.69 = 3.36''.$$

The nearest convenient working dimensions would naturally be used in practice.

With regard to the difference in the allowable working pressures for constant, as compared with intermittent, loads, Mr. Babcock made the following statement:* "I found that in crank-pins with good fitting I could allow as high as 1200 pounds maximum to the square inch; pins, perhaps 4 to 6 inches diameter, running up to 60 or 70 revolutions, would stand that continuously without getting warm. The main journal of the same engine would not stand over 300 pounds to the square inch without getting warm."

Regarding the locomotive Lady of the Lake, Druitt Halpin states † that, at the beginning of the stroke, the total pressure on the crank-pin $3\frac{1}{2} \times 3$ inches is 28140 pounds, which gives $28140 \div (3 \times 3\frac{1}{2}) = 2680$ pounds per square inch of projected area. The modifying effect of the reciprocating parts is not considered. In

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* Trans. Amer. Soc. Mech. Engrs., vol. VI., 1885, p. 856.

† Minutes of Proceedings of the Inst. Mech. Engrs., 1888, p. 657.

the same engine the main bearings are $6 \times 6 = 36$ square inches, and carry 8 tons each, or 498 pounds per square inch.

Mr. Beauchamp Tower quotes Mr. Tomlinson as saying that 300 pounds per square inch is undesirable in locomotive-axle bearings, while 1000 pounds per square inch, and considerably more, can be used on the crank-pins.*

Mr. Henry Davy states that in pump-bearings, for speeds of rubbing up to 12 feet per minute, 600 pounds per square inch for continuous pressure in the same direction, and 1000 pounds per square inch for intermittent pressures, can be satisfactorily used.†

In a freight locomotive built by the Schenectady Locomotive Works for the Boston & Maine Railroad Company, the bearing-pressures, due to the weight of the parts supported, calculated from data given in *Engineering Review* of March 26, 1898, are: 215 pounds per square inch of projected area for the driving-wheel axles, and about 250 pounds per square inch for the tender-bearings. The driver-axle bearings are 8 inches diameter by 10 inches long; the tender-journals $4\frac{1}{4}$ inches diameter by 8 inches long.

Dr. C. B. Dudley states that in railway practice, bearing-pressures as high as 350 to 400 pounds per square inch are used.‡

President Joseph Tomlinson of the Institute of Mechanical Engineers is recorded as saying: "The practical limit at which he had arrived was $2\frac{1}{2}$ cwt. per square inch; and if more than this pressure were allowed to the bearings of a locomotive engine, notwithstanding their freedom to wabble from side to side, they would not run cold. . . . Whenever he had an engine which would not run cold, and in which the weights upon the bearings could not be changed, he had put a bigger axle in and thereby cured the heating directly." §

Several of the leading builders of large engines, such as are used for direct driving of electrical generators, state that the highest pressure they consider safe for crank-shaft bearing is 150 pounds per square inch of projected area of the journal. The same value

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* Proc. Inst. Mech. Engrs., 1884, p. 30.

† *Ibid.*, 1883, p. 654.

‡ Journal of the Franklin Institute, 1892, p. 88.

§ Proc. Inst. Mech. Engrs., 1891, p. 131.

is given as the safe limit for small high-speed engines by the builders of this class of machinery.

Mr. Edwin Reynolds, of the Allis-Chalmers Co., is quoted as follows regarding journal-bearings: * " The square root of the speed in feet per second multiplied by the pressure per square inch of projected area, should not exceed 500." In reply to an inquiry, he is further quoted in the same place as saying: " It is true I have used the rule you mention for a limit and never go up to 500, except in vertical engines where the steam pressure in the cylinder is sufficient to lift the shaft against the cap. 350 to 375 should be the limit for horizontal engines. This method for determining size and proportions has proved satisfactory in a very large number of machines."

Step-bearings and Button Thrust-bearings.

17. When the weight of a vertical shaft and the parts attached to it, together with whatever end thrust may come on it, are supported by a box which bears against its end, and at the same time prevents it from moving sidewise, the whole combination of the rubbing parts and others in the immediate neighborhood is called a step-bearing.

18. Forms of step-bearings.—The simplest form of step-bearing is shown in Fig. 35, where the step B is bored out to fit the end of
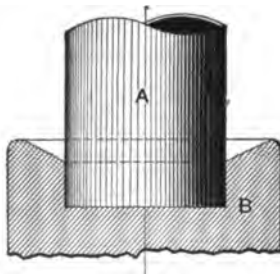
FIG. 85.

the vertical shaft A. In this case the bearing is made up of only two parts. The sloping part at the top of the box holds the oil that is used for lubrication. Radial grooves are generally cut across the

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* *American Machinist*, Oct. 16, 1898, p. 879.

bottom of the shaft to allow the oil to gain access to all parts of the horizontal bearing surface. Bearings of this class are commonly used where either the speed of rotation is low, or the pressure is light. Pillar-cranes are ordinarily supplied with such a bearing for stepping the mast. In this application the pressure may be high, for the speed of rotation is very low.

When the shaft in Fig. 35 makes one revolution, the parts on the end farthest from the centre rub over the box through a distance equal to the circumference of the shaft, while those near the centre rub over a much smaller distance, and the geometrically central point does not have any motion over the box. On account of this inequality of rubbing there will be a corresponding unevenness of wear, so that if the parts are fitted together accurately when new, so as to make the pressure uniform over the end of the shaft, the outer portion will wear away most rapidly in service, thus causing the pressure to become heaviest at the centre, and lightest at the outer part, of the rubbing surfaces. The pressure at the centre may become so intense as to crush the material at that point. Even if this does not occur, abrasion and cutting are likely to take place. The inequality of wear and pressure may be partly obviated by removing some of the material at the centre of the rubbing parts, leaving a pair of annular rings for the rubbing surfaces. This is advisable in nearly all cases.
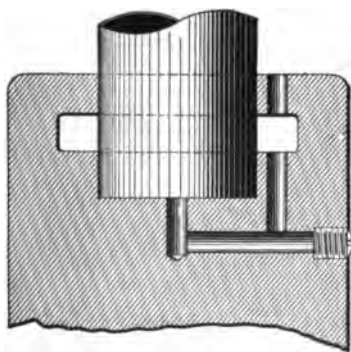
Fig. 36.

If the speed of the shaft is high, difficulty will be experienced in lubricating it on account of the centrifugal action of the rotating

part throwing the oil from the centre and not allowing it to return again, unless some special provision is made for its doing so. Such provision can be readily made, however, as shown in Fig. 36, by making an oil-passage from the top of the step to the centre of the bottom of the bearing. This arrangement forms a small centrifugal pump, which draws the oil in at the bottom of the bearing through the oil-passage, throws it to the outer part of the bore, and forces it to the groove around the top of the step, so that it will again be ready to start on the same circuit. Complete and free circulation may be thus secured.

For heavier duty, either on account of increased speed or pressure, this form of bearing may be made more durable by placing a number of disk-shaped washers between the end of the shaft and the box, as shown in Fig. 37. One set of these washers is generally made
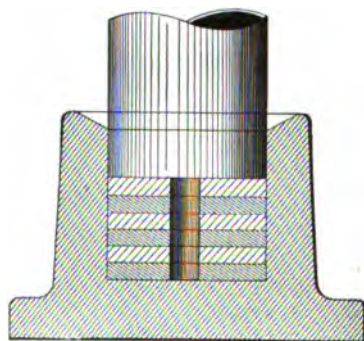
Fig. 37.

of some hard material, such as steel, and the other set of a softer material, as brass or bronze. The two kinds are then placed alternately, so that each washer rubs against a material different from itself. If the shaft is of mild steel and the box of cast iron, which is a common construction, the top washer is often fastened to the shaft, and the bottom one to the box, thus making all of the wear come upon the washers. The number of pairs of bearing surfaces over which the wear is distributed is one more than the number of free washers. The series of washers permits a slower speed of rubbing between each pair of the surfaces, and, in case abrasion should begin between any pair, the rubbing motion will cease there until the oil

has an opportunity to get between them, or until repair can be made, without serious injury to the machine or the necessity of stopping it. The washers generally have a hole bored through the centre, and are grooved radially for oil. The same device for securing circulation of the oil as is shown in Fig. 36 can be used, of course. Hardened and ground tool-steel, or case-hardened and ground mild-steel, washers running on brass or bronze give most excellent service.

In machinery where the shaft and box cannot be accurately aligned, or where they may get out of line from some cause, such as settling of the supporting parts or springing of the shaft, lenticular washers with spherical faces of the form shown in Fig. 38 may be used. By making them smaller in diameter than the
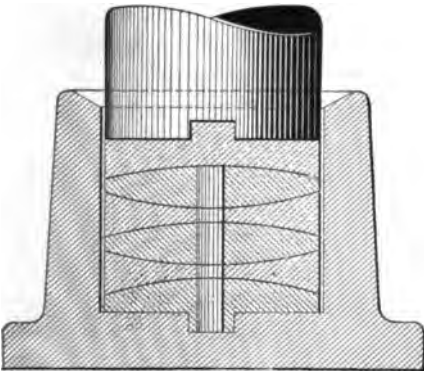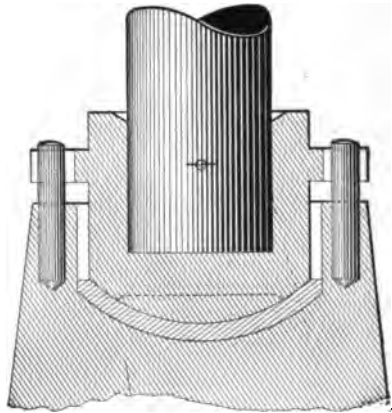
FIG. 38.   FIG. 38.1

bore of the box, they will adjust themselves to a perfect bearing for any relative position of the shaft and box, within the limits for which they are designed. As with the flat washers, however, the wear will be more rapid at the parts more remote from the centre.

Fig. 38.1 shows another method of allowing for self-adjustment of a step-bearing. It is less expensive than the preceding design. The step proper is spherical at the bottom and the base-plate is concaved to fit it. The step may be machined to form on the bottom and a "white-metal" alloy cast in the floor plate to fit it. The two pins are to prevent rotation of the step. If they press against the wings of the step at points on a horizontal diameter of the sphere, there will

be no tendency to throw the step from its place when a motion of adjustment takes place. The bottom of the step may be cut away as indicated by the broken line and only a corresponding portion of the socket used. It is not necessary to have both surfaces spherical, especially for light service. One, the support, may be conical or any other form that will hold the spherical surface and allow it to adjust itself.

The step-bearings for turbine water-wheels running on vertical shafts often have a lignum-vitæ step, Fig. 39, which supports the metallic shaft. The step is made crowning on top, and the end of the shaft cupped to fit it, the rubbing surfaces are spherical in form. Water lubrication is easily obtained by cutting radial grooves in the rubbing surfaces, for the bearing is surrounded by water, which will flow into the grooves. If the speed is high, the water can be made to circulate freely through the grooves by boring a hole through the centre of the wooden step from the bottom upward to the rubbing sur-
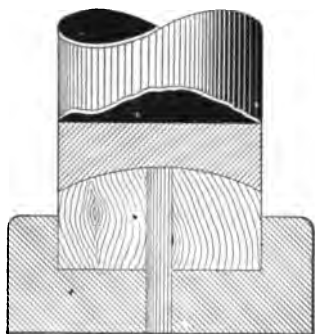
FIG. 39.
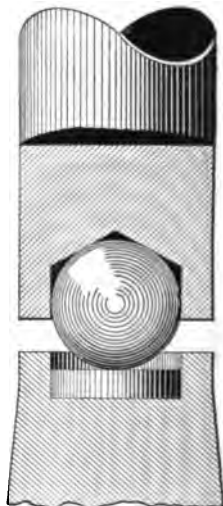
FIG. 39.1.

faces, as shown. The centrifugal action will throw the water out through the grooves to the circumference of the bearing, and at the same time draw it in through the hole in the centre of the step.

Exceedingly heavy service, both as to speed and pressure, is sometimes required of a step-bearing. It is not advisable, or even practicable in many cases, to meet this requirement by increasing the diameter of the bearing; for not only does the wear increase with the diameter, as has already been pointed out, but there is also

a corresponding increase of frictional resistance with its accompanying increase of power loss in the bearing. Moreover, the liability to abrasion and cutting is also increased. Some other means must therefore be adopted for securing the desired qualities.

Forced lubrication affords what seems to be the most satisfactory solution for securing the successful operation of step-bearings for heavy duty. This can be applied most easily and economically in the water-lubricated bearing with the wooden step, Fig. 39, when used in connection with a water-wheel. In such a case it is only necessary to connect a pipe to the bottom of the hole in the bearing, and lead it to the water in the fore-bay. Practically the whole head of water is thus made available for forcing the lubricant, which is the water itself, in between the rubbing surfaces. This assumes that the bearing is below the wheel, so that the lubricating water can flow freely from the bearing to the tail-race. The oil-grooves, if used, should extend from the centre only part way to the circumference. If extended clear across the rubbing surfaces, the lubricant would be forced through them without performing its function.

To secure a higher pressure, for forcing the lubricant into the bearing, than the head of water will give, a force-pump may be attached to the pipe connected with the bearing.

Any of the step-bearings shown in the preceding figures can be lubricated with oil in a manner similar to that just described for using water, it being necessary to provide a reservoir for catching the oil which overflows from the top of the bearing, and a pump for taking up this oil and forcing it into the bearing again.

18.1. Button thrust-bearings are a suitable form for light work. They are even used for comparatively heavy pressures and high speeds in some cases.*

Usually a crowned surface presses and spins against a flat plate. Both are hardened for durability. Since steel balls have come into such common use that they can be obtained at a low cost, they have been utilized to some extent for this form of bearing.

Fig. 39.1 shows an easily constructed thrust-bearing of this type. The end of the shaft is bored and a steel ball fitted snugly into it. The step is recessed for a hardened steel disk. When used in a vertical position as shown, the rubbing surfaces may

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*See § 77, Table XVI, for worm-gears with button thrust-bearing.

be submerged in oil.   A high speed would throw it away from the rubbing parts.

Fig. 39.2 has a loose ball between two plane surfaces.   The ball is kept in place by a quill that has a loose fit over the two
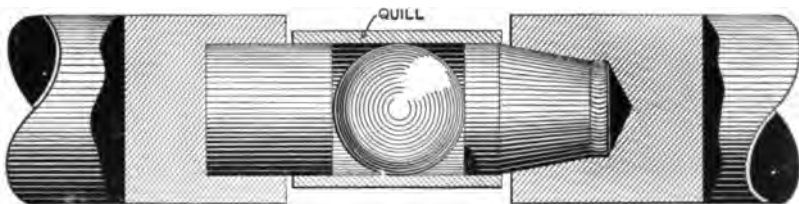
FIG. 89.2.

end plugs of the rotating parts.   This design has the advantage of two pairs of wearing surfaces.   The ball *may* rotate so as to present different parts of surface to the plugs.   One plug is tapered so as to be easily removable from the shaft, but is more expensive than the other, which is cylindrical.

Pivot-bearings.

19. Conical pivot-bearings of the form shown in Fig. 40 are extensively used in light machinery.   The ease with which they can be adjusted for wear is the chief factor in bringing them into use for light work.   The wedge-like action of the point, and the unequal wear on the rubbing surfaces, prevent their use for heavy machinery to any considerable extent.

The pressure acting over the conical bearing surfaces may be assumed as acting at two diametrically opposite points for the purpose of finding the amount of the total normal pressure between them.   Thus, in the figure, for the thrust P the normal pressure $N = (P \div 2)$ csc $(\theta \div 2)$, and the total normal pressure is

$$2N = P \csc \frac{\theta}{2}.$$

The intensity of pressure per unit area is the same as if the load were supported on a flat-ended shaft of the same area as the projection, on a plane normal to its axis, of the bearing surface of the pivot.   In other words, the angularity of the pivot-point does not affect the pressure per unit area on the bearing surfaces.   This is shown by dividing the total normal pressure $2N$ by the area of one of the rubbing surfaces.   The area A of the conical rubbing surface of the pivot, Fig. 40, is

$$A = 2\pi R \times \tfrac{1}{2}(VB) - 2\pi r \times \tfrac{1}{2}(VC)$$

$$= \pi R \times R \csc \frac{\theta}{2} - \pi r \times r \csc \frac{\theta}{2}$$

$$= \pi(R^2 - r^2)\csc\frac{\theta}{2}.$$
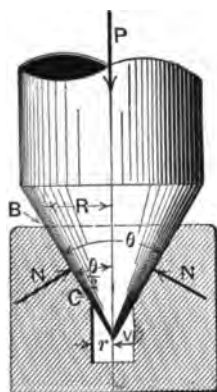
FIG. 40.

Hence for the pressure p per unit area

$$p = \frac{2N}{A} = \frac{P \csc \dfrac{\theta}{2}}{\pi(R^2 - r^2) \csc \dfrac{\theta}{2}} = \frac{P}{\pi(R^2 - r^2)}.$$

The angle θ does not enter the last member of this equation; hence p is not affected by the angle of the cone.

The wear will be more rapid on the parts having the greatest radial distances from the axis, and consequently the greatest rubbing action. The result is that after use the pressure between the rubbing surfaces will be greater near the point of the cone than near the base. For this reason a portion of the box is generally cut away at the centre, as indicated in the figure.

It is common practice, in light machinery, to use a conical bearing at each end of a shaft or spindle. It will then withstand side pressure as well as thrust.

20. The "tractrix" or "curve of constant tangent" is the only theoretically correct outline for a step- or pivot-bearing, since it is the only one that will wear uniformly over the rubbing surfaces, and thus maintain a uniform pressure per unit area between the surfaces. It is also called "Schiele's anti-friction curve," after its discoverer. The nature of the curve can best be described by the method of drawing it. This can be done on a piece of smooth horizontal paper in the following manner: In Fig. 41 the line AB is taken for the directrix of the curve. A beam-
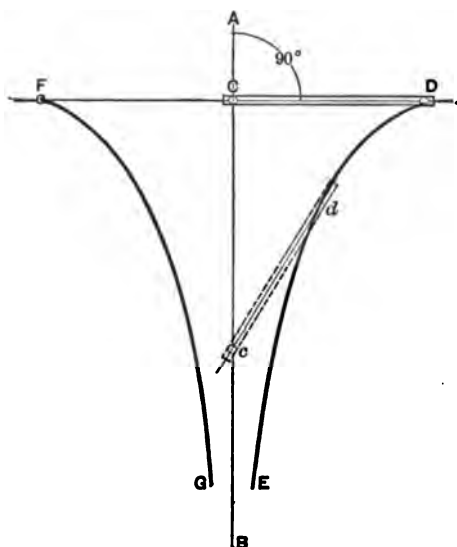
Fig. 41.

compass is placed so that one point, C, is on the directrix, and the other, D, lies on a line drawn normal to the directrix at C. C is then moved along AB toward B so that D trails freely after it, care being taken to hold the beam-compass by the point C so that D is not thrown out of the path it will naturally follow when there is nothing to press it aside. If D is a smooth, round pencil-point, it will trace the tractrix DE. A wedge-pointed pencil may also be used at D by placing it so that its edge takes the direction CD. When using this kind of a pencil-point, it is best to draw

another curve, FG, on the opposite side of the directrix, starting with C at the same point as before. They can then be tested for accuracy by folding the paper along AB, so that they will coincide if correctly drawn.

From the method of drawing the tractrix, it is evident that, when the compass is in any position cd, the line joining the points c and d must be tangent to the curve at d; the distance cd, being that between the points of the beam-compass, remains constant; therefore, on any line drawn tangent to the tractrix, the distance from the point of tangency to its intersection with the directrix is a constant. Hence the name "curve of constant tangent."

The tractrix can be continued to an indefinite length, but in practice only a portion of it is used. This is generally taken from D toward E as far as desired. Fig. 42 represents the end of a shaft
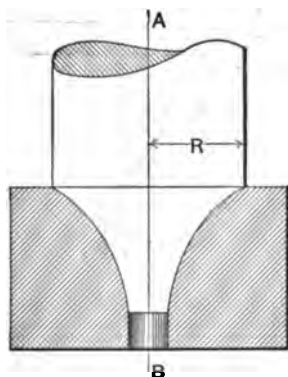
Fig. 42.

turned down to this form of profile, and resting upon the supporting part of the bearing. The directrix coincides with the centre-line of the shaft, and the largest radius is equal to the distance between the compass-points that were used for describing the curve.

The proof that the tractrix will wear away uniformly over the entire rubbing surface is as follows: In Fig. 43 a narrow band or ring of the bearing shown in Fig. 42 is represented. The width of the band, measured along the rubbing surface, is s; this width is assumed to be so small that the curved surface may be treated as if it were conical in form, but it is necessarily magnified in the draw-

ing so that the parts may be seen. The mean radius of the ring is r, and its horizontal thickness is t. The line R has the direction of the curve at its mean radius, and the length of R, between the point of tangency and the directrix AB, is the same as the greatest radius of the bearing, as shown in Fig. 42.

It has been shown in the preceding section that the pressure on a conical pivot-bearing is not affected in its intensity per unit area by the angle of the cone; therefore, in a correctly fitted tractrix-

FIG. 43.
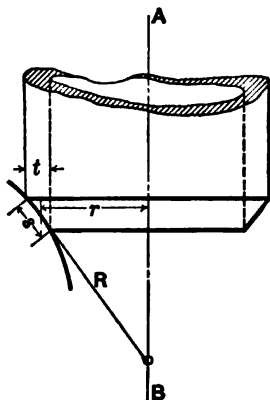
bearing, the pressure per unit area is the same over the entire rubbing surfaces. The wear upon each unit area of one surface may therefore be taken as proportional to the distance through which it rubs against the other while making one revolution. For convenience it may be assumed that

$v = $ volume of material worn from a unit area of one rubbing surface when the surfaces rub over each other through a unit distance under any given pressure.

The area of the rubbing surface of the ring is $2\pi rs$; the distance through which a point on the ring rubs during one revolution is $2\pi r$; therefore the volume of the material worn from the ring during one revolution of the shaft equals

$$V = 2\pi rs \times 2\pi r \times v.$$

In order to see more clearly how the shaft will settle vertically on its support when this amount of material is removed from the small band which has been selected, it may be assumed that this band is the end of a thin tube, of a thickness t, cut entirely free from the rest of the bearing. The sectional area of this tube, on a plane perpendicular to its axis AB, is $2\pi r t$. In the figure it can be seen that, in the similar right triangles, one having the hypothenuse s and side t, and the other the hypothenuse R and side r,

$t : s = r : R$, or $t = \dfrac{sr}{R}$; therefore the area of the cross-section of the tube equals

$$A = 2\pi r t = 2\pi r \frac{sr}{R}.$$

The amount K by which the tube is shortened during a revolution is the quotient found by dividing the quantity of material removed by the area of the cross-section of the tube, which gives

$$K = \frac{V}{A} = \frac{2\pi r s \times 2\pi r \times v}{2\pi r \dfrac{sr}{R}} = 2\pi v R.$$

This equation shows that the shortening of the tube is independent of its radius r, and is represented by the continued product of the constant 2π, and the quantities v and R, which are also constants for a given bearing, load, and solid lubricant. The shortening of this tube is therefore the same as that of any other that may be cut from the bearing, which shows that the complete bearing wears away so that a uniform pressure is maintained between the rubbing surfaces, and the bearing will retain its original form.

The equation of the tractrix, which may be used for obtaining points through which to draw the curve, can be developed as follows: In Fig. 44, AB is the directrix of the curve, p any point on the tractrix, and $R = CD$ the tangent at p; the coordinates of p are x and y. Taking the differential portion ds of the curve at p, and the corresponding dy and dx, it can be seen that, in the similar right triangles, one having the hypothenuse ds and sides

dx and dy, the other the hypothenuse $R = CD$ and sides x and $\sqrt{R^2 - x^2}$,

$$\frac{dy}{dx} = \frac{\sqrt{R^2 - x^2}}{x}.$$
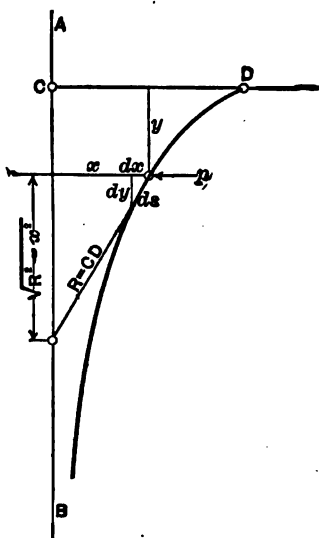
FIG. 44.

Integrating this expression, and taking into account only the positive signs, gives

$$y = R \log_\epsilon \frac{R + \sqrt{R^2 - x^2}}{x} - \sqrt{R^2 - x^2}$$

The natural system of logarithms must be used in this equation. This is indicated by the subscript ϵ. ($\epsilon = 2.7182818$.) The natural logarithm of a number may be found either in a table of natural logarithms, or by multiplying the common logarithm of the number (i.e., the logarithm to the base 10, which is written $\log_{10}$) by $\log_\epsilon 10 = 2.3025851$.

In the equation it is convenient to put $R = 1$ when solving for corresponding values of x and y. Any system of units or scale of

drawing can be adopted for plotting the curve so as to give it the required size. By putting $R = 1$ the equation becomes

$$y = \text{natural log } \frac{1 + \sqrt{1 - x^2}}{x} - \sqrt{1 - x^2},$$

or

$$y = 2.3025851 \text{ common log } \frac{1 + \sqrt{1 - x^2}}{x} - \sqrt{1 - x^2}.$$

For a point having $x = .6$, the solution for y is, with natural logarithms,

$$y = \text{nat. log } \frac{1 + \sqrt{1 - (.6)^2}}{.6} - \sqrt{1 - .36}$$

$$= \text{nat. log } \frac{1 + \sqrt{.64}}{.6} - .8$$

$$= \text{nat. log } 3 - .8$$

$$= 1.0986 - .8 = .2986.$$

The solution by common logarithms differs from this only in taking the logarithm of 3. Thus, by common logarithms,

$$y = 2.3025851 \text{ com. log } 3 - .8$$

$$= 2.3025851 \times .4771213 - .8$$

$$= 1.0986 - .8 = .2986.$$

Other values of the coordinates, calculated as above, are given in Table IV.

TABLE IV.

COORDINATES OF TRACTRIX.

x	y	x	y
1.0	0	.4	.650
.9	.031	.3	.920
.8	.093	.2	1.313
.7	.182	.1	1.997
.6	.298	0	∞
.5	.451		

The tractrix-bearing can be lubricated, when running at high speed, in the same manner as the step-bearing, Fig. 36. It is possible that, when oil-lubrication is used, the wear is not uniform on account of the variation of the coefficient of friction with speed of rubbing at the different radial distances.

This form of bearing is but little used, the only apparent reason being that it is difficult to make.

Collar-bearings.

21. Collars extending out from the surface of a shaft are generally provided for taking the end thrust of a shaft that cannot be supported by a step-bearing. Fig. 45 shows a shaft with such collars resting in a box of suitable form to receive it.

If, instead of a number of collars, a single one were used to resist the thrust, the area of its side pressing against the box would have to be equal to or greater than the combined areas on one side
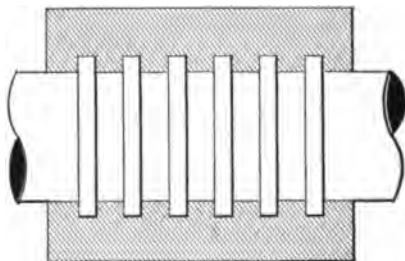
FIG. 45.

of all the collars shown, in order to keep the pressure between the rubbing surfaces the same per unit area. This would necessitate making the collar of a comparatively large diameter. This increase of diameter would bring into action the uneven wearing so marked in flat step-bearings, and also increase the frictional resistance to turning, in about the same proportion that the mean diameter of the rubbing surface of the single collar is greater than that of each collar when several are used. This is one reason, probably the most important one in most cases, for using the multiple-collar thrust-bearing instead of a single collar; another reason is that a greater economy of space can be secured with the smaller diameter of the multiple-collar bearing.

When the thrust is always in the same direction, as indicated by the arrow in Fig. 46, the bearing may be shortened, while still
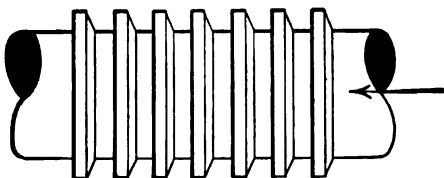
FIG. 46.

retaining equal strength and wearing surface, by making the collars of the form shown in this figure, where the flat faces are of the same area as in the preceding figure, but are brought nearer together.

In order to obtain the best service from a collar-bearing working under heavy pressure and at high rubbing speed, each collar should have its own individual bearing-ring, separately adjustable. By this means, which is the common practice for large machinery, each ring can be adjusted for equal bearing-pressure, even though the wear on the different ones may be unequal; and in case heating should occur at any collar, its ring can be adjusted to partly relieve the pressure between them until they run cool again, or it may even be removed for repair while the machinery is operating, the load being carried by the remaining collars in the meantime. Elaborate collar-bearings are used on the propeller-shafts of screw-propelled vessels.

When used for heavy service, collar-bearings are lubricated by an oil-bath, or by pipes leading to holes in the bearing-rings which open on the rubbing surface of the ring; four pipes and openings are often used on a single ring, placed at equal distances apart around the ring. Suitable oil-grooves are cut in the rubbing surface, starting from the oil-opening and running zigzag a short distance from it in a direction corresponding to that in which the collar rubs over it.

Table V represents the practice in thrust-bearings on merchant and naval vessels built in the yards of the Newport News Shipbuilding & Dry Dock Co. The horseshoe bearing has each part, which comes into contact with the collars of the shaft, made in the shape of the letter U, or a horseshoe. This form allows the bearing parts to be removed and replaced readily.

TABLE V.*

COLLAR THRUST-BEARINGS.

Steel collars running against white metal of bearing-rings; oil-bath lubrication.

	Gunboats.		Battle-ships.	Merchant Ships.				
Horse-power of one engine............	875	800	5000	600	250	8500	8800	400
Revolutions per minute...............	800	283	120	115	110	115	80	110
Diam. of shaft, inches	5⅞	5⅞	14	8	6	12¼	15½	7
Inner diam. of rubbing surface of collar, inches........	6	6	14¼	8¼	6¼	13¼	16	7¼
Outer diam. of rubbing surface of collar, inches........	8⅞	8⅞	21½	12½	9¼	19	24	11
Number of collars...	9	9	11	6	6	15	11	6
Estimated total thrust of one engine exerted on one collar-bearing, pounds ...	18750	18100	79800	12300	6890	55900	71200	9600
Method of cooling ...	Water circulation	Water circulation	Water circulation			Water circulation	Water circulation	
Type of bearing.....	Collar	Collar	Horse-shoe	Collar	Collar	Collar	Collar	

* Data kindly furnished by Mr. C. B. Orcutt, President of the Newport News Shipbuilding & Dry Dock Co., builders of the battleships Kearsarge, Kentucky, Illinois, etc.

The practice of the Marine Iron Works of Chicago, for their smaller work, is shown in Table VI and Figs. 47, 48, and 49. Fig. 47 is a steel shaft with seven collars integral with the shaft. The box has a Babbitt lining cast around the collars so as to fit them. An oil-trough at the top of the bearing, from which oil-holes lead to the top of each collar, furnishes a means of lubricating. The data relative to this bearing are given in Table VI. This bearing is designed for a 50-H.P. engine at 300 revolutions.

TABLE VI.*

COLLAR THRUST-BEARINGS.

Type of Bearing.................	Fig. 47.	Fig. 48.	Fig. 49.
Horse-power of engine	50	25	75
Revolutions per minute....	300	400	275
Diam. of shaft, inches......	2¼	2¼	4
Inner diam. of rubbing surface of collar, inches.....	2¼	2¼	4
Outer diam. of rubbing surface of collar, inches	4¼	4¼	9
Number of collars.........	7	1	1
Estimated total thrust on collar-bearing, pounds....	1630	1040	2160
Material of collar..........	Steel	Cast iron	Steel
Material of bearing-rings ..	Babbitt metal	Bronze washer	Bronze washer
Method of lubricating......	Oil-trough above journal†	Grease-cups. Solid oil in cups placed on oil-holes	Grease-cups ‡

* Data kindly furnished by the Marine Iron Works, Chicago, Ill., except the estimated total thrust on collar-bearing, which was calculated by the writer.
† Oil-holes lead from trough to edges of collars.
‡ Chamber in bottom part of box can be filled to make an oil-bath if desired.
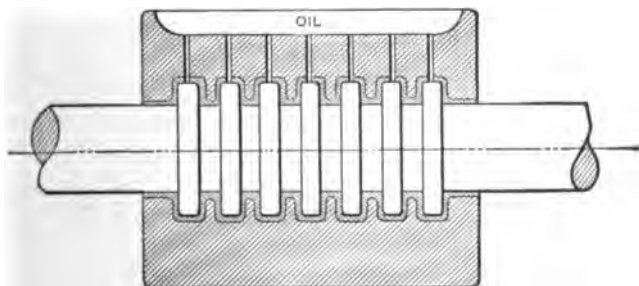
FIG. 47.

Fig. 48 is a thrust-bearing for a 25-H.P. engine making 400 revolutions per minute.   It consists of two cast-iron collars fastened to the shaft on each side of a cast-iron box-bearing.   Loose collars of bronze are interposed between each end of the box and the collar adjacent to it.   It is lubricated by means of grease-cups attached
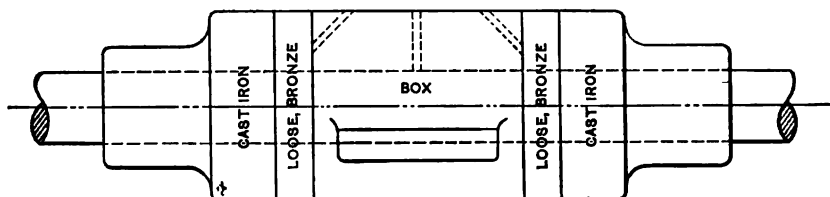
FIG. 48.

to oil-holes leading to the journals and the faces of the collars. The remaining data are given in Table VI.

Fig. 49 is for a 75-H.P. engine making 275 revolutions per minute.   It has a steel collar, fastened to the shaft, which bears
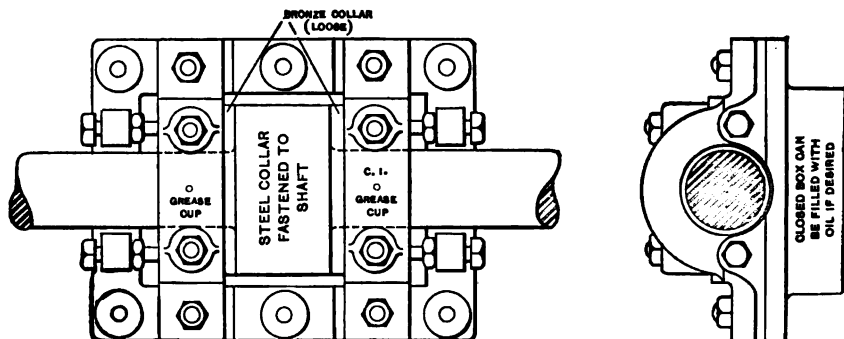
FIG. 49.

against loose bronze collars, which in turn bear against cast-iron boxes.   Lubrication is secured by grease-cups, and, if desired, by filling the reservoir at the bottom of the box with oil, thus obtaining bath lubrication.   Other data are given in Table VI.

Roller Journal-bearings.

22. Cylindrical roller-bearings for working conditions similar to those which an ordinary journal-bearing is designed to meet have

come into extensive use. Such a bearing is illustrated in Fig. 49.1. The shaft or journal is surrounded by several cylindrical rollers, which roll inside of an accurately bored casing or shell. The bore of the casing is slightly greater than the diameter of the journal plus twice that of the rollers, so that the latter will roll freely when not on the side of the journal where the load forces it against them. Under some conditions a steel sleeve is used over the shaft.

The journal, rollers, and casings must all be accurately cylindrical, and have their axes parallel, in order to work correctly. It is not possible to keep the rollers parallel to the journal without some additional device for that purpose.

A very effective method is to use a slotted tube somewhat longer than the rollers and bored to have a free running fit on the journal; the slots are cut longitudinally as long as the rollers, and each is wide enough to allow a roller to drop into and turn freely in it; a complete ring of metal is left at the ends of the slots, so that the whole slotted tube forms a rigid "cage" for holding the rollers in position. Such a cage is shown in Fig. 49.1. The cage rotates less than half as fast as the journal.

SLEEVE   CAGE AND ROLLERS   SHELL
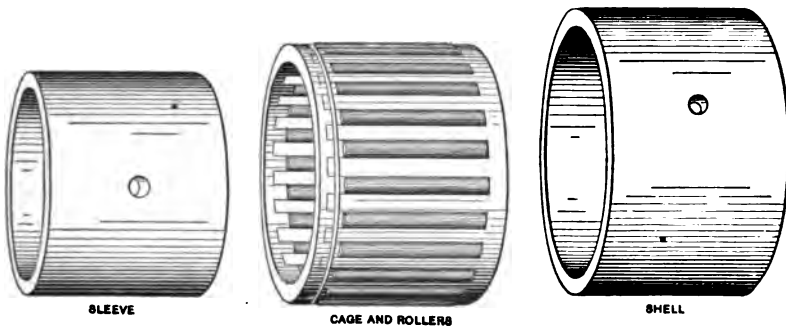
FIG. 49.1.*

The rollers rub against the sides of the slots with a pressure that depends on the accuracy of construction and, after the bearing has been in use for some time, the durability of the materials used. If any of the bearing surfaces wear out of their true cylindrical form the roller will have a tendency to twist about out of line with the shaft and to move lengthwise. This will cause an

* Made by Mossberg & Granville Mfg. Co , of Providence, R. I

end pressure of the roller against the cage at the end of the slot. Rapid wear and the early destruction of the bearing are apt to follow.

Various devices other than the plain slotted cage are used for, retaining the rollers in position. One method is to drill a hole in each end of the roller and insert a ball after the manner of Fig. 39.1. The cage used in connection with this device consists of a pair of rings each with as many countersinks as there are rollers in the bearing. One of the rings is placed at each end of the rollers so that the balls are in the countersinks. The two rings are then fastened together with rods in such a way as to hold the rollers in place by the balls fitting into the countersinks. The ball is sometimes let into the ring and the rolls countersunk at the ends to bear against the balls. Another method is to groove the ends of each roller circumferentially and place two balls between each adjacent pair of rollers, one ball at each end of the rolls, so that the balls run in grooves. Twice as many balls as rollers are used. The balls are in turn held in place by retaining rings at each end of the bearing. These rings are rigidly fastened together. Still another method is to place small rollers between the ones that carry the load. These small separating rollers are kept in place by a retaining cage or other device.

There is another trouble that is liable to occur if a roller gets out of alignment, especially if it is long and the material is brittle. As soon as the roller gets out of position the line of its contact with the journal is curved instead of straight. The roller must therefore bend and become liable to fracture by the bending. A broken roller is almost certain to cause rapid destruction of the bearing. Several short rollers, lying end to end in the same slot of the cage, are sometimes used for long bearings; also a special form of flexible roller (see Fig. 50).

Solid rollers of tool steel hardened and ground to form and running upon bearing surfaces of the same material are successfully used for heavy pressures and high speeds; mild steel case-hardened and ground has also proved satisfactory. Mild steel and tool steel unhardened are used to a large extent. Wrought iron (puddle iron) and cast iron are unsuitable for any service but the lightest pressures and slow speeds, and are uncertain for that.

The frictional resistance of a properly constructed roller-bearing is exceedingly low, especially at slow speeds.

A journal supported on rollers starts from rest with a very small proportion of the effort required for a journal-bearing, even of the best construction.

The wear is very slight in a well-constructed roller-bearing of durable materials, hence its friction remains low for a very long time—as long as the wear is inappreciable. In poor bearings with consequent rapid wear the friction increases rapidly and reaches a high value before they are worn out.

Roller-bearings should always be lubricated with a liquid lubricant. A single application lasts a long time, as long as two years or more on shafting at 400 revolutions per minute or slower speeds, when an oil pocket is provided.

Great improvement has been made in the construction of roller-bearings since their extensive use began in recent years. This has been accomplished by more careful construction and the use of suitable materials. Roller-bearings that are thoroughly reliable are now obtainable.

The avoidance of rapid wear in some classes of machinery where accurate adjustment is necessary under high pressure, as in calenders and cold-metal rolling-machines, is often of equal or greater importance than saving of power. Both have been successfully obviated in practice by the use of roller-bearings.

The load which a roller-bearing will carry is greater for high speeds than can be put on the common journal-bearing with sliding surfaces, practically the same for medium speeds, and less for very low speeds.

The safe load in pounds for a well-made solid roller journal-bearing with six or more rollers is

$$P = 100,000\ D^2\ \frac{NL}{3S} \quad \begin{array}{l} \text{(for values of } S \text{ not less} \\ \text{than 50 feet per minute)}; \end{array}$$

in which D = diameter of rollers, inches;

L = length of one roller, inches;

N = number of rollers;

P = total safe load or pressure on bearing, pounds;

S = linear velocity of convex bearing surface relatively to concave bearing surface, feet per minute.

For values of S less than 50 feet per minute the pressure may be taken the same as for $S = 50$. For exceedingly slow speeds the pressure may be taken somewhat higher, however.

This equation is based upon the assumption that the safe load per inch of total effective length of rollers is 2,000 D^2 pounds, considering one-third of the rollers under pressure.

Fig. 49.2 and Table VIA show the form and give the dimensions and loads for solid roller journal-bearings that have been success-
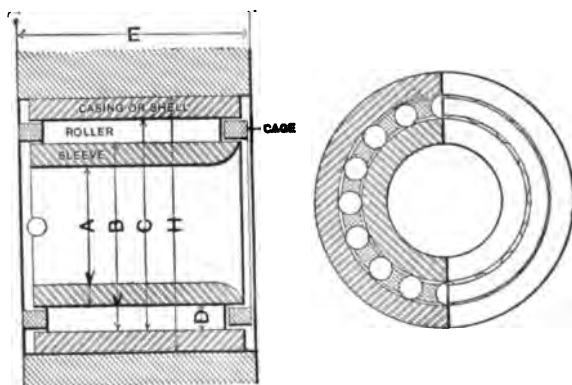
FIG. 49.2.

ful in practice. They refer to the bearing illustrated in Fig. 49.1. The effective length of the rollers is taken as one-third of the aggregate length of all the rollers in the bearing.

A roller journal-bearing for a 6-inch shaft has the following values given in Table VIA: $D = 11/16$ inch; $NL/3 = 72$; and, in column B, diameter of convex bearing surface = 7 inches, corresponding to a circumference of 22 inches = 1.83 feet.

At 200 revolutions per minute the linear velocity of the convex bearing surface is $S = 200 \times 1.83 = 366$ feet per minute.

The safe load for this bearing is, by the preceding equation,

$$P = 100,000 \left(\frac{11}{16}\right)^2 \times \frac{72}{366} = 9300 \text{ pounds.}$$

This value comes within less than one per cent. of that given in the table.

TABLE VIa.

SAFE LOADS FOR ROLLER JOURNAL-BEARINGS OF THE TYPE FIG. 49.2.

Diameter of Shaft or Bore of Sleeve, Inches.	Outer Diameter of Sleeve, Inches.	Inner Diameter of Casing or Shell, Inches.	Outer Diameter of Casing or Shell, Inches.	Length of Bearing, Inches.	Diameter of Rollers, Inches.	Number of Rollers.	Length of Each Roller, Inches.	Effective Length of All Rollers, Inches.	Safe Pressure on Journal, Pounds, Total = P.			
A	B	C	H	E	D	N	L	$\frac{NL}{3}$	Rev. per Min. 0–50	Rev. per Min. 200	Rev. per Min. 400	Rev. per Min. 800
¾	¾	1½	1½	1¾	9/16	8	1.25	3.33	233	233	150	75
7/16	15/16	1⁵⁄₁₆	1⁹⁄₁₆	1¾	9/16	10	1.25	4.16	290	290	180	90
½	1	1⅝	2	2¼	¼	10	1¹¹⁄₁₆	5.63	700	700	350	175
⅝	1⅛	1⅝	2¼	2¼	¼	10	1¹¹⁄₁₆	5.63	700	700	350	175
¾	1¼	1¾	2¼	2¼	¼	10	2.00	6.66	830	700	350	175
⅞	1¾	1⅞	2¾	2½	¼	12	2.00	8.00	1,000	800	400	200
1	1½	2	2¼	2¾	¼	14	2.5	11.66	1,460	900	450	225
1⅛	1⅝	2¼	2⅝	2¾	¼	14	2.5	11.66	1,460	900	450	225
1¼	1¾	2¼	2¾	3	¼	16	2.5	13.33	1,660	830	415	210
1⅜	1⅞	2⅛	2⅞	3	¼	16	2.5	13.33	1,660	830	415	210
1½	2	2½	3	3¼	¼	18	2¾	16.5	2,100	1,000	500	250
1⅝	2⅛	2⅝	3¼	3¼	¼	18	2.75	16.5	2,100	1,000	500	250
1¾	2¼	2¾	3¼	3½	¼	20	3.00	20.0	2,500	1,060	530	265
1⅞	2⅜	2⅞	3⅜	3½	¼	20	3.00	20.0	2,500	1,060	530	265
2	2½	3	3½	3¾	¼	22	3.25	23.63	2,960	1,110	555	280
2⅛	2⅝	3½	4	4¼	5/16	20	3.50	23.33	4,630	1,630	815	410
2¼	2⅞	3½	4¼	4½	5/16	20	3.50	23.33	4,630	1,630	815	410
2⅜	3	3⅝	4¼	4½	5/16	20	3.75	25.0	4,900	1,600	800	400
2½	3⅛	3¾	4½	4½	5/16	22	3.75	27.5	5,390	1,650	825	415
2⅝	3¼	4	4⅝	5	5/16	22	4.25	31.16	6,100	1,850	925	460
2¾	3⅜	4	4⅝	5	5/16	22	4.25	31.16	6,100	1,850	925	460
2⅞	3⅝	4⅜	5⅛	5¼	⅜	22	4.5	33	9,240	2,540	1,270	635
3	3¾	4½	5¼	5⅝	⅜	22	4²³⁄₃₂	34.6	9,690	2,540	1,270	635
3⅛	3⅞	4½	5½	5⅝	⅜	22	4²³⁄₃₂	34.6	9,690	2,540	1,270	635
3¼	4	4¾	5½	6	⅜	24	5.0	40	11,200	2,800	1,400	700
3⅜	4⅛	4⅞	5⅝	6	⅜	24	5.0	40	11,200	2,800	1,400	700
3½	4⅛	5	5¾	6¼	7/16	24	5.5	44	11,200	2,800	1,400	700
3⅝	4½	5¼	6¼	6¼	7/16	24	5.5	44	11,200	2,800	1,400	700
3¾	4⅝	5½	6⅜	6¾	7/16	24	5.5	44	11,200	2,800	1,400	700
3⅞	4¾	5⅝	6½	7	7/16	24	5.75	46	14,000	3,500	1,750	875
4	4⅞	5¾	6⅝	7	7/16	24	5.75	46	14,000	3,500	1,750	875
4⅛	5	6	6⅞	7½	½	24	6.25	50	18,750	4,690	2,345	1,170
4¼	5¼	6⅛	7	7½	½	24	6.25	50	18,750	4,690	2,345	1,170
4⅜	5¼	6¼	7½	8	½	24	6.75	54	19,600	4,900	2,450	1,225
4½	5⅝	6¼	7¼	8	½	24	6.75	54	19,600	4,900	2,450	1,225
4⅝	5½	6¼	7⅜	8¼	½	24	7.0	56	19,600	4,900	2,450	1,225
4¾	5⅝	6⅝	7½	8¼	½	24	7.0	56	19,600	4,900	2,450	1,225
4⅞	5⅞	7	8	8½	9/16	24	7.5	60	24,600	6,150	3,100	1,550
5	6	7½	8½	8½	9/16	24	7.5	60	24,600	6,150	3,100	1,550
5⅛	6¼	7⅜	8⅜	9½	⅝	22	8³⁄₁₆	60	30,400	7,600	3,800	1,900
5¼	6¼	7¼	8¼	9½	⅝	22	8³⁄₁₆	60	30,400	7,600	3,800	1,900
5⅜	6⅜	7⅝	8⅝	9½	⅝	22	8³⁄₁₆	60	30,400	7,600	3,800	1,900
5½	6¾	7¾	8¾	9¾	⅝	24	8.5	68	31,800	7,950	3,975	1,990
5⅝	6⅝	7½	8½	9¾	⅝	24	8.5	68	31,800	7,950	3,975	1,990
5¾	6½	8½	9½	10	11/16	24	8.5	68	35,200	8,800	4,400	2,200
5⅞	6¾	8¼	9¼	10	11/16	24	8.5	68	35,200	8,800	4,400	2,200
6	7	8⅜	9⅜	10½	11/16	24	9.0	72	37,400	9,350	4,675	2,340
6⅛	7¼	8½	9½	10½	11/16	24	9.0	72	37,400	9,350	4,675	2,340
6¼	7½	8⅝	9⅝	10½	11/16	24	9.0	72	37,400	9,350	4,675	2,340
6⅜	7⅞	8⅞	11½	11½	¾	22	9.25	72	37,400	9,350	4,675	2,340
6½	7⅝	9	10	11½	¾	22	9.25	72	37,400	9,350	4,675	2,340
6⅝	7⅞	9⅛	10⅛	11½	⅞	24	10.0	80	45,000	11,250	5,625	2,810
6¾	7¼	9¼	10¼	11½	⅞	24	10.0	80	45,000	11,250	5,625	2,810
6⅞	7⅞	9⅜	10¾	11½	¾	24	10.0	80	45,000	11,250	5,625	2,810
7	8⅛	9⅝	10⅝	12½	13/16	22	10.75	80	52,800	13,200	6,600	3,300
7⅛	8⅛	9¾	10¾	12½	15/16	22	10.75	80	52,800	13,200	6,600	3,300
7¼	8¼	9⅞	10⅞	12½	15/16	24	10.75	86	56,800	14,200	7,100	3,550
7⅜	8¼	10	11	12½	15/16	24	10.75	86	56,800	14,200	7,100	3,550
7½	8½	10½	11¼	12½	15/16	24	10.75	86	56,800	14,200	7,100	3,550

TABLE VIA.—*Continued.*

SAFE LOADS FOR ROLLER JOURNAL-BEARINGS OF THE TYPE FIG. 49.2.

Diameter of Shaft or bore of Sleeve, Inches.	Outer Diameter of Sleeve, Inches.	Inner Diameter of Casing or Shell, Inches.	Outer Diameter of Casing or Shell, Inches.	Length of Bearing, Inches.	Diameter of Rollers, Inches.	Number of Rollers.	Length of Each Roller, Inches.	Effective Length of All Rollers, Inches.	Safe Pressure on Journal, Pounds, Total = P.			
A	B	C	H	E	D	N	L	$\frac{NL}{3}$	Rev. per Min. 0-50	Rev. per Min. 200	Rev. per Min. 400	Rev. per Min. 800
7⅝	8⅝	10⅜	11⅜	13½	⅞	22	11.75	86	59,400	14,850	7,425	3,710
7¾	8¾	10½	11½	13½	⅞	22	11.75	86	59,400	14,850	7,425	3,710
7⅞	8⅞	10⅝	11⅝	13½	⅞	22	11.75	86	59,400	14,850	7,425	3,710
8	9	10¾	11¾	13½	⅞	22	11.75	86	59,400	14,850	7,425	3,710
8¼	9⅜	11¼	12½	14½	15⁄16	22	12.25	90	63,200	15,800	7,900	3,950
8¼	9⅜	11⅜	12⅝	14½	15⁄16	22	12.25	90	63,200	15,800	7,900	3,950
8⅜	9⅝	11½	12¾	14½	15⁄16	22	12.25	90	63,200	15,800	7,900	3,950
8½	9¾	11⅝	12⅞	14½	15⁄16	24	12.5	100	63,200	15,800	7,900	3,950
8⅝	9⅞	11⅞	13⅛	15	1	24	12.5	100	75,000	18,750	9,325	4,660
8¾	10	12	13¼	15	1	24	12.5	100	75,000	18,750	9,325	4,660
8⅞	10⅛	12⅛	13⅜	15	1	24	12.5	100	75,000	18,750	9,325	4,660
9	10¼	12¼	13½	15½	1	24	12.75	102	75,000	18,750	9,325	4,660
9¼	10½	12½	13¾	16	1	24	13.75	110	82,500	20,625	10,310	5,155
9½	10¾	12¾	14	16	1	24	13.75	110	82,500	20,625	10,310	5,155
9¾	11	13	14¼	16	1	26	15.0	130	91,000	22,750	11,325	5,660
10	11¼	13¼	14½	16	1	26	15.0	130	91,000	22,750	11,325	5,660
10¼	11¾	14	15½	16	1⅛	26	15.0	130	91,000	22,750	11,325	5,660
10½	12	14¼	15¾	18	1⅛	26	15⅝	136	103,200	25,800	12,900	6,450
10¾	12¼	14½	16	18	1⅛	26	15⅝	136	103,200	25,800	12,900	6,450
11	12½	14¾	16¼	18	1⅛	26	15¾	136	103,200	25,800	12,900	6,450
11½	13	15¼	16¾	18	1⅛	26	15⅝	136	103,200	25,800	12,900	6,450
12	13½	16	17½	18	1¼	26	15⅝	136	110,000	27,500	13,750	6,875
12½	14	16½	18	18	1¼	26	15⅝	136	110,000	27,500	13,750	6,875
13	14½	17	18½	18	1¼	28	15.0	140	119,900	29,975	14,990	7,495
13½	15	17½	19	20	1¼	28	17.0	160	125,000	31,250	15,625	7,810
14	15½	18	19½	20	1¼	28	17.0	160	125,000	31,250	15,625	7,810
14½	16	18½	20	20	1¼	30	17.5	175	136,500	34,125	17,100	8,550
15	16½	19¼	20¾	22	1¾	30	19.0	190	160,000	40,000	20,000	10,000
15½	17	19¾	21¼	22	1⅜	30	19.0	190	160,000	40,000	20,000	10,000
16	17¾	20½	22¼	22	1⅜	30	19.0	190	160,000	40,000	20,000	10,000
16½	18¼	21	22¾	22	1⅜	30	19.0	190	160,000	40,000	20,000	10,000
17	18¾	21½	23¼	22	1⅜	32	18.75	200	160,000	40,000	20,000	10,000
17½	19¼	22	23¾	22	1⅜	32	18.75	200	160,000	40,000	20,000	10,000
18	19¾	22½	24¼	22	1⅜	32	18.75	200	160,000	40,000	20,000	10,000
19	20¾	23¾	25½	24	1½	34	21.0	238	175,000	43,750	21,825	10,900
20	22	25	27	24	1½	36	21.0	252	175,000	43,750	21,825	10,900
22	24	27	29	26	1½	40	22.5	300	210,000	52,500	26,250	13,125
24	26	29	31	30	1½	42	27.0	380	256,500	64,125	32,000	16,000

A special roller, of which a group is shown in Fig. 50, has been designed to obviate the danger arising from the fracture of a roller,

FIG. 5ʋ.*

either from getting out of line or unevenness of the surfaces of the bodies in contact, and also to secure contact along the entire length of the roller even if the surfaces on which it rolls are not exactly true. It is made by winding a steel ribbon about a mandrel, in the same manner that a strip of paper may be wound about a round pencil, so that the edges of successive convolutions just clear each other. The cross-section of the ribbon is rectangular, and is varied according to the speed and load under which a bearing is to operate.

A box made up of these flexible steel ribbon rollers, as applied to a line shaft, is shown in Fig. 50.1. The cage, frame, or yoke is shown both in place among the rollers and removed from the box. In this particular box the cage is made up of slightly more than half an annular ring at each end, connected by three bars that are cast together with the part rings. In some designs steel stampings are used instead of castings. Some of the rollers lie in the spaces between the bars of the cage, and the others complete the circumference of the journal outside of the cage. In a box for a shaft about two inches in diameter, having fourteen rollers, four

* Made by the Hyatt Roller Bearing Co.

of them lie in each pocket of the cage and the remaining six outside of it. These six rollers are longer than the eight in the pockets.

FIG. 50.1.

In other styles of cages the rollers are all of the same length, and the cage is made up of two halves resembling the cage illustrated. The box is fitted with a pad at each end to prevent the oil from flowing out along the shaft.

Figs. 50.2 and 50.3 show a form of the Hyatt roller-bearing adapted to parts that have a hole bored in them to receive the box. The outer broken ring of sheet metal fits into the bore and forms a shell or casing inside of which the rollers run. A similar inside ring of metal, shown in Fig. 50.3, is used as a sleeve for iron shafts, or when the shaft is not suitable for the rollers to bear directly upon.

On account of the flexibility of the roller, hardened surfaces are not necessary for either the rollers or the bearing surfaces upon which they run. The shell and sleeve are made of sheet or

plate steel bent to form and used without finish by grinding or other
operation.

FIG. 50.2.

As to lubrication and the rubbing or grinding action of the
rollers against each other and the cage, the following statements
are made:

"The rollers do not grind against each other when under load
for the reason that they are separated, or, in other words, when
one reaches the zone of pressure it has a tendency to stop an instant
and separate from the preceding one.   The yokes or cages are usu-
ally made of very soft brass.   In bearings that have been in use
for years doing severe work, the file marks in the yokes are just as
plain apparently as the day the yoke was finished.   Many cases
are on record where these bearings have operated for two years on
shafting running at 400 revolutions per minute without attention
beyond the original lubrication, and without perceptible wear." *

Lubrication is especially effective on account of the reservoir
capacity of the rollers and the alternation of right- and left-hand

* Communication from Mr. Alfred P. Sloan, Jr., General Manager Hyatt
Roller Bearing Co., June, 1908.

TABLE VIB.

APPROXIMATE DIMENSIONS OF HYATT ROLLER JOURNAL-BEARINGS.

For speeds less than 50 rev. per minute. The diameter of roller and hence that of the casing vary for different speeds. The higher the speed the larger the roller. The ratio of length of bearing and diameter of shaft is practically the same as for other forms of journal bearings.

The outside diameter of casing is slightly smaller than the diameter of hole into which the bearing fits to give suitable clearance. On account of variation in the thickness of the steel the outside diameter of the casing is only approximate. The sleeve fits directly upon the shaft. It is intended for shafts of materials other than steel.

Diameter of Shaft.	Load, 200 Lbs. per Sq. In. of Projected Area of Journal.					Load, 400 Lbs. per Sq. In. of Projected Area of Journal.				
	Diam. of Roller.	Thickness of Sleeve.	Thickness of Shell.	Outside Diam. with Sleeve.	Outside Diam. with Sleeve and Shell.	Diam. of Roller.	Thickness of Sleeve.	Thickness of Shell.	Outside Diam. with Sleeve.	Outside Diam. with Sleeve and Shell.
1¼	½	⅟₁₆	⅛	2⅜	2⅝	⅝	⅟₁₆	⅛	2⅝	2⅞
1½	½	⅟₁₆	⅛	2⅝	2⅞	⅝	⅟₁₆	⅛	2⅞	3⅛
1¾	½	⅟₁₆	⅛	2⅞	3⅛	⅝	⅟₁₆	⅛	3⅛	3⅜
2	½	⅟₁₆	⅛	3⅛	3⅜	⅝	⅟₁₆	⅛	3⅜	3⅝
2¼	½	⅛	⅛	3½	3¾	⅝	⅛	⅛	3¾	4
2½	⅝	⅛	⅛	4	4¼	¾	⅛	⅛	4¼	4½
2¾	⅝	⅛	⅛	4¼	4½	¾	⅛	⅛	4½	4¾
3	¾	⅛	⅛	4¾	5	⅞	⅛	⅛	5	5¼
3¼	¾	⅛	⅛	5	5¼	⅞	⅛	⅛	5¼	5½
3½	¾	⅛	⅛	5¼	5½	⅞	⅛	⅛	5½	5¾
3¾	¾	⅛	⅛	5½	5¾	⅞	⅛	⅛	5¾	6
4	⅞	⅛	⅛	6	6¼	1	⅛	⅛	6¼	6½
4¼	⅞	⅛	⁵⁄₁₆	6¼	6⅝	1	⅛	⁵⁄₁₆	6½	6⅞
4½	⅞	⅛	⁵⁄₁₆	6½	6⅞	1	⅛	⁵⁄₁₆	6¾	7⅛
4¾	⅞	⅛	⁵⁄₁₆	6¾	7⅛	1	⅛	⁵⁄₁₆	7	7⅜
5	1	⅛	⁵⁄₁₆	7¼	7⅝	1⅛	⅛	⁵⁄₁₆	7½	7⅞
5¼	1	⅛	⁵⁄₁₆	7½	7⅞	1⅛	⅛	⁵⁄₁₆	7¾	8⅛
5½	1	⅛	⁵⁄₁₆	7¾	8⅛	1⅛	⅛	⁵⁄₁₆	8	8⅜
5¾	1	⅛	⁵⁄₁₆	8	8⅜	1⅛	⅛	⁵⁄₁₆	8¼	8⅝
6	1⅛	⅛	⁵⁄₁₆	8½	8⅞	1¼	⅛	⁵⁄₁₆	8¾	9⅛
6¼	1⅛	⅛	⁵⁄₁₆	8¾	9⅛	1¼	⅛	⁵⁄₁₆	9	9⅜
6½	1¼	⅛	⁵⁄₁₆	9¼	9⅝	1⅜	⅛	⁵⁄₁₆	9½	9⅞
6¾	1¼	⅛	⁵⁄₁₆	9½	9⅞	1⅜	⅛	⁵⁄₁₆	9¾	10⅛
7	1¼	⅛	⁵⁄₁₆	9¾	10⅛	1⅜	⅛	⁵⁄₁₆	10	10⅜

TABLE VIE.—*Continued.*

Diameter of Shaft.	Load, 600 Lbs. per Sq. In. of Projected Area of Journal.					Load, 800 Lbs. per Sq. In. of Projected Area of Journal.				
	Diam. of Roller.	Thickness of Sleeve.	Thickness of Shell.	Outside Diam. with Sleeve.	Outside Diam. with Sleeve and Shell.	Diam. of Roller.	Thickness of Sleeve.	Thickness of Shell.	Outside Diam. with Sleeve.	Outside Diam. with Sleeve and Shell.
1¼	¾	1⁄16	⅛	2⅞	3⅛	⅞	1⁄16	⅛	3⅛	3⅜
1½	⅞	1⁄16	⅛	3⅛	3⅜	1	1⁄16	⅛	3⅜	3⅝
1¾	¾	1⁄16	⅛	3⅜	3⅝	1	1⁄16	⅛	3⅝	3⅞
2	¾	1⁄16	⅛	3⅝	3⅞	1	1⁄16	⅛	3⅞	4⅛
2¼	¾	⅛	⅛	4	4¼	1	⅛	⅛	4¼	4½
2½	⅞	⅛	⅛	4½	4¾	1	⅛	⅛	4½	5
2¾	⅞	⅛	⅛	4¾	5	1	⅛	⅛	5	5¼
3	1	⅛	⅛	5¼	5½	1⅛	⅛	⅛	5½	5¾
3¼	1	⅛	⅛	5½	5¾	1⅛	⅛	⅛	5¾	6
3½	1	⅛	⅛	5¾	6	1⅛	⅛	⅛	6	6¼
3¾	1	⅛	⅛	6	6¼	1⅛	⅛	⅛	6¼	6½
4	1⅛	⅛	⅛	6¼	6¾	1¼	⅛	⅛	6¾	7
4¼	1⅛	⅛	⅛	6¾	7⅛	1¼	⅛	3⁄16	7	7⅜
4½	1⅛	⅛	⅛	7	7¾	1¼	⅛	3⁄16	7¼	7½
4¾	1⅛	⅛	⅛	7¼	7⅝	1¼	⅛	3⁄16	7½	7⅞
5	1¼	⅛	3⁄16	7¾	8⅛	1¾	⅛	3⁄16	8	8¼
5¼	1¼	⅛	3⁄16	8	8⅜	1¾	⅛	3⁄16	8¼	8⅝
5½	1¼	⅛	3⁄16	8¼	8⅝	1¾	⅛	3⁄16	8½	8⅞
5¾	1¼	⅛	3⁄16	8½	8⅞	1¾	⅛	3⁄16	8¾	9⅛
6	1¾	⅛	3⁄16	9	9¾	1½	⅛	3⁄16	9¼	9⅝
6¼	1¾	⅛	3⁄16	9¼	9⅝	1½	⅛	3⁄16	9½	9⅞
6½	1½	⅛	3⁄16	9¾	10⅛	1⅝	⅛	3⁄16	10	10⅜
6¾	1½	⅛	3⁄16	10	10⅜	1⅝	⅛	3⁄16	10¼	10⅝
7	1½	⅛	3⁄16	10¼	10⅝	1⅝	⅛	3⁄16	10½	10⅞

spirals, the latter serving to distribute the oil from end to end of the bearing.

The dimensions that are used for roller-bearings of the form of Fig. 50.2 are given in Table VIB for convenience of reference and to show what loads may be used for speeds not greater than 50 revolutions per minute.

Table VII gives the results of tests made by Wm. Sellers & Co. on several Hyatt roller-bearings.

TABLE VII.

TESTS OF HYATT ROLLER-BEARINGS.*

Coefficient of friction *μ* given in the body of table.

Dimensions of Bearings.	Total Load, Pounds.	Revolutions per Minute.				
		5	25	48	128	214
		μ	μ	μ	μ	μ
Diameter of journal, 1 15/16". Length of bearing, 3". Bearing bored .009" smaller than sum of two liners, two rollers, and shaft.	1,000 2,000 3,000	....	....	.16964 .08974 .06739		
Diameter of journal, 1 15/16". Length of bearing, 3". Bearing bored .004" larger than sum of two liners, two rollers, and shaft.	1,000 2,000 3,000	.02958 .01874 .01249	.01578 .00986 .00789	.00789 .01080 .01020	.00785 .00789 .00789	.00789 .00592 .00526
Diameter of journal, 8". Length of journal, 12". Bore of bearing .023" larger than sum of two liners, two rollers, and shaft.	10,000 20,000 30,000 40,000 50,000	.02399 .01923 .01856 .01805 .01708	....	.03826 .02386 .01959 .01795 .01692	.02540 .01560 .01280 .01240	
Hyatt commercial, 2" shafting-box.	500 1,000 1,500 2,000	.03156 .02762 .01841 .01282	.03156 .01973 .01710 .01775	....	.02367 .01578 .01578 .01578	.01972 .01381 .01315 .01282

* *Age of Steel*, April 10, 1897, p. 17; *American Machinist*, June 24, 1897, p. 20.

Two street-railway cars, one fitted with Hyatt roller-bearings and the other with common journal-bearings, were tested at Providence, R. I. The roller-bearings showed a saving of 13% of the total power used to drive the car with common bearings.*

Fig. 50.3 is a flexible roller journal-bearing for heavy duty. It is 24 inches in diameter inside and 12 inches long over all. There are 54 rollers 1½ inches in diameter and 11½ inches long. The rollers are held in position by a cage with rods passing through every sixth roller.

* *American Machinist*, Oct. 21, 1897, p. 794.

24 × 12 in. special bushing for heavy duty.
Safe working load 50,000 lbs. at 60 revolutions per minute.
FIG. 50.3.
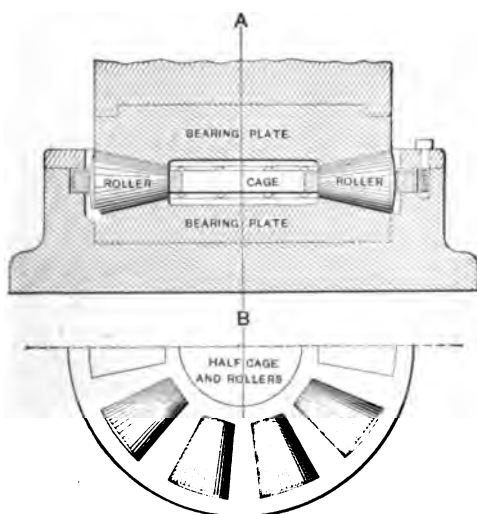
FIG. 51.

Roller Thrust-bearings.

23. Conical roller-bearings are used to resist the end thrust of a shaft or other part, performing the same function as the more common forms of collar-bearings and of step-bearings already described. Fig. 51 shows the typical form of such a bearing. The rollers are truncated cones of such a taper and so placed that their vertices, as well as those of the conical surfaces on which they roll, all coincide on the axis *AB* of the shaft. When so constructed there is true rolling action between the parts; slipping is entirely absent. It is not necessary that the rollers shall have their axes at right angles to that of the shaft, but they may be inclined at any angle within practical limits. It is often convenient to make either the step or the end of the shaft flat on the surface where the cones roll and the other bearing surface coned to suit the rollers.

On account of the taper form of the rollers, there is a tendency to force them out radially from between bearing plates.

The rollers should be small in comparison with the diameter of the bearing in order to keep their tendency to move out radially as small as possible. It would hardly be advisable to make the apex angle, embraced between two diametrically opposite elements of the conical surface, greater than 15° in any case; 10° or less is a more suitable value. It should not be forgotten, however, that the rolling resistance of the cones increases as the diameter decreases.

In order to prevent radial movement of the rollers, and also to hold them from twisting around so that their axes do not intersect that of the shaft, a retaining "cage" of some form is used. Such a cage is shown in Fig. 51. It is a very simple design, but not the best form for heavy service. As shown it consists of a flat ring perforated to form pockets to receive the rollers. The conical surface of each roller has a snug-running fit in its pocket. By making the crowned outer end of the roller more convex than the corresponding end of the pocket, the frictional resistance is kept lower than if these rubbing surfaces have the same radius of curvature. The cage has a free-running fit in the thrust-block or step of the bearing.

The angular speed of rotation of the cage is half that of the moving bearing plate when the bearing plates have equal angles.

Figs. 51.1 and 51.2 show a conical roller thrust-bearing whose dimensions, loads, and speeds are given in Table VIIB. This bearing is guaranteed to work under pressures as high as 5000 pounds per square inch on the bearing surface of one bearing ring at slow

FIG. 51.1.

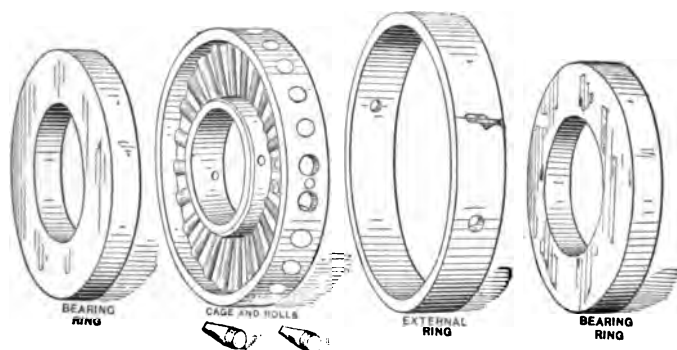speeds. At high speeds the bearing pressures must, of course, be lower. Each roller is coned on the part that comes into contact
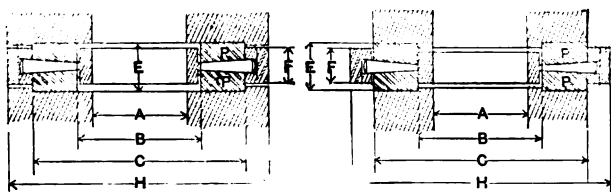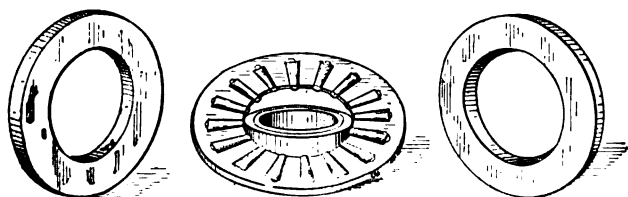

FIG. 51.2.

FIG. 51.3.

with the bearing rings. The larger end is cylindrical beyond the bearing surface, and of a diameter equal to that of the larger end of the cone frustum. The cage is one solid piece of metal bored radially with combined conical and cylindrical holes to fit

TABLE VIIB.

SAFE LOADS FOR CONICAL ROLLER THRUST-BEARINGS OF THE TYPE FIG. 51.2.

If the thrust-box or thrust-block is large enough to fit over plates P, use construction 1; if not, use construction 2.

Diameter of Shaft. Inches.						Number of Rollers.	Angle of Rollers. Deg.	Projected Bearing Area of Plate P. $\frac{\pi}{4}(C^2 - B^2)$.	Safe Pressure on Bearing. Pounds.	
A	B	C	H	E	F	N		Sq. In.	Rev. per Min. 0-50	Rev. per Min. 100
1 to 1¼	1⅝	2⅝	3¼	¾	½	28	6½	3.338	420	420
1⁵⁄₁₆ " 1½	1⅞	3	3⅝	1³⁄₁₆	⁹⁄₁₆	28	6½	4.307	600	525
1⁹⁄₁₆ " 1¾	2⁷⁄₁₆	3⁹⁄₁₆	4⁵⁄₁₆	⅞	⅝	30	6½	6.21	1,113	835
1¹³⁄₁₆ " 2	2½	4	4⅞	⅞	⅝	30	6½	7.66	1,530	960
2¹⁄₁₆ " 2¼	2¹³⁄₁₆	4⁹⁄₁₆	5⁹⁄₁₆	1⁵⁄₁₆	1¹⁄₁₆	30	6	10.137	1,940	1,000
2⁵⁄₁₆ " 2½	3⅛	5	6⅛	1	¾	30	6	11.966	2,520	1,260
2⁹⁄₁₆ " 2¾	3½	5⅝	6⅝	1⅛	⅞	30	6	15.23	3,340	1,670
2¹³⁄₁₆ " 3	3¹³⁄₁₆	6⁹⁄₁₆	7⅛	1⅛	⅞	30	6	19.83	4,650	2,325
3¹⁄₁₆ " 3¼	4¹⁄₁₆	6⁹⁄₁₆	8	1¼	1	30	6	20.862	5,120	2,560
3⁵⁄₁₆ " 3½	4⅜	7⅛	8¹¼₆	1½	1¼	30	6	24.84	5,600	2,800
3⁹⁄₁₆ " 3¾	4⅝	7⅞	8¹⁵⁄₁₆	1⅝	1³⁄₈	30	6	28.86	6,900	3,450
3¹³⁄₁₆ " 4	4⅞	7⅞	9½	1⅝	1³⁄₈	30	6	30.5	7,500	3,750
4¹⁄₁₆ " 4¼	5¼	8½	10⅝	1¾	1½	30	6	35.097	8,000	4,000
4⁵⁄₁₆ " 4½	5½	8⅞	10¹³⁄₁₆	1⅞	1⅝	30	6	38.104	9,100	4,550
4⁹⁄₁₆ " 4¾	5¾	9¼	11⁵⁄₁₆	1⅞	1⅝	30	6	41.234	10,000	5,000
4¹³⁄₁₆ " 5	6	10	11⅞	1⅞	1⅝	30	6	50.266	11,000	5,550
5¹⁄₁₆ " 5¼	6⅝	10⁷⁄₁₆	12⅜	2¹⁄₁₆	1¾	30	6	54.2	11,850	5,925
5⁵⁄₁₆ " 5½	6¾	10⅞	12⅞	2¹⁄₁₆	1¾	30	6	57.101	12,500	6,250
5⁹⁄₁₆ " 5¾	7	11⁵⁄₁₆	13⁷⁄₁₆	2⁵⁄₁₆	2	30	6	62.52	14,900	7,450
5¹¹⁄₁₆ " 6	7¼	11¾	14	2⁵⁄₁₆	2	30	6	67.148	16,900	8,450
6¹⁄₁₆ " 6½	7¾	12⅝	14⅞	2⁷⁄₁₆	2¼	30	6	78.017	18,900	9,450
6⁹⁄₁₆ " 7	8¼	13½	15¾	2⁹⁄₁₆	2¼	30	6	89.684	22,400	11,200
7³⁄₁₆ " 7½	9	14⅝	17	2¾	2⁷⁄₁₆	32	6	104.373	25,600	12,800
7⁷⁄₁₆ " 8	9½	15½	17⅞	2⅞	2⁹⁄₁₆	32	6	117.808	31,600	15,800
8¹⁄₁₆ " 8½	10	16³⁄₁₆	18¾	3	2⅝	32	6	132.96	36,600	18,300
8¹⁄₁₆ " 9	10½	17¼	19⅝	3¼	2¾	32	6	147.12	42,000	21,000
9¹⁄₁₆ " 9½	11	18⅛	20½	3¼	2⅞	32	6	162.98	48,000	24,000
9⁹⁄₁₆ " 10	11½	19	21½	3⅜	3	32	6	179.6	54,000	28,000

the rollers. The radial thrust of the rollers is taken by the external ring. The apex angle of the rollers is generally 6°; never more than 6½° in this design.

Fig. 51.3 illustrates a thrust-bearing for comparatively light duty not exceeding a bearing pressure of 250 pounds per square inch of bearing surface when the speed is as high as 250 revolutions per minute.

Another form of cone-roller thrust-bearing is shown in Fig. 51.4. The radial pressure of the roller is resisted by a ball between two conically cupped surfaces. The apex angle of the rollers does not

Fig. 51.4.   Fig. 51.5.

exceed 15° in this design as manufactured. The stock forms of rollers are shown in Fig. 51.4a.

A case is cited where a step-bearing with conical rollers working under a pressure of 104.5 pounds per square inch had a coefficient of friction $\mu = .0025$ when running on steel.*

23.1. Cylindrical roller thrust - bearings are used to a considerable extent, and for very heavy duty in some cases. Such a bearing is shown in Fig. 51.5. The bearing plates have flat surfaces. The rollers are, of course, all of the same diameter. Several rollers are placed end to end in the same pocket. The pockets are at different distances from the centre of the bearing, so that the rollers will travel in different paths and have less tendency to wear grooves in the bearing plates than if several rollers followed the same path. The axes of the rollers are radial to the bearing. The ends of the rollers are crowned.

A bearing 22 inches outside diameter of the type Fig. 51.5 has been successfully tested under 80,000 pounds pressure at 300 revolutions per minute.† Another 18 inches outside and 8 inches inside diameter has been operating 800 days of 16 hours under a

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* *Cassier's Magazine*, May, 1897, p. 66.   Several forms of roller-bearings are illustrated and described in this article.
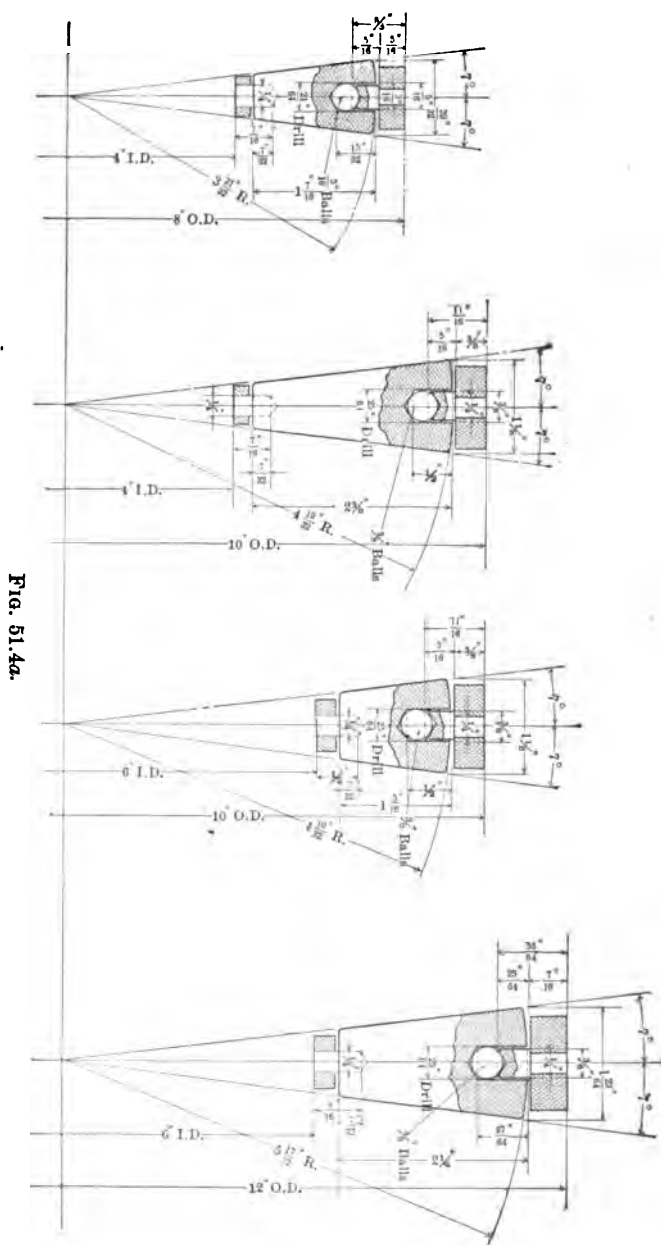
† *Machinery*, April, 1903, page 898.

FIG. 51.4a.

constant pressure of 100,000 pounds at 125 revolutions per minute. The parts were of case-hardened steel in both bearings.*

In thrust - bearings with cylindrical rollers running on flat plates there is a combined rolling and slipping or spinning action between the rollers and plates. The natural path of the roller is straight. It is made to follow a circular path by the cage. Each time a roller passes over its complete circular path it spins around once, in relation to the plate, in addition to rolling over the latter a distance equal to the mean circumference of its path. With the very short rollers used in practice the rubbing of the rolling surfaces over each other is of small consequence.

The ends of the rollers should be well rounded at the edges to prevent small pieces from breaking off.

The wear of the bearing surfaces of cylindrical roller thrust-bearings operating under heavy pressure is first shown by a rough appearance of the surfaces. The roughness is caused by the flaking off of a thin layer or skin of the metal. It is readily perceptible to both the eye and touch. Once started, rapid destruction is almost certain to follow. There is also a wear of the outer end of the roller against the cage due to the pressure of the latter necessary for guiding the roller in its path against its tendency to move in a straight line, and, at high speeds, some appreciable centrifugal action.

A bearing with 184 cylindrical rollers each $\frac{1}{2}$ inch in diameter by $\frac{1}{4}$ inch long running on a path $10\frac{3}{8}$ inches outside and $4\frac{5}{8}$ inches inside diameter, on a pair of bearing plates 1 inch thick and 11 inches diameter, lasted well. It was kept in service till the plates were worn away unevenly to an average depth of about $\frac{1}{30}$ of an inch.

The cage pockets at the outer radius were worn most on the side next the centre of the bearing. This was caused by the tipping of the roller as the depression was worn in one plate while the other remained flat, and its tendency to crowd toward the centre on account of the sloping edge of the depression. Some of the pockets were worn through into the next one radially. Many of the rollers were fractured. The bearing was used for passenger-

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* *Machinery*, May, 1908, page 489.

elevator service.   The thrust on it when the elevator was loaded
and at rest or moving uniformly was about 4500 pounds.   The
entire weight to be accelerated, including the counterweight to
the passenger cage, etc., was about 9500 pounds.   This was
brought up to a speed of from 350 to 400 feet per minute within
a distance of 20 feet in regular service.   The full speed of rotation
was more than 350 revolutions per minute.   The case is cited to
show how much such a bearing can be worn and still operate.

By supporting a step- or thrust-bearing on a spherical base, the
pressure may be kept uniformly distributed among the rollers.

The combined speeds and pressures that roller thrust- and step-
bearings will carry are from four or more times as great as for ball
bearings occupying the same space, except for very low speeds.

Ball Bearings.

**23.2. Hardened steel balls rolling on hardened steel sur-
faces** are used for machine bearings to a considerable extent.
Their chief field is for light service, especially for light pressures.
They have also been used successfully under both high speed and
pressure with certain forms of bearing surfaces.   For such duty
the material must be extremely and uniformly hard, yet strong
and free from microscopic flaws and cracks, and the surfaces
must be highly polished without scratches.   Accuracy of form is
also essential.   The same is desirable for light service, but is not
so absolutely necessary.

Wear is not to be taken into consideration in the design of
ball bearings for heavy service, and no adjustments for it are
required.   When wear becomes perceptible, the life is practically
ended.   On account of the great number of failures that have
occurred in improperly made ball bearings, there is often undue
prejudice against them, even for light service.   In a great many
of the forms there is not pure rolling motion between the ball and
race, but a combined rolling and spinning motion instead.   On
account of the localization of pressure upon a small area at the
place of contact, the sliding or rubbing action due to spinning is
apt to produce abrasion even under moderate pressures; it is
almost certain to do so under heavy loads on the bearing.   In

ball bearings for heavy service pure rolling motion is especially desirable.

23.3. Ball journal-bearing with two-point contact.—Fig. 51.6 shows a ball bearing in which the balls have a pure rolling action
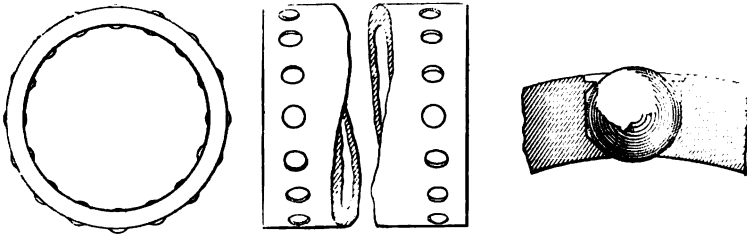
FIG. 51.6.

on cylindrical bearing surfaces. The ball cage is a hollow circular cylinder with radial perforations which serve as ball pockets. The latter are so distributed that no two balls travel in the same path. A simple method of preventing the balls from falling out of the cage when the latter is removed from between the bearing surfaces is shown. The pockets are made by drilling radially toward the axis of the cage with an ordinary cone-pointed drill. By stopping the drill feed before the lips have passed completely through the shell, a flange of metal is left that prevents the ball from falling inside the cage when put into the pocket. After the ball is in the pocket the outer edge of the hole is pressed down by a hollow punch or set so as to flange in the outer end of the pocket and thus retain the ball.

This form of bearing offers no resistance to thrust and end motion. It is easily made without special appliances, and is very useful.

23.4. Ball journal-bearing with three-point contact.—Fig. 51.7 is a bearing in which all the balls travel over the same path on the races. Each race is a solid ring. One is cylindrical and the other grooved. Both sides of the groove should make the same angle with the centre line of the bearing, so that the two points of contact of a ball against its sides shall be equidistant from the axis of the bearing. A ball then has pure rolling on the cylindrical surface. In the groove it has a combined rolling and spinning

motion, the latter about the axis passing through the points of contact a and b. This form of bearing does not resist end motion except to the extent of the frictional resistance to sliding between the balls and the cylindrical bearing surface.
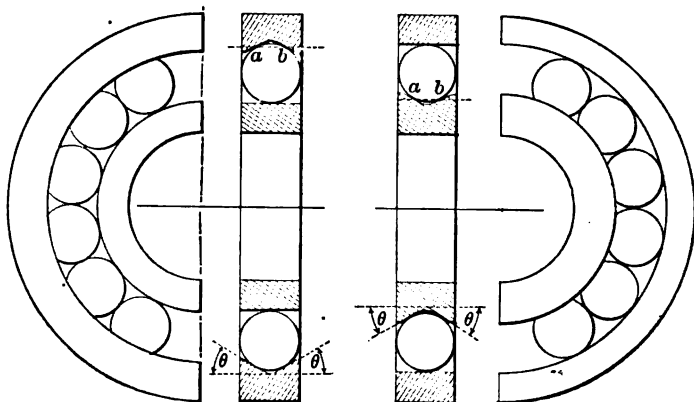
FIG. 51.7.   FIG. 51.8.

If the angle θ between the sides of the groove and a parallel to the axis of the bearing is too small, the ball is apt to bind or cramp between the cylindrical bearing surface and one side of the groove, especially if there is any end motion of the shaft. Such cramping generally results in the fracture of one of the parts, usually the ball. The more highly finished the surfaces, the smaller the angle θ that can be safely used. Probably 25° is as small as is safe, and 30° is advisable.

The normal pressure of a ball against each side of the groove (Fig. 51.7) is found by the expression

Pressure against each side of groove = ($\frac{1}{2}$ Radial pressure on ball) sec θ

Fig. 51.8 is similar to the preceding one, except that the inside ring is grooved instead of the outside one.

23.5. Ball journal-bearing with grooved race, two-point contact. —Fig. 51.9 is a form of bearing which has proved satisfactory both experimentally and in commercial and military service. The balls of a single unit all roll in the same grooves of a pair of races. The profile of the groove is an arc of a circle with a radius

somewhat greater than that of the ball. One of the races or bearing rings is a single piece. The other is made up of three parts: the ring itself, a small removable piece, and the screw for
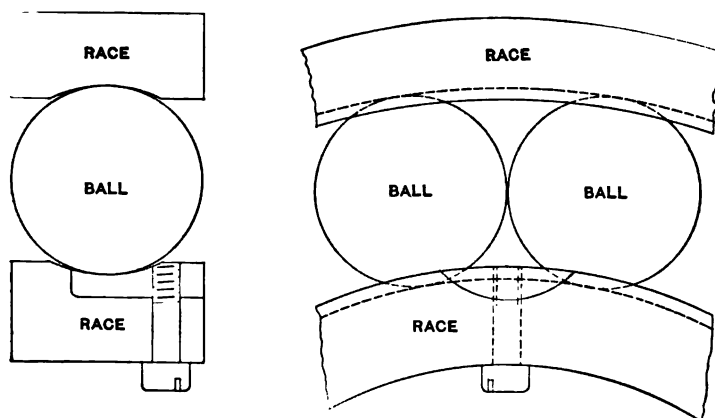
FIG. 51.9.

holding the latter in place. The removable piece on one ring is necessary for introducing the balls and taking them from the bearing. It should be placed either on the inner or outer race where no pressure due to the load comes upon it. This form of bearing will resist end thrust such as comes on the hubs of automobiles, on engine shafts, etc., as well as carrying the load normal to the axis of the bearing.

Professor Stribeck * gives for this form of bearing, when the radius of the grooves is $9/8$ times that of the ball,

Safe load in kilograms for one ball $\Big\} = 150 \times (\text{diameter of ball in centimetres})^2$

and, if $z =$ the number of balls when there are not less than ten,

†Safe load in kilograms on bearing $\Big\} = \dfrac{z}{5} 150 \times (\text{diam. of balls in centimetres})^2$.

By expressing these values in pounds and inches, and using the following notation for a ball journal-bearing of the form Fig. 51.9,

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* Glasers Annalen für Gewerbe und Bauwesen, 1901.

† See tables for safe loads, pages 420–420e.

P = total safe load on ball journal-bearing, pounds;
N = number of balls when not less than ten;
d = diameter of balls, inches,

the following approximate expressions are deduced:

Safe load in pounds for one ball = $2100d^2$ pounds (d in inches)

and

$$P = 420Nd^2 \text{ pounds.}$$

The loads for bearings with 14 balls are given in Table VIIA in accordance with the last formula.

TABLE VIIA.

SAFE LOADS FOR BALL BEARINGS WITH FOURTEEN BALLS OF THE
TYPE FIG. 51.9.

Diameter of balls, inches	$\frac{1}{2}$	$\frac{3}{4}$	1	$1\frac{1}{2}$	2
Load, pounds	1,470	3,300	5,880	13,200	23,500

23.6. Ball thrust-bearings, two-point contact. — The end pressure or thrust of a shaft may be taken by balls placed between two flat circular plates or disks whose planes are perpendicular to the axis of rotation. The plates must be hardened and ground to a smooth, accurate surface. A retaining cage is generally used for keeping the balls in position. If no cage is used, and there are enough balls to cover the bearing surfaces of the plates, when the radius of the latter is several times the diameter of the balls the latter crowd hard together and offer much frictional resistance to the motion of the parts. The crowding is due to the tendency of each ball to roll on the bearing surfaces in a straight path normal to a line joining the centre of the ball and axis of the bearing. This tendency is the same as with a short roller. Since the balls all try to move away from the centre of the disks, they crowd together when stopped by the casing that necessarily surrounds the bearing.

By the use of a retaining cage with individual pockets for the balls, the frictional resistance of the bearing and the wear of the balls are both reduced. Such a cage may be made by perforating a disk with circular holes as in Fig. 51.10. After a ball is in its pocket, the ends of the hole may be flanged in so as to retain it.

This may be done by the use of a cup-end punch or die, as already described for the cylindrical journal-bearing. No two balls should travel in the same path. The cage should be of some comparatively soft material such as brass or bronze. The motion of a ball relatively to a bearing disk is combined rolling and spinning about an axis through the two points of contact with the disks. The pressure of the balls against the cage in a direction radial to the bearing plates is apt to be so great as to wear the pockets unduly in heavy service. By the use of conical bearing surfaces similar to those for a cone-roller thrust-bearing, for the balls to roll upon, the radial pressure and spinning of the balls are both reduced, always provided that the bearing plates do not differ so much.

FIG. 51 10.

from flat surfaces as to cause the balls to slip, not roll, out radially on account of the wedge-like action of the divergent surfaces. By coning the plates so that the two apexes are at the same point on the axis of the bearing (the condition for cone-roller bearings) the tendency of the ball is to roll in a circle about the axis of the bearing. The ball will itself seek and remain in the position which brings the cone apexes into coincidence. If placed too far from the axis, it will creep toward the centre of the bearing; if placed too near, it will creep from the centre. When the bearing plates are coned, and there is no retaining ring or other device to prevent the balls from moving radially, the pressures between the ball and plates are not normal to the conical surface, or radial to the sphere. The direction of the pressure must be along the line passing through the two places of contact between the ball and

plates. On account of its deformation under pressure, a ball acts very much like a short cone roller between coned plates.

If the balls are all of the same diameter, they must all be at the same distance from the centre of the bearing, thus forming a ring of balls. They should be held in place by a retaining ring.

23.7. Ball thrust-bearings, three-point contact. — The single groove-ball thrust-bearing (Fig. 51.11) makes use of one ring of balls without the necessity of a retaining ring. The use of the latter may be advisable, however, to keep the balls from rubbing against each other. Pure rolling of the ball on the flat bearing plate can be secured by making the sides of the groove at such angles that the two points of contact of a ball placed in the groove will lie
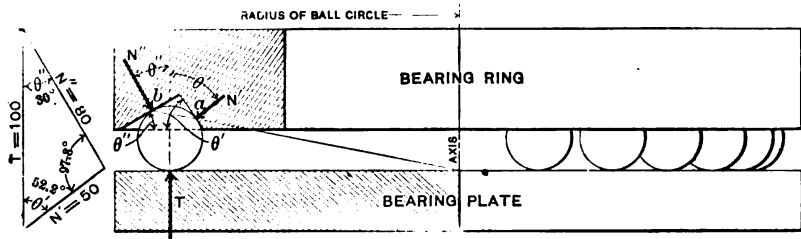
FIG. 51.11.

on a line whose intersection with the axis of the bearing lies in the plane of the flat bearing surface, as shown in the figure.

In a bearing with 1-inch diameter balls with their centres in a circle 9 inches in diameter, or $4\frac{1}{2}$ inches in radius, if the outer side of the groove makes an angle $\theta'' = 30°$ with the surface of the flat bearing ring, then the corresponding angle of the inner side of the groove is $\theta' = 52.2°$, in order to cause the line passing through the points of contact a and b to meet the flat bearing surface of the plate at its centre. The angle of 30° for the outer side of the groove is probably as small as should ever be used, on account of the tendency of the balls to wedge or jam between the plate and side of the groove. On the other hand, larger values of θ'' give a greater spinning action of the ball against the sides of the groove, as well as higher pressures at the same places.

The relation between the pressures against the bearing surfaces in a bearing of the form of Fig. 52.11 are expressed by the equations

$$N' = \frac{T \sin \theta''}{\sin [180° - (\theta' + \theta'')]};$$

$$N'' = \frac{T \sin \theta'}{\sin [180° - (\theta' + \theta'')]}.$$

By substituting in these equations the values $\theta'' = 30°$ and $\theta' = 52.2°$, the pressures against the sides of the groove, Fig. 51.11, for $T = 100$ pounds, are:

$$N' = \frac{100 \sin 30}{\sin [180 - (52.2 + 30)]} = \frac{100 \times 5}{\sin 97.8} = \frac{50}{.99075} = 50.4 \text{ lbs.:}$$

$$N'' = \frac{100 \sin 52.2}{\sin [180 - (52.2 + 30)]} = \frac{100 \times 79012}{99075} = 79.8 \text{ lbs.}$$

Fig. 51.12 has the same size of balls and ball circle as the preceding design, viz., 1″ diameter of ball for a 9″ diameter ball circle. The groove has the same bottom angle as before, 97.8°,.
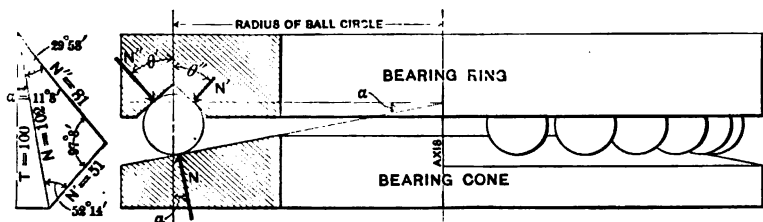
FIG. 51.12.

but the sides make equal angles with the flat face of the ring. The mating surface is coned to secure pure rolling of the ball upon it. The cone elements differ from a flat plate by the angle $\alpha = 11° 8'$. A force $T = 100$ pounds, acting parallel to the axis of the bearing, will cause a pressure $N = 102$ pounds between the ball and bearing cone. The angle between N, normal to the sphere, and the two normals at the places of contact in the groove, differ but very slightly from those in the preceding figure, only 0° 2′ in each case. The pressures N' and N'' also differ very slightly from those of the flat plate bearing.

In the matter of construction, there is little ground for choosing either of the two bearings, Fig. 51.11 or 51.12, in preference to the

other.   The former, on account of having a flat plate instead of a cone, will work with very fair satisfaction when the axis of the plate does not coincide with, but is parallel to, that of the ring.   The latter, on account of the conical surface, must have the ring and cone co-axial.

23.8. **Ball thrust-bearing, four-point contact.**—Fig. 51.13 is a form of bearing that is much used, and which frequently gives trouble.   When the two bearing rings are grooved so that two lines passing through the points of contact of the ball with the grooves meet on the axis of the bearing as shown, then the best kinematic conditions for four-point contact are secured.   The difficulty of making the grooves so that the pressure will be properly distributed is great.   Thus, suppose that the two rings are to
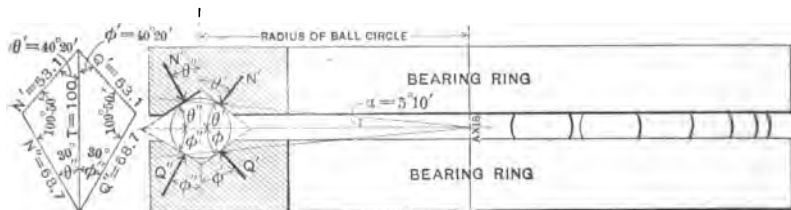
Fig. 51.13.

be exactly alike, as in the figure, and that the outer sides of the grooves are made correctly.   If the inner sides are not then so made that the intersection of the sides of the groove lies at the same distance from the centre of the plate in both cases, a construction difficult to make exact, then the pressure will not be properly distributed, for the side of the groove that is "low" will not carry its due load.   In fact, it will carry none at all until the deformation of the parts under pressure is sufficient to bring the ball against this side of the groove.   The unduly heavy pressure on the other side of the groove will cause destruction of the bearing much sooner under heavy loads than it should occur.   The bearing may be designed with unlike rings, but the difficulties of construction still remain.

The axes of the two bearing rings must coincide in the four-point contact thrust-bearing.   If they are not coincident, although

still parallel, the pressures will be thrown unduly on one side of a groove.

The figure is drawn to scale and shows the proper distribution of pressure in a bearing with balls 1 inch in diameter with their centres in a circle 9 inches in diameter, as for the two preceding bearings.

23.9. Ball journal-bearing, four-point contact. — Fig. 51.14 shows a ball bearing whose chief function is to resist pressure normal to its axis of rotation. It will also resist thrust and end motion. Each groove should have the same angle on both sides.
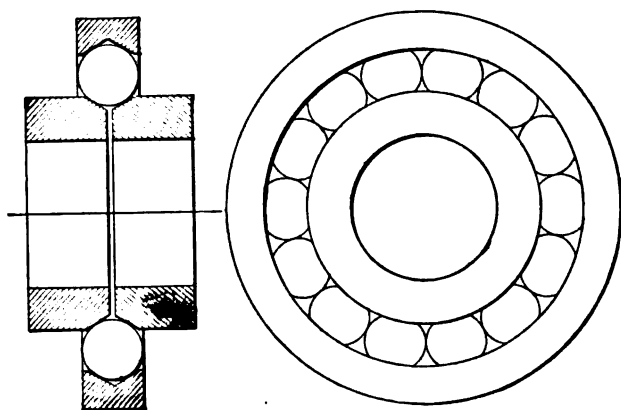
FIG. 51.14.

The two grooves do not have to be of the same angle, however. Two rigidly supported bearings of this form should not be used on the same shaft, for any change of length of the shaft between the bearings would force one side of the groove hard against the balls in each bearing. It can, of course, be used satisfactorily in connection with other bearings that do not resist end motion. The grooves can be made flatter than when only one part is grooved. An angle less than 30° between any two adjacent sides of the grooves is not advisable.

23.10. Cup-and-cone two-point contact hub ball bearing.—Fig. 51.15. This is the very familiar form of ball bearing so extentensively used for the wheels of road vehicles. Each ball runs

in a pair of races whose surface profiles in a cross-section are concave conoids with a radius of curvature somewhat greater than the radius of the ball. The actual locations of the places of contact on the ball are indeterminate. For the purpose of determining the approximate values of the bearing pressures under a given load, the centres of the bearing areas have been assumed to be at the ends of a diameter of the ball in the figure. This is not a stable condition, however, for as the parts move, the ball, when in this initial position, will try to roll farther out from the axis of the bearing. This tendency will be resisted by the increasing slope of the sides of the races, at the places of contact, and
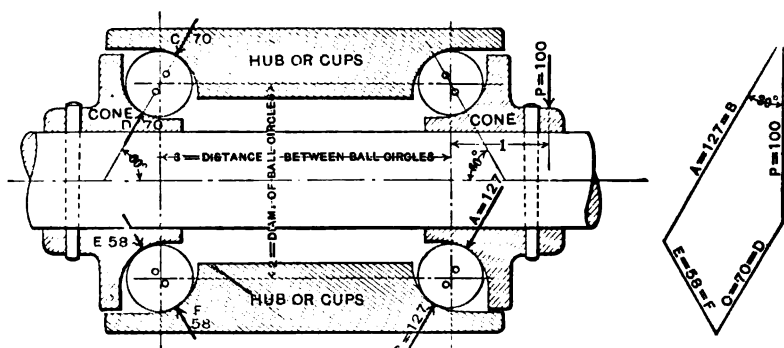
FIG. 51.15.

the ball kept nearly in the position shown. On the other hand, if the ball happens to get in closer to the axis than shown, so that a pair of lines tangent to the two areas of contact meet before intersecting the axis of the bearing, then the rolling of the ball will have a tendency to crowd it still further in between the surfaces. This tendency of the ball to force itself farther in either direction out of its true position when it has once started, and the fact that it is always trying to leave the correct position when under pressure, make it necessary to lubricate the bearing to prevent the ball from wedging in the race. The radius of curvature of the races must, on the same account, be made sufficiently small to give the ball but little room for side motion. The curvature should be to a radius between $1\frac{1}{8}$ and $1\frac{1}{2}$ times the radius of the ball.

With the proportions shown in Fig. 51.15, in which the diameter of the ball circle is taken as 2, and the distance between the ball circles as 3, units, a force $P = 100$ pounds normal to the axis of the bearing and at a distance of 1 unit outside of the right-hand ball circle produces the pressures indicated between the balls and races. These forces are obtained by the force diagram to the right of the bearing. It is assumed for convenience of illustration that the pressure is all carried by one ball when forces are shown acting upon a ball. The pressure is actually distributed among several balls in an ordinary commercial bearing.

If the cones were reversed in position and placed with their bases next to each other, and the cups also reversed to correspond with the new position of the cones, leaving the diameter of and distance between the ball circles unchanged, the pressures against the right-hand lower and left-hand upper balls would be nearly doubled with the force P of the same value and acting the same as before.

23.11. Three-point contact hub ball bearing.—Fig. 51.16 is of the same general form as the preceding one. The difference is
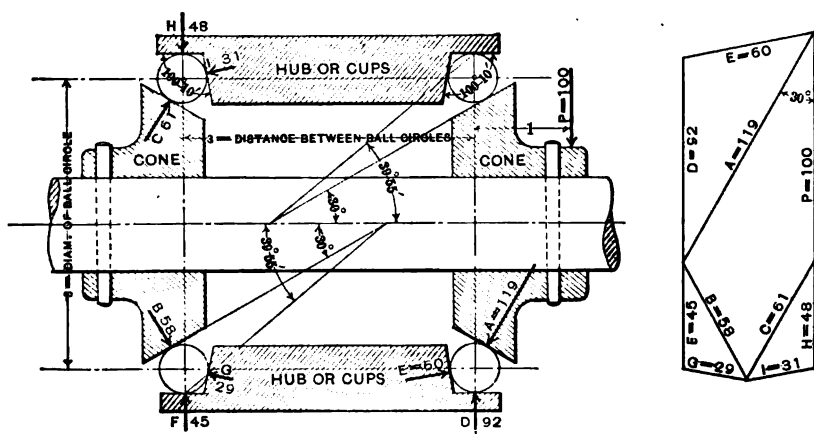

FIG. 51.16.

that true conical surfaces are used for the inner bearing surfaces and grooves for the outer ones. The ratio of the diameer of the ball circles to the distance between them must be made greater in

this form than in the cup-and-cone design in order to secure a sufficiently large cone apex angle to prevent the balls from wedging between the cone and outer side of a groove. The half-apex angle of 30° is about as small as should be used for bicycle wheels where there is no great side pressure against the tire. In three- or four-wheeled stably balanced vehicles, or elsewhere when two wheels are placed on the same axle, it is better to make the apex angle 60° or larger, 30° or more for the half angle.

The force diagram and pressures against the balls are shown for a pressure $P = 100$ pounds normal to axis of the bearing and at a distance of one unit outside of the right-hand ball circle. Reversing the cones in this case would have the same effect as for the cone-and-cup bearing.

23.12. Four-point contact hub ball bearing.—Fig. 51.17 is intended for the same use as the preceding two. It has the same
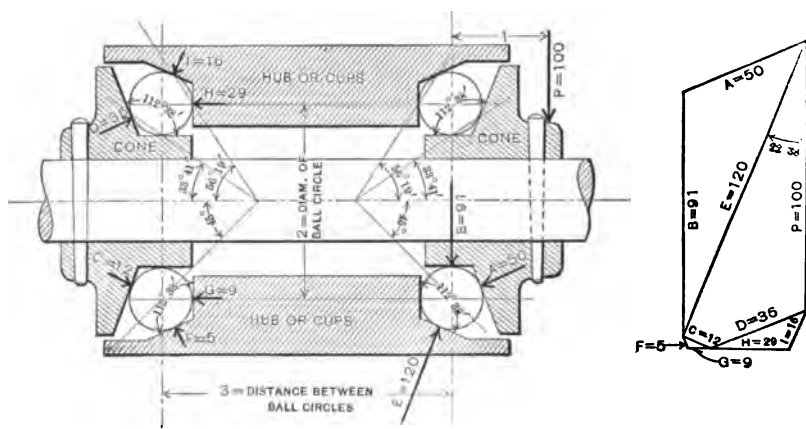
FIG. 51.17.

ratio between the diameter of and distance between ball circles as the two-point bearing. These two have the advantage of compactness over the three-point bearing.

The adjacent sides of the grooves should not have an angle much smaller than 30° between them. The force diagram and pressures upon the balls are shown in the figure for a force $P = 100$

pounds acting at a distance of 1 unit outside of the right-hand ball circle.

The objections to the four-point contact are not so serious in this design as in the thrust-bearing.

Special Forms of Bearings.

24. The bearing shown in Fig. 52 is often used on light machinery, such as circular saws, wood-planing machines, etc. It
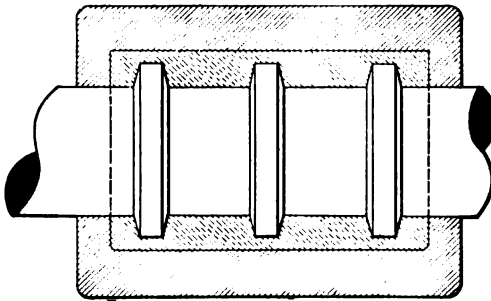
Fig. 52.
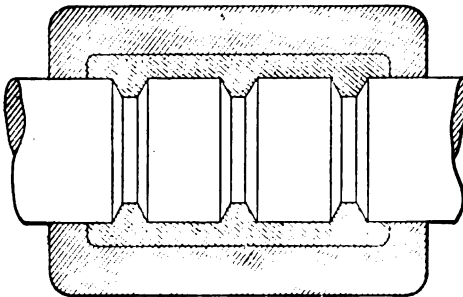
Fig. 52.1.

is something of a combination between a journal- and thrust-bearing, and serves in a measure the purpose of both.

The collars are integral parts of the shaft. The box is divided

lengthwise and lined with a soft bearing metal, which is cast in while the journal is in place.

A bearing (Fig. 52.1) of a similar nature is made by cutting grooves into the shaft to serve the same purpose as the collars. It answers very well when only light service is required, but the shaft is weakened so as to be unfit for heavy service.
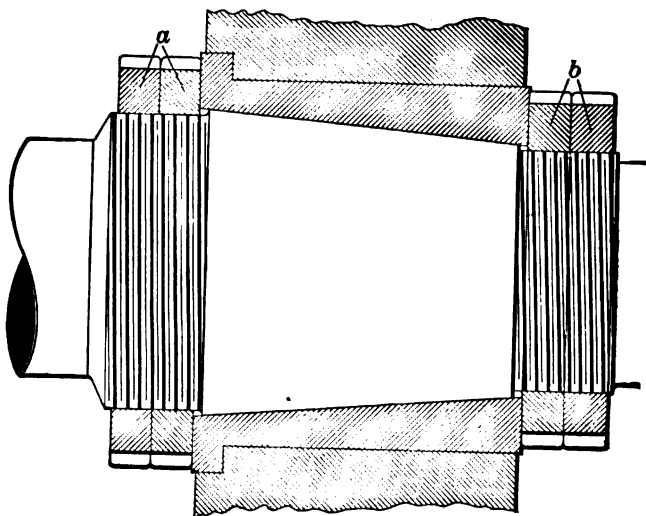
FIG. 53.

Fig. 53 is a taper bearing provided with lock-nuts *a* and *b* for adjusting it and taking end thrust. The box surrounding it is solid. The taper is commonly about $1\frac{1}{2}$ inches in diameter per foot of length. This form of bearing is chiefly used on light machinery running under light journal-pressure.

CHAPTER II.

SPUR-GEARS.

25. Strength of spur-gear teeth.—When a rotating spur-gear transmits power to its mate, there must be pressure between the pairs of teeth that are in contact. If the teeth were correctly formed and accurately spaced, the pressure necessary to turn the driven gear would be nearly uniformly distributed between the pairs of teeth in contact. The elasticity of the teeth might theoretically affect the uniformity of the distribution, but not to an extent to be worthy of consideration in practice.

On account of the inaccuracy of spacing that always exists to some extent, and the loss of the correct form of the teeth by wear, it is customary to assume, for purposes of designing, that the entire force that is required to rotate the driven gear is applied by a single

FIG. 54.

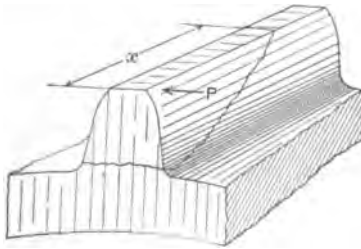tooth of the driver to its mate on the driven. If there is a probability that the teeth may be thrown out of alignment by the springing of the shafts or movement of the supports, or that foreign substances may come between the teeth, the pressure may be localized at the top of a tooth at one end, as at P in Fig. 54, and break it off as shown by the irregular line of fracture. To allow for such

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a condition, it is customary to assume that, however wide the face
of the gear, the strength of the tooth is no greater than if the gear-
face had only a width x measured across the face of the gear.   The
value of x is different for teeth of the same pitch and height, but
of different profile, and cannot be accurately determined in any
case.   For the more common forms of gears having teeth whose
height is roughly .7 of the circular pitch, it is generally safe to take
x as 1.5 to 2 times the circular pitch, according to whether the
teeth are narrow or broad at the base.   The wider the face of the
gear, the less rapidly will it wear; and if accurately made and
rigidly supported, the strength will be about proportional to its
width for widths as great as six or more times the circular pitch.

When a pair of teeth first come together, there is a shock caused
by the blow as they strike each other.   The intensity of the shock
increases with the speed at which the teeth travel; hence gears
running at high speed will not stand as much pressure between their
teeth as those running slower.   There may also be shocks caused by
loads suddenly applied to the driven gear, as in rock- and ore-
crushers, and rolling-mill machinery.   The gear should be propor-
tioned to resist such shocks.

The pressure between a pair of teeth would be normal to their
surfaces along the line of contact, if there were no frictional resist-
ance acting to prevent the combined motion of rolling and slipping
of the one over the other.   This frictional resistance always does
exist, however, to an extent depending on the material of the teeth,
the finish of their working surfaces, and whether they are lubricated
or dry, clean or covered with dirt and grit.

Fig. 55 shows the profiles of a pair of teeth A and B just as they
come into contact when A is driving B, the rotation about the
centres of the gears being in the direction indicated by the arrows.
The point of contact is at C, and the common normal to the tooth
curves passes through the pitch-point P.   The pressure between
the teeth would be in the direction CP if there were no friction
between them.   The friction causes the line of pressure to be
inclined to CP by some angle PCH, whose value depends on the
coefficient of friction of the tooth surfaces.   The lever-arm, about
the centre of the driven gear, of the force acting along CH, is shorter
than that of a force acting along CP; hence the force that acts along

CH to turn the driven gear against a given resistance must be proportionally greater than that which would act along *CP*.

E and *D* are the positions of the same pair of teeth just as they are on the point of separating. As before, the common normal to the curves at the point of contact *F* passes through *P*, and the frictional resistance causes the pressure between them to make some
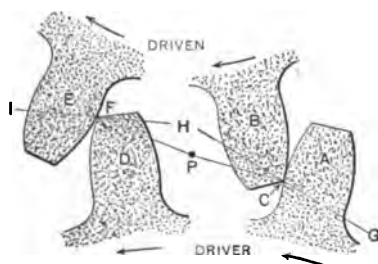

Fig. 55.

angle *PFH* with this normal. In gears that are not lubricated, or are lubricated and covered with dirt and grit, it is possible that the coefficient of friction may be large enough to increase the angle *PFH* to such an extent that the direction of pressure *FH* will become tangent to the gear-face at the top of the tooth; or, what is practically the same, normal to the radial line which passes through the middle of the tooth profile. This radial line will be hereafter referred to as the median line of the tooth.

A tooth of the driven gear is subjected to the greatest stress when in the position *B*, just as it comes in contact with its mate, for the pressure is then applied at the greatest distance from its base; and a tooth of the driving gear is working under the greatest stress when in the position *D*, just as the pair are separating.

For the tooth *B*, on the driven gear, it can be seen that the pressure against it, acting along a line inclined to the median line of the tooth, produces both radial compression and bending stresses in the material. This is true whether friction is considered or not. Friction increases the proportion of compressive to bending stress.

The tooth *D*, on the driver, also has both compressive and bending stresses caused by the pressure against it when friction is not considered. Friction reduces the ratio of compressive to bending

stress, and if the friction becomes great enough to bring the line of pressure normal to the median line of the tooth, the compressive stress is eliminated, and the action is a purely bending one.

If such a material as cast iron, which is much weaker in tension than compression, is used for gears having equal diameters and the same form of teeth, D will be more apt to break than B; if the material is one that is equally strong in both tension and compression, B would probably fail first.

The force P that will break a gear-tooth when exerted normal to its median plane at the top of the tooth, and uniformly distributed across the face of the gear, as in Fig. 56, may be found by the
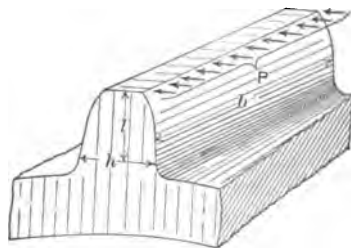
Fig. 56.

formula for a cantilever. The quantities entering into this formula when applied to a gear-tooth are:

$P =$ force applied tangent to top of tooth;
$S =$ maximum fibre-stress per unit area in the material;
$v =$ shearing stress per unit area in the material;
$b =$ breadth of gear-face;
$h =$ thickness of tooth at breaking section;
$l =$ distance from top of tooth to breaking section.

The formula is

$$P = \frac{Sbh^2}{6l}.$$

On account of the filleting which is commonly used at the bottom of the tooth, the distance l of the breaking section from the top of the tooth is less than the total height of the tooth. The location of the breaking section can be found for any tooth,

whether the profiles on both sides of the median line are similar or not, by taking sections normal to the median plane and at different distances from the top of the tooth, introducing the l and h of each in the formula, until the one that gives a minimum value of P is found. When the profile of the tooth is symmetrical about the median line, the value of P can be obtained more directly by drawing a parabola whose vertex is at the centre of the top of the tooth, and whose sides are tangent to the tooth curves. The value of P for any section parallel to the direction of P is the same as for all other sections, this being the property of a cantilever of parabolic profile and uniform breadth.

In any system of interchangeable gears, such as the cycloidal with constant size of generating circle for a given pitch, or the involute with constant angle of obliquity, the thickness of the teeth at the base is less for gears of small diameter than those of large; consequently the teeth of the smaller gears are weaker.

The curve, Fig. 57, was obtained by calculating, according to the formula for the cantilever just given, the strength of cycloidal teeth generated by a describing circle having a diameter half that of a 15-tooth gear of the pitch adopted, and a fillet at the bottom of the tooth of a radius equal to one sixth of the width of the space at the addendum circle.* This is practically the form of tooth adopted by the Brown and Sharpe Mfg. Co. for their interchangeable gears. The length of the tooth from top to bottom is 0.6866 times the circular pitch, and the thickness on the pitch-circle equals half the pitch. It can be seen by the diagram that the teeth on a 120-tooth gear are twice as strong as those on one having 12 teeth. By multiplying or dividing the reading on the scale of " pounds pressure at tooth-point " by a suitable constant, the curve can be made to apply to any pitch, width of gear-face, and fibre-stress.

With regard to the effect of the position of the line of contact between a pair of engaging gears, it is shown by construction that, in a pair of perfectly made and aligned gears having 13 teeth each, at the instant contact changes from one to two pairs of teeth, or

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* " Diagrams for the Relative Strength of Gear Teeth," by Forrest R. Jones. Trans. Amer. Soc. Mech. Engrs., vol. XVIII., p. 766.

FORM, STRENGTH, AND PROPORTIONS OF PARTS.

TABLE VIIc.

GEAR WHEELS: TABLE OF TOOTH PARTS.

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch.	Circular Pitch.	Thickness of Tooth on Pitch Line.	Addendum $\frac{1''}{P}$. or	Working Depth of Tooth	Depth of Space below Pitch Line.	Whole Depth of Tooth.
P	P'	t	s	D''	$s+f$	$D''+f$
½	6.2832	3.1416	2.0000	4.0000	2.3142	4.3142
¾	4.1888	2.0944	1.3333	2.6666	1.5428	2.8761
1	3.1416	1.5708	1.0000	2.0000	1.1571	2.1571
1¼	2.5133	1.2566	.8000	1.6000	.9257	1.7257
1½	2.0944	1.0472	.6666	1.3333	.7714	1.4381
1¾	1.7952	.8976	.5714	1.1429	.6612	1.2326
2	1.5708	.7854	.5000	1.0000	.5785	1.0785
2¼	1.3963	.6981	.4444	.8888	.5148	.9587
2½	1.2566	.6283	.4000	.8000	.4628	.8628
2¾	1.1424	.5712	.3636	.7273	.4208	.7844
3	1.0472	.5236	.3333	.6666	.3857	.7190
3½	.8976	.4488	.2857	.5714	.3306	.6163
4	.7854	.3927	.2500	.5000	.2893	.5393
5	.6283	.3142	.2000	.4000	.2314	.4314
6	.5236	.2618	.1666	.3333	.1928	.3595
7	.4488	.2244	.1429	.2857	.1653	.3081
8	.3927	.1963	.1250	.2500	.1446	.2696
9	.3491	.1745	.1111	.2222	.1286	.2397
10	.3142	.1571	.1000	.2000	.1157	.2157
11	.2856	.1428	.0909	.1818	.1052	.1961
12	.2618	.1309	.0833	.1666	.0964	.1798
13	.2417	.1208	.0769	.1538	.0890	.1659
14	.2244	.1122	.0714	.1429	.0826	.1541

⊢ TABLE VIID.

GEAR WHEELS: TABLE OF TOOTH PARTS—*Continued.*

DIAMETRAL PITCH IN FIRST COLUMN.

Diametral Pitch. P	Circular Pitch. P'	Thickness of Tooth on Pitch Line. t	Addendum 1″ or $\frac{1}{P}$. s	Working Depth of Tooth. D″	Depth of Space below Pitch Line. s + f	Whole Depth of Tooth. D″ + f
15	.2094	.1047	.0666	.1833	.0771	.1438
16	.1963	.0982	.0625	.1250	.0723	.1348
17	.1848	.0924	.0588	.1176	.0681	.1269
18	.1745	.0873	.0555	.1111	.0648	.1198
19	.1653	.0827	.0526	.1053	.0609	.1185
20	.1571	.0785	.0500	.1000	.0579	.1079
22	.1428	.0714	.0455	.0909	.0526	.0980
24	.1309	.0654	.0417	.0833	.0482	.0898
26	.1208	.0604	.0385	.0769	.0445	.0829
28	.1122	.0561	.0357	.0714	.0413	.0770
30	.1047	.0524	.0333	.0666	.0386	.0719
32	.0982	.0491	.0312	.0625	.0362	.0674
34	.0924	.0462	.0294	.0588	.0340	.0634
36	.0873	.0436	.0278	.0555	.0321	.0599
38	.0827	.0413	.0263	.0526	.0304	.0568
40	.0785	.0393	.0250	.0500	.0289	.0539
42	.0748	.0374	.0238	.0476	.0275	.0514
44	.0714	.0357	.0227	.0455	.0263	.0490
46	.0683	.0341	.0217	.0435	.0252	.0469
48	.0654	.0327	.0208	.0417	.0241	.0449
50	.0628	.0314	.0200	.0400	.0231	.0431
56	.0561	.0280	.0178	.0357	.0207	.0385
60	.0524	.0262	.0166	.0333	.0193	.0360

TABLE VIIe.

GEAR WHEELS: TABLE OF TOOTH PARTS.

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch.	Threads or Teeth per Inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum or $\frac{1''}{P}$.	Working depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P'	$\frac{1''}{P'}$	P	t	s	D''	$s+f$	$D''+f$	$P' \times .31$	$P' \times .335$
2	¼	1.5708	1.0000	.6366	1.2732	.7366	1.3732	.6200	.6700
1⅞	⁸⁄₁₅	1.6755	.9375	.5968	1.1937	.6906	1.2874	.5818	.6281
1¾	⁴⁄₇	1.7952	.8750	.5570	1.1141	.6445	1.2016	.5425	.5863
1⅝	⁸⁄₁₃	1.9333	.8125	.5173	1.0345	.5985	1.1158	.5088	.5444
1½	⅔	2.0944	.7500	.4775	.9549	.5525	1.0299	.4650	.5025
1⁷⁄₁₆	¹¹⁄₁₆	2.1855	.7187	.4576	.9151	.5294	.9870	.4456	.4816
1⅜	⁸⁄₁₁	2.2848	.6875	.4377	.8754	.5064	.9441	.4262	.4606
1⁵⁄₁₆	¹¹⁄₁₄	2.3986	.6562	.4178	.8356	.4834	.9012	.4069	.4397
1¼	⅘	2.5133	.6250	.3979	.7958	.4604	.8583	.3875	.4188
1³⁄₁₆	¹⁴⁄₁₃	2.6456	.5937	.3780	.7560	.4374	.8156	.3681	.3978
1⅛	⁸⁄₉	2.7925	.5625	.3581	.7162	.4143	.7724	.3488	.3769
1¹⁄₁₆	¹⁴⁄₁₅	2.9568	.5312	.3382	.6764	.3913	.7295	.3294	.3559
1	1	3.1416	.5000	.3183	.6366	.3683	.6866	.3100	.3350
¹⁵⁄₁₆	1¹⁄₁₅	3.3510	.4687	.2984	.5968	.3453	.6437	.2906	.3141
⅞	1⅐	3.5904	.4375	.2785	.5570	.3228	.6007	.2713	.2931
¹³⁄₁₆	1⁴⁄₁₃	3.8666	.4062	.2586	.5173	.2993	.5579	.2519	.2722
¾	1⅓	4.1888	.3750	.2387	.4775	.2762	.5150	.2325	.2513
¹¹⁄₁₆	1⁵⁄₁₁	4.5696	.3437	.2189	.4377	.2532	.4720	.2181	.2303
⅝	1⅗	4.7124	.3333	.2122	.4244	.2455	.4577	.2066	.2238

TABLE VIIF.

GEAR WHEELS: TABLE OF TOOTH PARTS—*Continued.*

CIRCULAR PITCH IN FIRST COLUMN.

Circular Pitch	Threads or Teeth per Inch Linear.	Diametral Pitch.	Thickness of Tooth on Pitch Line.	Addendum or $\frac{1''}{P}$.	Working Depth of Tooth.	Depth of Space below Pitch Line.	Whole Depth of Tooth.	Width of Thread-Tool at End.	Width of Thread at Top.
P'	$\frac{1''}{P'}$	P	t	s	D''	$s+f$	$D''+f$	$P' \times .31$	$P' \times .385$
⅝	1⅝	5.0265	.3125	.1989	.3979	.2301	.4291	.1938	.2094
🙾	1⅞	5.5851	.2812	.1790	.3581	.2071	.3862	.1744	.1884
½	2	6.2832	.2500	.1592	.3183	.1842	.3433	.1550	.1675
🙾	2¼	7.1808	.2187	.1393	.2785	.1611	.3003	.1356	.1466
⅜	2½	7.8540	.2000	.1273	.2546	.1473	.2746	.1240	.1340
⅜	2¾	8.3776	.1875	.1194	.2387	.1381	.2575	.1163	.1256
⅓	3	9.4248	.1666	.1061	.2122	.1228	.2289	.1033	.1117
🙾	3¼	10.0531	.1562	.0995	.1989	.1151	.2146	.0969	.1047
⅜	3½	10.9956	.1429	.0909	.1819	.1052	.1962	.0886	.0957
⅓	4	12.5664	.1250	.0796	.1591	.0921	.1716	.0775	.0838
⅜	4½	14.1372	.1111	.0707	.1415	.0818	.1526	.0689	.0744
⅓	5	15.7080	.1000	.0637	.1273	.0737	.1373	.0620	.0670
🙾	5¼	16.7552	.0937	.0597	.1194	.0690	.1287	.0581	.0628
⅓	6	18.8496	.0833	.0531	.1061	.0614	.1144	.0517	.0558
⅐	7	21.9911	.0714	.0455	.0910	.0526	.0981	.0443	.0479
⅛	8	25.1327	.0625	.0398	.0796	.0460	.0858	.0388	.0419
⅑	9	28.2743	.0555	.0354	.0707	.0409	.0763	.0344	.0372
🙾	10	31.4159	.0500	.0318	.0637	.0368	.0687	.0310	.0335
🙾	16	50.2655	.0312	.0199	.0398	.0230	.0429	.0194	.0209

vice versa, which is the time when the greatest stress is brought upon the material of the tooth, the pressure is applied at such a distance outside of the pitch-line of the tooth subject to the greater bending moment, as to allow an increase of pressure of more than 70% of that which could be applied at the top of the tooth.   With
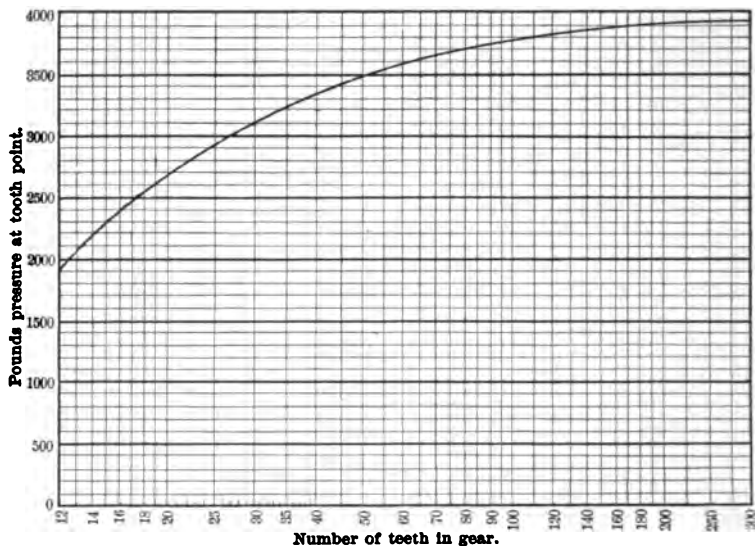

Diagram showing the pressure at the point of a single tooth in a gear of ⅙ diametral (= 6.28 inches circular) pitch and 1 inch face, which will produce a fibre stress of 0000 pounds per square inch in the material of the gear.

FIG. 57.

a greater number of teeth in either or both gears, a greater increase is allowable.

In calculating the strength of a tooth by the cantilever formula, no account is taken of the shearing stress on the section under consideration.   The value of this shearing stress per unit area of the section normal to the median plane is

$$v = P \div \text{area of section.}$$

Combining this with the fibre-stress due to bending gives for the maximum tension or compression, both being the same in value,

$$\text{Maximum stress} = \frac{S}{2} + \sqrt{v^2 + \frac{S^2}{4}}.$$

The value of the " maximum stress " for a given pressure P is but 4% greater than that of S obtained by the formula for the cantilever.   This 4% excess is for a rack-tooth, which is the strongest of all, internal gears excepted; it grows smaller as the number of teeth decreases.   On account of its comparatively small value, when compared with the uncertain elements which enter into any attempt to calculate the strength of gear-teeth, it may be safely neglected.

The maximum shearing stress, according to the formula for combined shear and flexure,

$$\text{Maximum shear} = \sqrt{v^2 + \frac{S^2}{4}},$$

is about 54% of the maximum tension.

The curves of Figs. 58 and 59 were laid out from Fig. 57 by making use of the fact that, when the number of teeth and width of a gear remain constant, the strength of the teeth varies as the circular pitch; and by taking account of the fact that, when the number of teeth remains constant, and the width of the gear-face bears a constant ratio to the circular pitch, the strength of the teeth varies as the square of the pitch, the curves of Figs. 60 and 61 were plotted.

In the diagrams, Figs. 58, 59, 60, and 61, each of the diagonal lines, some of which are straight and some broken, represents a different fibre-stress in the material of the tooth.   The break in some of the lines has no significance, being made only as a means of shortening the diagrams to a convenient length.   It should be noted, however, that a change of pounds per inch is thus made necessary on the pressure-scale, the change occurring at the pressure-line where the angle is made in the diagonals.   Figs. 58 and 59 are for gears having a face 1 inch wide.   They are of exactly the same nature, one for small and the other for large pitches, and might have been combined in a single diagram.   Such a combination would make it difficult to read values for the smaller pitches unless the whole diagram were excessively large.   The same is true of Figs. 60 and 61.

The diagrams, Figs. 58, 59, 60, and 61, are for use in finding any one of the four factors, circular pitch, pitch diameter, pressure
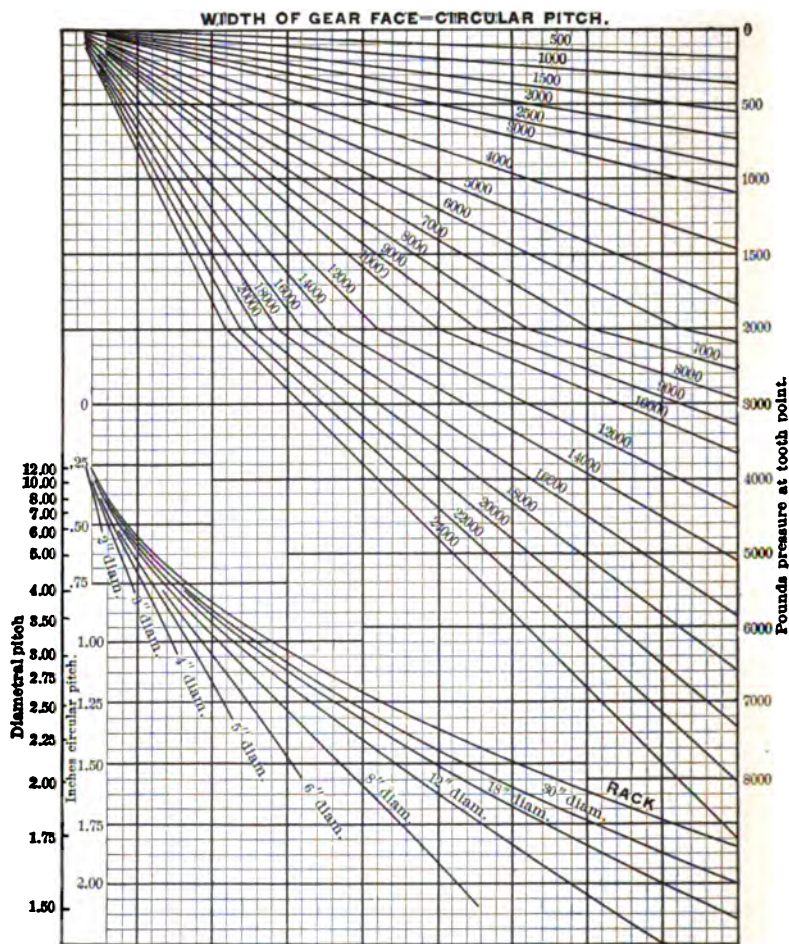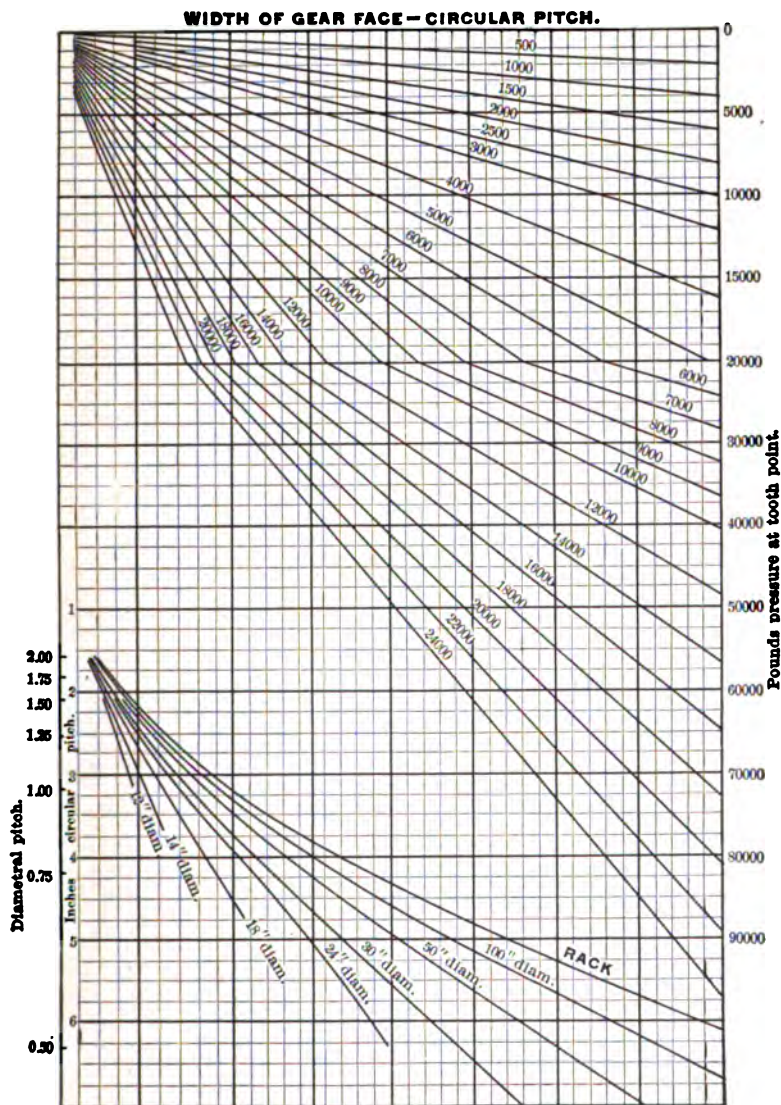
Diagram showing relation between pressure at tooth point, stress in outer fibre, and pitch of a gear tooth.   Figures in diagram above "**RACK**" indicate fibre stress—those below, diameter of gear.

FIG. 60.

Diagram showing relation between pressure at tooth point, stress in outer fibre, and pitch of a gear tooth. Figures in diagram above "**RACK**" indicate fibre stress—those below, diameter of gear.

Fig. 61.

at tooth-point, or fibre-stress, when the other three are known or assumed.

The following formula has been developed from Fig. 57 by Prof. John H. Barr.*

The notation, slightly changed from his, is:

$C =$ circular pitch, inches;
$D =$ pitch diameter of gear, inches;
$F =$ width of gear-face, inches;
$N =$ number of teeth in the gear;
$P =$ pressure on teeth, pounds;
$S =$ tensile or compressive fibre-stress in the material of the gear, pounds per square inch.

$$P = CFS\left(0.106 - \frac{0.678}{N}\right). \quad \ldots \ldots \ldots \ldots \quad (8)$$

From this the following equations may be deduced:
For a gear-face 1 inch wide,

$$P = CS\left(0.106 - \frac{0.678}{N}\right); \quad \ldots \ldots \ldots \ldots \quad (9)$$

For use when N is known.

$$C = \frac{PN}{S(0.106N - 0.678)}. \quad \ldots \ldots \ldots \ldots \quad (10)$$

$$P = CS\left(0.106 - 0.215\frac{C}{D}\right); \quad \ldots \ldots \quad (11)$$

For use when D is known.

$$C = D\left[0.246 - \sqrt{.0605 - 4.65\frac{P}{SD}}\right]. \quad \ldots \ldots \quad (12)$$

For a width of gear-face $=$ circular pitch,

$$P = C^2S\left(0.106 - \frac{0.678}{N}\right); \quad \ldots \ldots \ldots \quad (13)$$

For use when N is known.

$$C = \sqrt{\frac{PN}{S(0.106N - 0.678)}}. \quad \ldots \ldots \ldots \quad (14)$$

$$P = C^2S\left(0.106 - 0.215\frac{C}{D}\right) \text{ For use when } D \text{ is known.} \quad \ldots \ldots \quad (15)$$

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* Trans. Amer. Soc. of Mech. Engrs., vol. XVIII., 1897, p. 776.

When the width of the gear-face is equal to the circular pitch,. or bears a given ratio to it, the equation involving the diameter of the gear contains both the cube and square of C; hence no equation having a general solution for obtaining C when D, P, and S are known can be obtained from the original form given above. The last equation given can be used tentatively, of course, by substituting assumed values of C in it and solving, continuing until a satisfactory value is found.

In designing gears it is customary to assume that the pressure against the teeth is the same in amount as would have to be applied tangent to the pitch-circle of the gear in order to drive it. The difference between the pitch radius and the radial distance to the point where the pressure is applied to the tooth surface is not great enough, in any ordinary system of gears, to need consideration.

When, for a gear, the pitch radius R in inches, and the greatest torsional moment M in inch-pounds to be exerted upon it, are known, the force P that must act tangent to the pitch-circle can be found by the equation

$$P = \frac{M}{R} \text{ pounds.} \quad \ldots \ldots \quad (16)$$

If the highest rate of working, expressed in horse-power (H.P.), or in inch-pounds of energy E transmitted per minute, together with the number of revolutions per minute V, and pitch radius R inches, are known, P can be found by the equation

$$P = \frac{33000 \times 12 \times \text{H.P.}}{2\pi R V} \text{ pounds,.} \quad \ldots \quad (17)$$

or

$$P = \frac{E}{2\pi R V} \text{ pounds.} \quad \ldots \ldots \ldots \quad (18)$$

Table VIII, representing experiments made by the Brown & Sharpe Mfg. Co. upon cut gears of their own manufacture, shows the observed breaking pressure of the teeth and the calculated fibre-stress which would exist if the pressure were all applied at the top of one tooth, normal to its median plane, and uniformly distributed across the face of the gear.

Example.—Let it be required to find the pitch of a gear 18 inches in diameter, 4 inches face, working at 4000 pounds maximum stress in the material, to withstand 2000 pounds pressure at its point. This gives a pressure of 2000÷4=500 pounds per inch of width of face. In Fig. 58 on the right-hand scale, marked "pounds pressure at tooth-point," find the 500-pound line and follow it to its intersection with the 4000-pound fibre-stress line; from this point drop down to the curve marked "18 inches diameter," and thence to the scale on the left of the diagram, thus obtaining a reading of 2.25 diametrical pitch, or about 1.4 inches circular pitch.

Example.—A pressure of 4550 pounds is to act on a gear of 8 inches pitch diameter and 7 inches face, the limit of fibre-stress being 6000 pounds per square inch. The pressure per inch of width in this case is 4550÷7=650 pounds. By the same method as before, following the 650-pound pressure-line to its intersection with the 6000-pound fibre-stress line, and thence toward the bottom of the diagram, it is found that the vertical line does not cut the 8-inch diameter-line, but falls to the right of it, the latter terminating at 1.5 diametrical pitch, this being the greatest pitch that can be used when the number of teeth is not less than twelve, which is the lower limit in the diagrams. The fact that the vertical and diameter lines do not intersect shows that no gear having twelve teeth or more can be designed to fulfil the conditions given. With a fibre-stress slightly greater than 6000 pounds per square inch, however, a gear of 1.5 diametrical pitch will answer.

Example.—For a width of face equal to the circular pitch, Figs. 60 and 61 are used. The method is the same as above, except that the pressure for a width of face equal to the circular pitch is read on the scale of pressures at tooth-point, instead of the pressure per inch of width, as was done before. Thus, for a gear 30 inches in diameter, whose face width is to be three times the circular pitch, to work at a fibre-stress of 14,000 pounds per square inch, under a pressure of 90,000 pounds at the tooth-point, we have 90,000÷3=30,000 pounds pressure on a width of face equal to the circular pitch. Following the 30,000-pound line from the right-hand side of Fig. 61 to its intersection with the 14,000-pound diagonal line,

and thence to the 30-inch diameter curve, gives 5.75 inches circular pitch, which is something less than .5 diametrical pitch.

Problem.—Find the horse-powers that can be transmitted by a pair of gears consisting of a steel pinion and bronze gear. The gear is run at 25 revolutions per minute. The pinion is 6 inches pitch diameter, has 12 teeth of 2 diametrical pitch, and its limiting fibre-stress is 12,000 pounds per square inch. The gear is 30 inches pitch diameter, has 60 teeth, and its limiting fibre-stress is 6000 pounds per square inch. The gear faces are 4 inches wide.

By Fig. 58 the pressure at the tooth-point that will produce 12,000 pounds per square inch stress in the pinion is 960 pounds per inch of width of gear face. And, by the same figure, 900 pounds per inch of width of gear face will cause a fibre-stress of 6000 pounds per square inch in the gear.

· Since the two gears work together, the lower of the two tooth pressures, i.e., 900 pounds, is one that must be used for calculating the capacity of the gears for transmitting power. The total pressure for the gear 4 inches wide is

$$P = 4 \times 900 = 3600 \text{ pounds.}$$

The horse-powers that can be transmitted may be found by equation (17). The pitch diameter and revolutions per minute of the gear will be used. By substituting in equation (17)

$$\text{H.P.} = \frac{2\pi 15 \times 25 \times 3600}{33000 \times 12} = 21.3.$$

Problem.—A gear whose pitch diameter is 6 inches = .5 of a foot is required to transmit 5 horse-powers at 120 revolutions per minute. It is to be assumed that, on account of inaccuracy of form and poor supports, the teeth may bear on one corner only. The working stress is to be 6000 pounds per square inch. It is required to find the greatest allowable number of teeth for this gear which will fulfil the above conditions.

The pressure exerted against a tooth according to the assumed conditions is, by equation (17),

$$P = \frac{33000 \times 5}{.5\pi 120} = 875 \text{ pounds.} \quad \cdot$$

The diagram, Fig. 60, for a width of gear face = to the circular pitch may be used for the solution. Since the pressure is at a corner of the tooth, a width of gear face equal to only 1.5 times the circular pitch will be taken as effective. The pressure for a width of face = circular pitch will therefore be 875 ÷ 1.5 = 583 pounds. Find 583 on the "pounds pressure at tooth-point" scale. Follow this horizontally toward the left to its intersection with the 6000-pound oblique line, then vertically downward to the 6-inch diameter line and then horizontally to the left. The reading thus found on the diametral pitch scale is about 2.5. The corresponding circular pitch is 1.28 inches. The number of teeth for 2.5 diametral pitch is 15.

Problem. — Design a pinion and gear to transmit 50 horse-powers when the pinion runs 100 revolutions per minute and the gear 25. The gear faces are to have a width three times as great as the circular pitch, and the gears to be of the smallest diameter possible, using nothing less than 12 teeth, that will transmit the power with a fibre-stress not greater than 3000 pounds per square inch in the material of the teeth, according to the assumptions made for the diagrams, Figs. 57 to 61, and equations 8 to 15. The gears are to be accurately cut and rigidly supported. The Brown & Sharpe system, having a describing circle of a diameter half as great as that of the pitch circle of a 15-tooth gear of the pitch under consideration, to be used.

The total turning moment that must be exerted by each gear is, by equation (17),

$$\text{Total turning moment} = \frac{33000 \times 12 \times \text{H.P.}}{2\pi V} = PR$$

$$= \frac{33000 \times 12 \times 50}{2\pi 100}$$

$$= 31600 \text{ inch-pounds.}$$

The turning moment for a width of gear face equal to the circular pitch is, since the face is to be three times as wide as the circular pitch,

$$PR = \frac{31600}{3} = 10530 \text{ inch-pounds.}$$

Equation (14) can be used for determining the circular pitch. When both terms of the fraction under the radical are multiplied by R the equation becomes

$$C = \sqrt{\frac{PRN}{SR(0.106N - 0.678)}}.$$

In this equation N must be taken equal to 12, for, of all gears of the same diameter and not less than 12 teeth, the 12-tooth one is the strongest. This can be seen by inspection of the diagrams Figs. 58–61. R may be substituted in the last equation in terms of C, in accordance with its value,

$$R = \frac{6C}{\pi} \text{ for a 12-tooth gear.}$$

By making these substitutions, and squaring both sides,

$$C^2 = \frac{10530 \times 12 \times \pi}{3000 \times 6C(0.106 \times 12 - 0.678)};$$

$$C^3 = \frac{10530 \times 12\pi}{3000 \times 6 \times 0.594} = 37.12;$$

$$C = 3.336 \text{ inches.}$$

Pitch diameter of the pinion $= \dfrac{12 \times 3.336}{\pi} = 12.75''$.

Pitch diameter of the gear is $4 \times 12.75 = 51$ inches.

Width of gear face is $3 \times 3.336 = 10''$, about.

Total pressure at tooth-point $= \dfrac{31600 \times 2}{12.75} = 4970$ pounds.

Problem.—Same as preceding one, except that the greatest fibre-stress shall not exceed 10,000 pounds per square inch, instead of 3000.

This solution could be made, of course, in the same manner as before. It can be done much more readily, however, by making use of the fact that, in similar gears for transmitting equal amounts of energy at the same rotative speed, the fibre-stresses have a ratio to each other which is the inverse of the ratio of the cubes of the diameters of the gears, or of their other similar linear dimensions. (The proof of this statement is given after the solution of the problem.) Accordingly, putting:

$D =$ diameter of gear working at 3000 pounds per square inch fibre-stress;

D' = diameter of gear working at 10,000 pounds per square inch fibre-stress;

the value of D' is expressed by the equation

$$D' = D \sqrt[3]{\frac{3000}{10000}} = D \; \overline{0.3} = .6694D.$$

The value of D for the pinion in the preceding example is 12.75 inches.

Therefore the diameter of the pinion is

$$D' = .6694 \times 12.75 = 8.53 \text{ inches.}$$

Diameter of gear $= 4 \times 8.53 = 34.12''$;

Pitch $= .6694 \times 3.336 = 2.233''$;

Width of face $= 3 \times 2.233 = 6.7''$;

Total pressure at tooth-point $= \dfrac{31600 \times 2}{8.53} = 7410$ pounds.

The proof of the property of gears just made use of can be given by the aid of the equation on page 78, which, by multiplying both its terms by R, becomes

$$PR = \frac{Sbh^2 R}{6l}.$$

Since the turning moment must remain the same, whatever the size of a gear, when transmitting a fixed amount of power at a given speed of rotation, the following is true for two similar gears, represented by the two terms of the equation respectively:

$$\frac{Sbh^2 R}{6l} = \frac{S'b'(h')^2 R'}{6l'}.$$

Corresponding linear dimensions of the two gears must all have the same ratio since they are similar. Calling this ratio x, the last equation may be written

$$\frac{Sbh^2 R}{6l} = \frac{S'(bx)(hx)^2 Rx}{6lx} = \frac{S'bh^2 R}{6l} x^3,$$

whence

$$\frac{S}{S'} = x^3.$$

Therefore

$$\frac{R'}{R} = x = \sqrt[3]{\frac{S}{S'}}; \quad \text{or} \quad \frac{S}{S'} = \frac{(R')^3}{R^3}.$$

Which may be read, for this particular problem,

$$\frac{\text{Stress in larger pinion}}{\text{Stress in smaller pinion}} = \frac{(\text{Radius of smaller})^3}{(\text{Radius of larger})^3}.$$

TABLE VIII.

BREAKING LOADS FOR CUT CAST-IRON GEARS, EXPERIMENTALLY
DETERMINED.

Diametral Pitch.	Circular Pitch, inches.	Width of Face, inches.	Number of Teeth.	Pitch Diam., inches.	Revolutions per Min.	Velocity at Pitch-circle, feet per min.	Observed Breaking Pressure at Pitch-circle, pounds.	Stress in Outer Fibre, pounds per sq. inch.
10	.3142	1$\frac{1}{16}$	110	11	27	78	1060	33000
8	.3927	1$\frac{1}{4}$	72	9	40	94	1460	29000
6	.5236	1$\frac{7}{16}$	72	12	27	85	2220	24000
5	.6283	1$\frac{1}{2}$	90	18	18	85	2470	20500

The following formulas, 19 to 32, based upon the assumption that the pressure is always normal to the tooth surface, have been developed from investigations made by Mr. Wilfred Lewis.* In them the force is assumed as being effective at the point where the normal to the top of the tooth curve intersects the median line of the tooth. The pressure between the teeth is resolved into two components at this point—one radial, and the other perpendicular to the median line of the tooth. The radial component is neglected, only the one normal to the radius and exerting a purely bending action on the tooth, being considered.

Formulas 19 to 32 are applicable to cycloidal gears developed by a generating circle whose diameter is half that of a 12-tooth gear of the same pitch as that under consideration, and to involute gears in which the normal to the tooth curve at the pitch point makes an angle of 75° with the line drawn from the pitch point to the centre of the gear.

The proportions of the teeth to which the following formulas apply, taking the circular pitch C as the unit of measurement, are: thickness on pitch-line $= .47C$; addendum $= .3C$; clearance $= .05(C + 1)$.

Two methods of filleting the bottom of the teeth are represented. In one the fillets are as large as will just clear the tops of the teeth

* Proceedings Engineers' Club of Philadelphia, vol. x., 1893, p. 16 ; *American Machinist*, May 4, 1893, p. 3. and June 8, 1893, p. 7; Trans. Amer. Soc. Mech. Engrs., vol. XVIII., 1897, pp. 776, 781.

of an intermeshing rack, and in the other the radius of the fillets is equal to the clearance between the top of a tooth and the bottom of a space.

For convenience of application, the formulas are given for both a gear-face one inch wide, and for a width of face equal to the circular pitch. It is assumed that the pressure is uniformly distributed across the face of the gear. The notation is the same as in the preceding equations.

The formulas deduced from Mr. Lewis's investigation of strength of gear-teeth are:

FOR A GEAR-FACE ONE INCH WIDE:

Large Fillets.

$$P = CS\left[0.124 - \frac{0.684}{N}\right]; \qquad \text{........ (19)}$$

For use when N is known.

$$C = \frac{PN}{S(0.124N - 0.684)}. \qquad \text{........ (20)}$$

$$P = CS\left[0.124 - 0.218\frac{C}{D}\right]; \qquad \text{..... (21)}$$

For use when D is known.

$$C = D\left[0.284 + \sqrt{.0807 - 4.6\frac{P}{SD}}\right]. \qquad \text{..... (22)}$$

Small Fillets.

$$P = CS\left[0.124 - \frac{0.888}{N}\right]; \qquad \text{......... (23)}$$

For use when N is known.

$$C = \frac{PN}{S(0.124N - 0.888)}. \qquad \text{......... (24)}$$

$$P = CS\left[0.124 - 0.282\frac{C}{D}\right]; \qquad \text{..... (25)}$$

For use when D is known.

$$C = D\left[0.22 - \sqrt{0.049 - 3.57\frac{P}{SD}}\right]. \qquad \text{..... (26)}$$

FOR A WIDTH OF GEAR-FACE = CIRCULAR PITCH:

Large Fillets.

$$P = C^n S\left[0.124 - \frac{0.684}{N}\right];$$

$$C = \sqrt{\frac{PN}{S(0.124N - 0.684)}}.$$

For use when N is known. . . . . (27)
. . . . (28)

$$P = C^n S\left[0.124 - 0.218\frac{C}{D}\right].$$ For use when D is known. . . . . (29)

Small Fillets.

$$P = C^n S\left[0.124 - \frac{0.888}{N}\right];$$

$$C = \sqrt{\frac{PN}{S(0.124N - 0.888)}}.$$

For use when N is known. . . . . (30)
. . . . (31)

$$P = C^n S\left[0.124 - 0.282\frac{C}{D}\right].$$ For use when D is known. . . . . (32)

Cases that require a large amount of power to be transmitted, when the diameter and speed of a gear are so limited by the conditions to be fulfilled as to require an excessive and objectionable breadth of face, can sometimes be met by using two or more gears of the required diameter, on the same shaft, with faces of such a breadth that the sum of all their breadths is equal to that which would be required for a single gear rigidly supported and perfectly aligned. When several gears are used on the same shaft in this manner, they should not be placed so that their teeth are in line, but "stepped" by placing the teeth of each successive gear in advance of (or behind) those of its neighbor by a distance equal to the circular pitch divided by the number of gears on the shaft; such an arrangement gives smoother running and less liability to breakage.

26. Methods of strengthening gears.—For cases where very strong teeth are required, and where the pressure against the teeth

is usually or always in the same direction, the form of tooth shown in Fig. 62 can be used, in which *A* is the working side. When
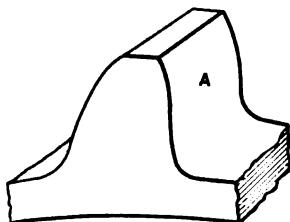
FIG. 62.

used for such a purpose as hoisting, where the machinery is reversed and driven backward to lower the hook or other device for suspending the load, the side opposite *A*, as well as *A*, should be made of some correct tooth outline; but when *A* is the only side that works, the opposite side may be made of any outline that will give strength and, at the same time, clear the tops of the teeth of the intermeshing gear.

For cycloidal gears this **buttressed tooth** can be obtained by using a large describing circle for the face and flank of *A*, and a small one for the corresponding parts of the opposite side, when it is desired to make the latter of a form suitable for backward driving. Involute gears of this form require a large base-circle for the working side *A*, and a small base-circle for the opposite side.

The strength of such a tooth, as compared with that of the ordinary form having the same pitch, height, and breadth, can be determined quite approximately by developing a tooth of each form, and comparing their thicknesses at or near the bottom; when a large fillet is used at the bottom of the tooth, the weakest plane lies a short distance above the bottom. The strength of the teeth is approximately proportional to the squares of their thickness at the sections lying the same distance from the top. Having once obtained the ratio of strengths, it can be used for all teeth of the same form, as long as the pitch and breadth of gear face of those compared are equal to each other.

"**Shrouding**" is another method of adding strength to teeth. It consists of adding an annular ring or disk to one or both ends of a gear. This **shroud** may extend either partly or entirely to the

tops of the teeth. It forms an integral part of the gear-casting or forging. Since the shroud forms a rigid support for the ends of the teeth, shearing of the metal between the ends of the tooth and the shroud must occur at this point in order to allow the tooth to break near the bottom as a cantilever. A full shroud at each end of a gear evidently increases its strength more than when part of the tooth stands above the shroud. The thickness of a shroud, measured parallel to the axis of the gear, is generally at least as great as that of a tooth at the pitch-line.

The strengthening effect of a shroud depends upon the breadth of the gear, a narrow one being more strengthened than a wide one of the same pitch and diameter; for, while in both cases the shearing resistance added by the shrouds is the same, it forms a greater ratio to the cantilever resistance as the gear grows narrower.

When a pinion engages with a large gear, the former wears more rapidly on account of having its teeth come into mesh more frequently; and, if they are of the same material, the pinion consequently grows weaker more rapidly than the spur-gear, in addition to having been the weaker of the two at first, on account of the form of its teeth; hence the pinion is the one to be shrouded if this device is used at all. Two meshing spur-gears of the same size and material should both have equal shrouds extending about half-way to the top of the teeth, provided it is necessary to strengthen them.

27. Short gear-teeth are much stronger than those having the proportions commonly used, in which the height of the tooth is roughly 0.7 of the circular pitch. The shorter teeth also run more quietly. Modern practice is adopting them to a considerable extent when the gears are not intended to be interchangeable. A tooth height of about 0.4 of the circular pitch is commonly used. Such teeth are especially satisfactory for cast gears used without machine-finishing on the working surface of the teeth.

Mr. C. W. Hunt gives the following proportions and, in Table IX, working loads for the involute gears adopted by the company bearing his name.* They are used for coal-hoisting engines and similar machinery, which generally do not have solid foundations. The teeth are cast to form and used without machine-finishing.

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* Trans. Amer. Soc. of Mech. Engrs., vol. XVIII., 1897, p. 787.

The notation is changed to agree with that of the formulas given above.

TABLE IX.

WORKING LOADS FOR SHORT-TOOTH UNCUT CAST-IRON GEARS.

C Circular Pitch, Inches.	P' Working Load, pounds.	P'' Maximum Load, pounds.	C Circular Pitch, Inches.	P' Working Load, pounds.	P'' Maximum Load, pounds.
1	1820	1650	2¼	6700	8300
1¼	2300	2600	2½	8300	10500
1½	3000	3700	2¾	10000	12500
1¾	4100	5000	3	12000	14800
2	5300	6600			

The addendum and dedendum are each equal to 0.2 of the circular pitch C; the clearance equals $.05(C + 1)$ inches; the width of the gear-face is $2C + 1$ inches.

Non-metallic Spur-gears.

28. Mortise-gearing is made by keying or pinning wooden teeth into cast-iron rims designed to receive them. Fig. 63 shows end and
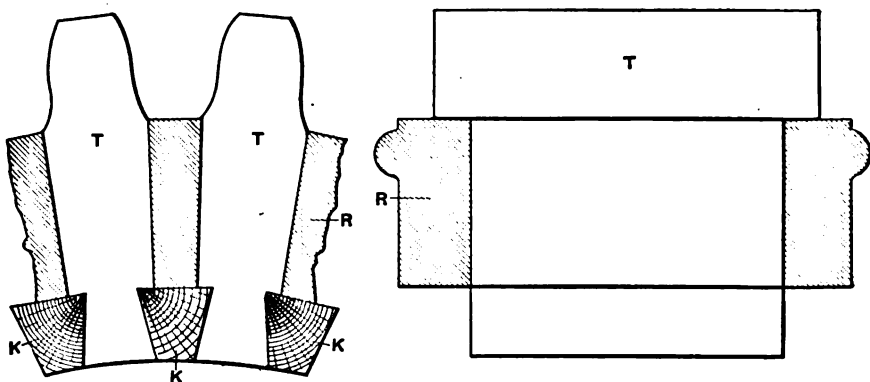
FIG. 63.

side views of a mortise-gear, the rim R being in section, so as to show full end and side views of the wooden teeth T and the wooden keys K which hold the teeth in the tapered openings through the

rims.   Fig. 64 is another form, the tooth-shank here being smaller
than the bottom of the tooth, thus making a shoulder which rests
against the rim and holds the tooth from dropping down into it
when the wood shrinks or is compressed by the pressure due to trans-
mitting power.   A pin P is used to hold the tooth in place, instead
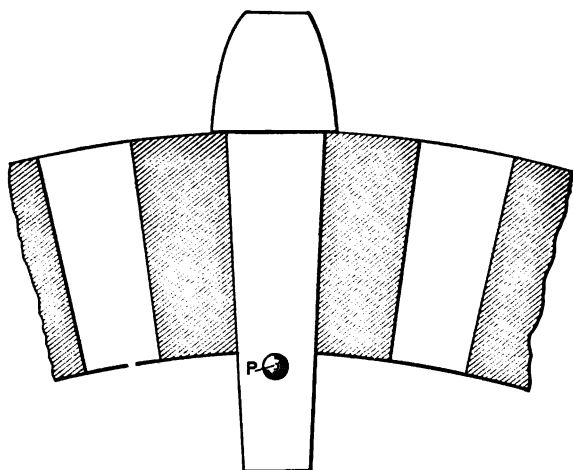
FIG. 64.

of the key of the preceding form; the key could be used in both
forms equally well, however.

The working side of a tooth of the form of Fig. 64 is generally
made with a deeper shoulder than on the opposite side, in order to
allow for the wear due to service, the tooth retaining nearly its
original strength till worn past the shoulder.   Bevel- and spur-gears
can both be made with wooden teeth when it is thought that the
existing conditions to be met can be more satisfactorily fulfilled by
them than with metal gears.

The more common practice is to mate a cast-iron and mortise
gear, the smaller being of iron when they are of different diameters,
the iron teeth having a thickness on the pitch-line of about 0.4 of
the circular pitch, that of the wooden teeth being the remaining
0.6 of the pitch, due allowance being made for backlash and rough
work by reducing these values slightly.   The smaller gear is made
of iron, because the work performed by any one tooth upon it is

greater than for a tooth of the larger one; consequently greater durability is obtained by thus making the smaller gear of a material better able to withstand wear; also, the gears are stronger, since the teeth of the smaller, whose form makes them weaker than those of the same thickness on the pitch-line would be on the larger, are made of the stronger material. Sometimes, but very infrequently, both gears are made with wooden teeth.

Comparatively noiseless running at high speeds is one of the good qualities of mortise-gearing, this being especially marked when comparison is made with a pair of cast gears; the elastic quality of wood makes them able to resist sudden shocks that might break cast-iron gears of the same size and running at the same speed; they are very durable when run under moderate pressure with proper care in lubricating the teeth.

In practice, the wooden tooth-blanks are first keyed in the rim and then machined to proper form in the same manner as metal gears, the only difference in the two processes being that different cutting edges are required for shaping wood and metal, as, for instance, a saw, running at high speed, is used in a bevel-gear planing-machine when cutting wooden teeth, instead of the sharp-cornered planer-tool required for metal.

The woods more commonly used for teeth are maple, hickory, and locust. In order to prevent swelling and shrinking as the atmosphere changes its humidity, the blanks are thoroughly saturated with paraffin or some other oily substance before putting in place.

29. Rawhide, indurated vegetable fibre, etc., are frequently used for small gears where quiet running is desired. The gears are usually made of a number of thin disks placed side by side and held together by a pair of metallic disks at the ends. Most of them are durable under light service, but are not strong. Under heavy service they may wear rapidly, if they do not break. Rawhide and some of the other materials of a similar nature are liable to shrink considerably when used in a dry place. This may be a serious objection on account of reducing the diameter of the gear and causing the disks to become loose.   .

30. Factor of safety for tooth-gears.—While, as in most cases in machine-designing, it is impossible to fix a factor of safety, since its value must depend upon the conditions of each individual case,

an examination of a few of the points to be considered may be of aid in selecting its value.

Thus, in a hand-driven crane, where the speed is slow and any unusual strain is not liable to occur, the factor used needs only to include an allowance for flaws, heterogeneous material, and internal stress, it not being necessary to include shocks and stresses that cannot be estimated, since neither of these last two really exist when proper safety-locking devices are used. If the change is made to power-driving, however, and high speeds are used, then an allowance must also be made for the shock due to the striking of the teeth together. The greatest stress that can come on the crane at any time is calculable within practical limits, being limited to the breaking strength of the chain, plus frictional resistances; hence no allowance for unknown stress need be made. Repeated stress certainly does occur in the teeth of a gear in service, but the allowance for flaws, heterogeneous material, and internal stresses is generally so large that the material does not regularly work near enough to its elastic strength to make necessary any allowance for this cause of fracture.

Rolling-mill machinery, stone- and ore-crushers, and other machines applied to similar purposes, are subjected to shocks and unknown stresses. In such cases the necessary factor of safety can be determined only by experience and some knowledge of the nature of the material to be operated upon.

31. The efficiency of spur-gearing depends very largely upon the frictional loss in the supporting bearings. The greater the loss of power in the bearings the lower the efficiency. The pressure between the teeth causes an equal amount of pressure upon the bearings supporting each gear if the bearings are on each side of the gear. The weight of the gear, as well as other weights and forces, must be taken into account when calculating the total pressure on the bearings.

The experiments made by Wm. Sellers & Co. upon a spur-gear 18.62 inches pitch diameter, having 39 teeth of $1\frac{1}{2}$ inches circular pitch, running on journals $2\frac{1}{8}$ inches diameter, and driven by a spur-pinion 5.73 inches pitch diameter, with 12 teeth of the same pitch, and running on one journal $2\frac{5}{16}$ inches and the other $1\frac{11}{16}$ inches diameter, both placed close to the hub of the pinion, show

the average efficiencies given in Table X for pressures of 430, 700, 1100, 1600, and 2500 pounds pressure between the teeth.*

TABLE X.

EFFICIENCIES OF SPUR-GEARS.

Revolutions of pinion per minute	8	5	10	20	50	100	200
Efficiency, per cent	90	92	94	95.6	97.3	98.2	98.6

If the same amount of power were transmitted with the larger gear driving the smaller, the efficiency would be lower. This may be more readily seen by supposing that the journals of both gears are of the same size. The pressures on the journals of the two gears are equal with or without friction. When the resisting moment is applied to the pinion-shaft, the increase of pressure between the teeth, due to friction, over that which would be required were there no frictional resistance, is greater than that when the larger gear is the driven. The ratio of the increase in the two cases is inversely as the radii of the gears. The power required to drive a gear is proportional to the pressure against its teeth if the coefficient of friction remains constant. The change of pressure on the bearings would probably cause a slight change in this coefficient, but even if this should occur, the amount of driving power to be applied to the large gear when driving the small one, in order to deliver a given amount of power to an operating-machine, would be greater than that necessary if the driving gear were smaller than the driven.

BEVEL-GEARS.

32. Strength of bevel-gear teeth.—In order to investigate the nature of the pressure between the teeth of an accurately constructed and adjusted pair of bevel-gears, and of the fibre-stress in the material, let it first be assumed that they are not rotating, and that there is no pressure between their teeth. Then assume that one is locked so as to prevent its turning, and that a turning force is applied to the other, thus producing pressure between the engaging teeth.

On account of the elasticity of the material, a slight deflection

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* "Experiments on the Transmission of Power by Gearing," by Wilfred Lewis. Trans. Am. Soc. Mech. Engrs., vol. VII., 1886, p. 273.

.of the teeth will be caused by the pressure on them. The correspondingly slight rotation thus allowed in the other parts of the gear to which the turning force is applied, will cause a point at the top of the large end of a free tooth to move through a linear distance whose ratio to the corresponding linear motion of a point similarly situated at the small end of the same tooth is equal to the ratio of the radial distances of the two points from the axis of the gear.

If the line of contact between a pair of engaging teeth is at the top of the working surface of a tooth on the locked gear, the top of the tooth will be deflected more at the large end of the gear than at the small, the deflection of any point along the top of the tooth being proportional to its radial distance from the axis of the gear. The linear dimensions of the tooth profile at the large and small ends of the tooth are proportional to the radii of the addendum-circles at the ends of the gear. The profiles are, of course, similar. Hence, in accordance with the property of cantilevers of similar profile that (referring to Fig. 56), when the breadth b remains constant, as well as the ratio of the length l to the height h, the linear deflection at the end is proportional to the load P, it can be seen that the distribution of pressure along the line of contact is in proportion to the radial distances from the axis of the gear, and also in proportion to the linear dimensions of the tooth sections. And, again in accordance with the property of cantilevers of similar profile, that, when the breadth b remains unchanged, the end deflection that will produce the same maximum fibre-stress S in each is proportional to the linear dimensions, it is evident that the maximum fibre-stress is the same in the tooth from end to end of the gear.

If H is the large, and h the small, addendum radius of the gear, then the resultant pressure against a tooth acts at a radial distance equal to $2(H^2 - h^2) \div 3(H^2 - h^2)$ from the axis of the gear.

The mean value of the pressure per unit length of the line of tooth contact equals the total or resultant pressure divided by the width of gear-face. In a tooth under pressure, this mean value is exerted at the middle of the gear-face. Therefore, a spur-gear having the same face width, and teeth of the same form and size as those of the bevel-gear at the middle of its face, is of the same

strength as the bevel-gear. If m is taken as the mean addendum-radius of the bevel-gear, and α is the angle between the axis of the bevel-gear and the elements of the addendum-cone, the radius of the spur-gear must be m sec α, in order to agree with the system of gear-teeth used for the bevel-gear.

As in the case of spur-gears, the pitch-surface dimensions may be used without serious error for practical forms of gears. The dimensions of the pitch-cone will therefore be used hereafter instead of those of the addendum-cone.

Fig. 65 is a section of a bevel-gear. The dimensions given are:
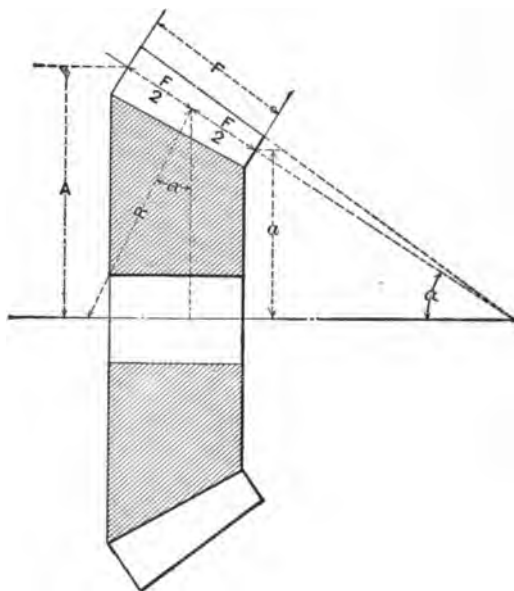
FIG. 65.

$A =$ largest pitch radius of bevel-gear;

$a =$ smallest pitch radius of bevel-gear;

$C = \dfrac{A + a}{2A} C' =$ circular pitch at middle of gear-face;

$C' =$ circular pitch at large end of gear;

$F =$ width of gear-face, measured on pitch-cone;

$R =$ length of a normal to pitch-cone at middle of gear-face, measured between pitch surface and intersection with axis

of gear; this is the radius of the pitch-circle upon which the teeth are generated for the middle of the gear-face; $\alpha =$ angle between gear-axis and pitch-cone elements.

The notation for the pressure is:

$P_{(mean)} =$ mean value of pressure on bevel-gear tooth per inch of length of contact-line;
$P_{(res)} =$ resultant of all elementary pressures on bevel-gear tooth;
$P' =$ pressure which, if applied at the greatest pitch radius of the gear, would produce the same torsional moment on the gear-shaft as the actual acting pressure.

In accordance with the assumption just made, the resultant pressure $P_{(res)}$ acts at a radial distance $2(A^2 - a^2) \div 3(A^2 - a^2)$ from the axis of the gear. This distance is inconvenient to use in designing, it being more satisfactory to use the pitch radius A of the large end of the gear. The relation between P' and $P_{(res)}$ is expressed by the formula

$$P'A = P_{(res)}\frac{2}{3}\frac{A^2 - a^2}{A^2 - a^2}.$$

Whence

$$P' = P_{(res)}\frac{2}{3}\frac{A^2 - a^2}{A(A^2 - a^2)}.$$

The relation between $P_{(mean)}$ and $P_{(res)}$ is given by the formula

$$P_{(res)} = FP_{(mean)}.$$

Therefore

$$P' = \frac{2}{3}\frac{A^2 - a^2}{A(A^2 - a^2)}FP_{(mean)}. \quad . \quad . \quad . \quad (33)$$

The value of $P_{(mean)}$ corresponds to those given on the scale of " pounds pressure at tooth-point " in Figs. 58 and 59. The value of $P_{(mean)}$ therefore can be found on one of these diagrams by using: gear diameter $2R$; circular pitch C, taken at the middle of the bevel-gear face; and any given or assumed working fibre-stress S.

If the force P' is given, and the pitch C' required for a given value of S and a specified gear-blank, the value of $P_{(mean)}$ is first found by the equation

$$P_{(mean)} = \frac{3}{2} \frac{A(A' - a')}{A' - a'} \frac{P'}{F}, \quad \ldots \quad (34)$$

and then, by the diagram, the value of C, from which C' can be determined.

Example.—Let $A = 10$ inches, $F = 4$ inches, $\alpha = 50°$, $C' = 1.0472$ inches (corresponding to 3 diametral pitch), and $S = 3000$ pounds per square inch.   Then:

$$a = 10 - 4 \sin 50° = 10 - 4 \times .766 = 6.936 \text{ inches};$$

$$C = \frac{A + a}{2A} C' = (16.94 \times 1.047) \div 20 = 0.887 \text{ inch};$$

$$R = \frac{(A + a)}{2} \sec \alpha = \frac{16.94}{2} 1.556 = 13.18 \text{ inches};$$

$$2R = 26.36 \text{ inches.}$$

The value of $P_{(mean)}$ for 0.887 of an inch pitch, a pitch diameter of 26.36 inches, and 3000 pounds per square inch fibre-stress is found on the diagram Fig. 58 to be 260 pounds.   Therefore, by equation (33),

$$P' = \frac{2}{3} \frac{(10)' - (6.94)'}{10[(10)' - (6.94)']} 4 \times 260 = 900 \text{ pounds.}$$

The horse-power that would be transmitted by the gear at 100 revolutions per minute is

$$\text{H.P.} = \frac{900 \times 2\pi \times 10 \times 100}{33000 \times 12} = 9.52.$$

Example.—Take $P' = 1000$ pounds, $S = 2000$ pounds per square inch, and the dimensions of the gear-blank the same as those in the preceding example.   The pitch C' is required.

By equation (34)

$$P_{(mean)} = \frac{3}{2} \frac{10(10)^2 - (6.94)^2}{(10)^2 - (6.94)^2} \frac{1000}{4} = 292 \text{ pounds,}$$

In the diagram Fig. 58, $C = 1.65$ inches. Therefore

$$C' = \frac{2AC}{A+a} = \frac{2 \times 10 \times 1.65}{10 + 6.94} = 1.95 \text{ inches.} \quad \text{(End of example.)}$$

The value of $P_{(mean)}$ in equation (33) can also be determined by formula (11), P for spur-gears and $P_{(mean)}$ for bevel-gears being identical. Substituting, in equation (33), the value of $P_{(mean)}$ as given in equation (11) changes (33) to the form

$$P' = \frac{2}{3} \frac{A' - a'}{A(A'-a')} CFS \left[0.106 - 0.215 \frac{C}{2R} \right]. \quad . \quad . \quad (35)$$

The value of $P_{(mean)}$ as given by formulas (21) and (25) can also be used for the system of gears that they represent.

The solution of the next to the last example by equation (35) gives

$$P' = \frac{2}{3} \frac{(10)^2 - (6.94)^2}{10(10)^2 - (6.94)^2} 0.887 \times 4 \times 3000 \left[0.106 - 0.215 \frac{0.887}{26.36} \right]$$
$$= 900 \text{ pounds.}$$

Problem.—Find the size of a pair of bevel-gears to transmit 40 horse-powers, and another pair to transmit 100 horse-powers, with a velocity ratio of 5 between a pair of shafts at right angles to each other; the pinion to have 16 teeth with a width of gear face equal to 2.5 times the circular pitch at the large end of the gear, to run at 100 revolutions per minute, and work at a fibre-stress of 10,000 pounds per square inch; the supports to be rigid and the teeth accurately formed; gears to be of the smallest possible dimensions to fulfil these conditions.

The direct solution for obtaining the required sizes of the gears is not as convenient as to assume some dimensions of the gears and find their capacity for transmitting power. Then,

by use of the data thus obtained, the size of the required pair of gears can be determined.

Accordingly, it will be assumed that the circular pitch (at the large end of the pinion) is 2 inches, then the width of the face is

$$F = 2.5 \times 2 = 5''.$$

The pitch radius at the large end of the pinion with 16 teeth is

$$A = \frac{16 \times 2}{2\pi} = 5.093''.$$

For a velocity ratio of 5 with shafts at 90°, the tangent of the half pitch-cone angle of the small gear is

$$\tan \alpha = .2; \text{ whence } \alpha = 11° \ 19'.$$

The pitch radius at the small end of the pinion is, for the face width $F = 5''$,

$$a = 5.093 - 5 \sin \alpha = 5.093 - 5 \times .1962 = 5.093 - .981 = 4.112''.$$

The mean circular pitch is

$$C = \frac{A+a}{2A}C' = \frac{5.093 + 4.112}{2 \times 5.093}2 = \frac{9.205}{5.093} = 1.807''.$$

The equivalent diameter of a spur-gear having the same form of tooth as that at the middle of the face of the bevel pinion is

$$2R = (A+a) \sec \alpha = 9.205 \sec 11° \ 19' = 9.205 \times 1.019 = 9.38''.$$

The mean value of the pressure against the point of one tooth, as found by equation (11), is

$$P_{(\text{mean})} = CS\left(.106 - .215\frac{C}{D}\right) = CS\left(.106 - \frac{.215 \times 1.807}{9.38}\right)$$
$$= 1.807 \times 10,000 \ (.106 - .0414) = 18070 \times .065 = 1170.$$

The equivalent pressure at the largest pitch diameter of the pinion is, by equation (33),

$$P' = \frac{2}{3} \times \frac{(5.09)^3 - (4.11)^3}{5.09[(5.09)^2 - (4.11)^2]} \times FP_{(\text{mean})}$$
$$= \frac{2}{3} \times \frac{131.87 - 69.43}{5.09 \ (25.91 - 16.89)} \times FP_{(\text{mean})}$$
$$= \frac{2}{3} \times \frac{62.44}{5.09 \times 9.02} \times 5 \times 1170 = 5300 \text{ pounds.}$$

The power that the gears will transmit is

$$\text{H.P.} = \frac{5300 \times 100 \times 2\pi \ 5.093}{33000 \times 12} = 42.8.$$

The dimensions of the pinion for transmitting a different amount of power can now be determined by making use of the fact that, for similar gears working at the same fibre-stress and speed of rotation,

$$\frac{\text{H.P. of smaller}}{\text{H.P of larger}} = \frac{(\text{Radius of smaller})^3}{(\text{Radius of larger})^3}.$$

Accordingly, for this case, the pitch radius of the pinion for transmitting 40 H.P. is

$$A' = A\sqrt[3]{\frac{40}{42.8}} = A\sqrt[3]{.935} = 5.093 \times .978 = 4.98'',$$

which corresponds to a pitch diameter of 9.96 inches at the large end of the pinion.

The circular pitch to agree with 9.96 inches diameter and 10 teeth is

$$C' = \frac{9.96}{2 \times 5.093} \times 2 = 1.96''.$$

Width of gear-face $= 5\sqrt[3]{\frac{40}{42.8}} = 5 \times .978 = 4.89''.$

To transmit 100 H.P. the dimensions of the pinion will be

$$A' = A\sqrt[3]{\frac{100}{42.8}} = A \times \sqrt[3]{2.336} = 5.093 \times 1.32 = 6.72'',$$

which corresponds to a diameter of 13.44 inches, and

$$C' = \frac{13.44}{2 \times 5.093} \times 2 = 2.64''.$$

Width of gear-face $= F = 1.32 \times 5 = 6.6''.$

The side and end wear of the supporting bearings of a pair of bevel-gears, caused by the pressure between their teeth, both tend to localize the pressure between the teeth at the large ends. In allowing for such wear, when calculating the strength of the teeth, it is therefore correct to assume that the load is carried by a part of the larger end of the tooth. Probably a width of gear-face equal to the circular pitch at the large end is as great as can be safely taken when the apex angle a of the pitch-cone approaches near to 90°; when this angle is very small the gear becomes more like a spur-gear, and the width of gear-face may be increased to 1.5 times the circular pitch. Allowance for localization of pressure at the small end of the tooth may be made in the same manner as for the large end.

Such localization may be caused by foreign matter between the teeth, settling of supports, etc.

Mortise bevel-gears are very commonly used. Shrouding the teeth is also practised to a considerable extent. Teeth corresponding to the double helical teeth of spur-gears are less frequently adopted; these do not, of course, have an end sliding motion like that of screw-bevel and screw gears. A machine for cutting bevel-gears whose elements do not intersect at the apex of the pitch-cone has recently been put into practical use.

33. **The efficiency of bevel-gearing** is less than that of spur-gearing, for the reason that the pressure between the teeth causes an end thrust on the bearings, in addition to the side pressure corresponding to that of spur-gears. The frictional resistance due to this end thrust causes a reduction of efficiency.

<div align="center">FRICTION-GEARS.</div>

34. **Friction-gears,** having smooth surfaces held in contact under pressure, and transmitting power by means of the frictional resistance between their surfaces, find a very considerable application in certain classes of machinery, notably that which is frequently stopped and started, and whose source of power is a constantly rotating shaft or pulley; they also afford a convenient method of obtaining different speeds when the primary shaft of a machine rotates uniformly. Less frequently they are used for transmitting power between two uniformly rotating shafts.

When a considerable amount of power is transmitted, and the pressure between the gears is heavy, one of a pair of gears which are in contact, is generally made of iron, and the surface of the other of some material such as wood, paper, leather, hard rubber, etc.

35. **Cylindrical friction-gears.**—The turning force P which can be exerted by one smooth cylindrical friction-gear upon another when they are pressed together with a force F, the coefficient of friction of the material being μ, is given by the formula

$$P = \mu F.$$

The coefficient of friction μ for paper friction-wheels, as determined by a series of experiments made by Prof. W. F. M. Goss,[*]

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[*] Trans. Am. Soc. Mech. Engrs., vol. XVIII., 1897, p. 102.

is given below.   In these experiments one of the pair of wheels in
contact was iron, and the other of compressed straw-board in " thin
disks cemented together under heavy pressure and strenghtened by
iron side-plates, or fitted over iron centres." The edges of the
disks were pressed against the iron wheel when power was trans-
mitted.

The experiments were made upon paper friction-wheels approxi-
mately $5\frac{1}{2}$, 8, 12, and 16 inches in diameter, all in contact with a
16-inch cast-iron wheel.   " The contact pressure was varied from
75 pounds per inch of width to more than 400 pounds, and the
speed limits gave a peripheral velocity varying from 450 to 2700
feet per minute."

It was found that:

The coefficient of friction μ increases as the rate of slipping
between the gears increases;

When the slip is as great as 3%, there is apt to be a sudden
increase in its value to 100%; i.e., motion ceases to be transmitted
to the driven wheel;

" The coefficient of friction is apparently constant for all pres-
sures of contact up to a limit which lies between 150 and 200
pounds per inch of width of wheel-face, beyond which limit its value
apparently decreases ";

" Friction-wheels of 8, 12, and 16 inches diameter give nearly
the same value for the coefficient, while results from a 6-inch wheel
are lower by about 10% ";

" Variations in peripheral speed between 400 and 2800 feet per
minute do not affect the coefficient of friction."

Fig. 66,* taken from Professor Goss's paper, shows the relation
between the slip and the coefficient of friction which he found could
be easily maintained with paper friction-wheels 8 inches or more in
diameter.

It is probable that the coefficients of friction for the other
materials that are most commonly used in contact with metal for
friction-gears operating under comparatively high pressures, are
lower than those generally found by the laboratory experiments
where plane surfaces are moved over each other at unit pressure

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* Trans. Am. Soc. Mech. Engrs.; vol. XVIII., 1897; p. 103.

very much lower in comparison. It is believed that the following values of μ are as high as can be safely used for pressures of 100 pounds or more per inch of width of gear-face:

Metal on metal	0.2
Leather on metal	0.3
Wood on metal	0.3

A system of cylindrical friction-gears that is quite commonly used where it can be applied, and which has many advantages in
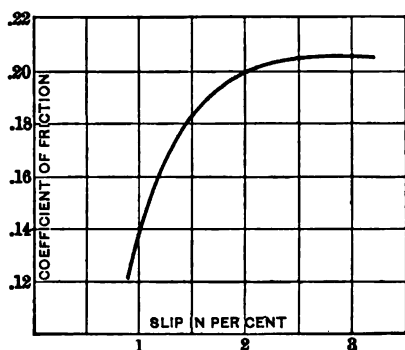
FIG. 66.

the way of economy of construction and ease of operation, is shown in Fig. 67. The larger gears, which are of iron, are placed on the driving and driven shafts, whose centres remain at a fixed distance apart. The small gear, intermediate between the other two, is so supported that it can be pressed against the others, or withdrawn from contact with them, at will. It is generally made of some of the so-called "friction materials," and thus affords a means of securing a high coefficient of friction, and a somewhat elastic surface in contact with the more rigid iron.

In a pair of friction-gears having different materials on their working faces, it is advisable to use the softer material on the driver; then, in case of excessive slipping, there is not so much danger that a flat place will be worn in the driven one.

Another system of cylindrical friction-pulleys that has been found satisfactory in some cases, where the distance between the centres of the shafts can be varied, consists of two pulleys, one on

the driving shaft and the other on the driven, and an endless belt of leather fitting loosely around one, which is flanged to hold it in place. The face of the second pulley is slightly narrower than the space between the flanges of the other, so that when the two are drawn together the belt is pressed between the pulley faces and forms a cushion against which they work. The coefficient of fric-
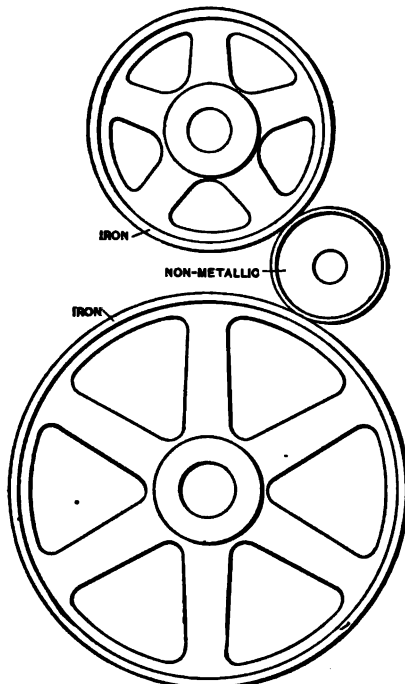
FIG. 67.

tion in such a system is, of course, that of leather on iron. This device has advantages in its simplicity and comparatively low cost of construction. It is hardly applicable for the transmission of large amounts of power.

The power that can be transmitted by a pair of cylindrical friction-gears in practical operation is about the same per inch of face width as that per inch of width of the kind of flat leather belt that would be used on the same machinery. It depends, of course,

upon the pressure and coefficient of friction. When a given amount of horse-power H.P. is transmitted, the value of P can be found by equation (17), § 25.

The efficiency of friction-gears depends very largely upon the frictional resistance of their supporting journals. The heavy pressures between them, necessary to produce the required turning force, brings a correspondingly high pressure upon the journals, with its attendant frictional loss. While no efficiency tests upon this class of transmission machinery seem to have been made public, it would appear probable that the friction loss in it is much greater than in spur-gears or belt-connected pulleys constructed with the same care.

36. Grooved friction-gears, having their surfaces grooved circumferentially, are used when it is desired to have a tight grip between them without excessive pressure upon the bearings. Fig. 68 shows a section through the rims of such a pair of gears in con-

FIG. 68.
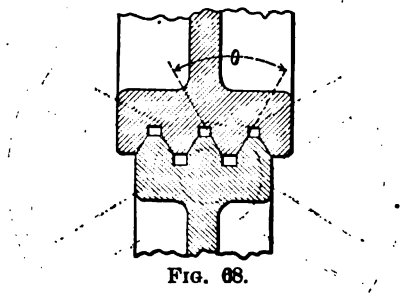
tact, cut by a plane passing through the axes of both gears. The wedge-like action gives them increased holding power.

There is a sort of rubbing or grinding action between a pair of grooved friction-gears in action which is detrimental to both their life and efficiency of service. The grinding action increases with the radial depth of the engaging surfaces; hence, if such gears are used, it is better to make them with a large number of shallow grooves than a few deep ones.

Both of a pair of grooved friction-wheels are frequently made of the same metal, cast iron being most used. Better service can be obtained by making them of such metals or alloys as work well together in journal- and step-bearings under heavy pressure.

The angle θ between the sides of a groove should seldom be smaller than 30°, but the increase of grip over that of a pair of smooth cylinders is not great if the angle exceeds 40°.

The total pressure acting between the sides of the grooves of a pair of gears with parallel axes, when the gears are pressed together with a force F normal to their axes, the angle of the groove being θ, is $F \csc \dfrac{\theta}{2}$; and the tangential or turning force acting at their pitch-point is approximately $\mu F \csc \dfrac{\theta}{2}$, in which $\mu =$ coefficient of friction between the materials of the grooves.
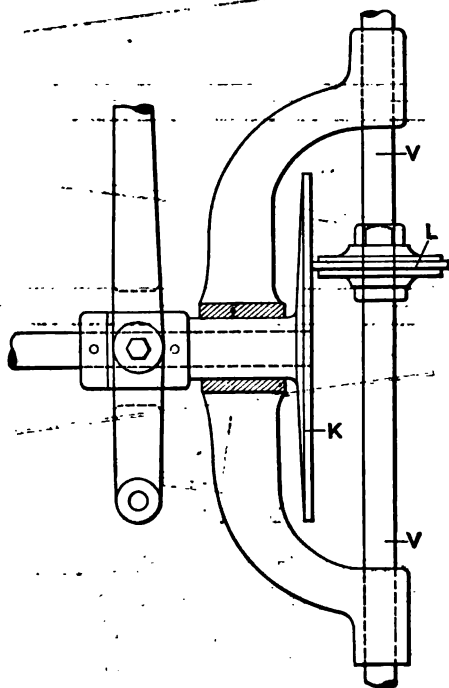
FIG. 69.

37. Friction bevel-gears are often used for connecting intersecting shafts where the same conditions of operation as have been mentioned for cylindrical friction-gears exist. In addition to the

side pressure on the bearings, there is an end thrust which may be so great as to require special provisions, in the way of a step- or collar-bearing, to withstand it when the apex angle α of the gear-cone is large and the service is heavy. If F is the normal pressure between the gears, and α the angle between the axis of the gear and its face, then the value of the end thrust E may be expressed by the equation

$$E = F \sin \alpha.$$

38. Crown friction-gears are used on light machinery where it is desired to vary the speed of the driven shaft while the driver runs
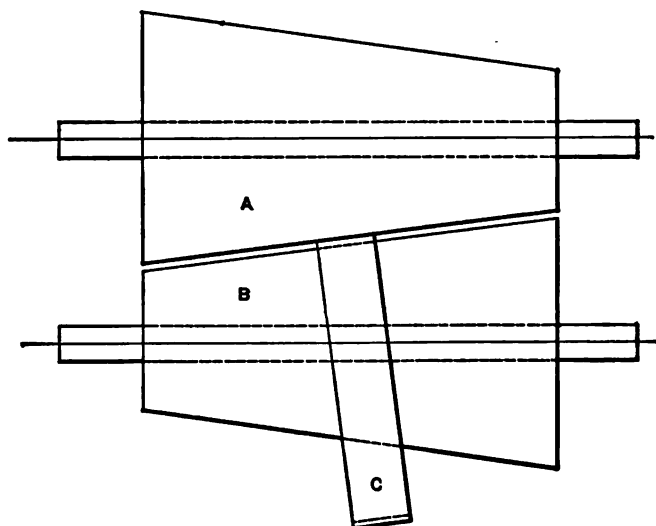
FIG. 70.

uniformly. Fig. 69 represents such a device. A ring L, of leather or other suitable material, is held between a pair of disks on the shaft V, which is in the same plane, and at right angles to the axis of the metal disk K, against which L presses.

If L is the driver, moving it across the face of K will change the speed of the latter, and the direction of K's rotation will be reversed by moving L across its centre. When possible L should be the driver, since it is faced with the softer material. When this method of driving is used on a drill-press, where the force required

to turn the drill-spindle increases as the speed of rotation decreases, as is the case when changing from a small to a large drill-bit, L should always be the driver, since its lever-arm about the axis of K increases in the same proportion that the speed of K decreases, the speed of L remaining constant.

39. Double-cone, variable-speed friction-gears.—A special form of friction-gears for variable speeds is shown conventionally in Fig. 70. The mechanism consists of a pair of similar cones, A and B, with smooth surfaces, placed side by side on parallel axes, and separated a slight distance to allow a short endless belt C to pass between them. Their pressure against the belt serves to transmit power between them as they rotate. The belt is held in position by a shifter, not shown; by moving it along from end to end of the cones when they are running, their speed ratio can be varied.

CHAPTER III.

FLAT BELTS.

40. When a belt is transmitting power by its frictional resistance against the face of a pulley over which it passes, there must necessarily exist a difference of tension in it, at the points of tangency with the pulley, equal to the torsional force exerted upon the pulley. The tensile stress gradually increases from the point where the belt first comes in contact with a driven pulley to where it leaves it. The torsional force that can be transmitted to the pulley depends jointly upon the belt tensions, coefficient of friction between the belt and pulley face, radius of pulley, velocity at which the belt travels, and the weight of the belt per unit of length. The last three items must be included on account of the centrifugal force exerted on the belt as it passes around the pulley, which force reduces the pressure against its face, and is the principal factor which limits belt speed. The centrifugal force becomes greater as the speed increases, until, at high velocities, the belt is partly lifted from the pulley, and consequently but little turning force can be transmitted by it.

41. Equations for power transmission by flat belts.—By the use of the equations developed below, the torsional force that can be exerted by a belt, and the relations between all quantities involved, can be determined. The following notation is used:

A = sectional area of belt, square inches;

H.P. = horse-power transmitted;

P = total turning force exerted by the entire width of belt against the pulley = $T_n - T_o$, pounds;

R = radius of pulley or sheave, feet;

T_n = total tension in belt on tight side, pounds;

T_o = total tension in belt on slack side, pounds;

T_r = total tension in belt when at rest, pounds;

V = velocity of belt, feet per minute;

  c = centrifugal force for 1 cu. in. of belt, pounds;

g = acceleration due to gravity, = 32.2 ft. per sec. per sec.;

p = turning force transmitted by a belt having a cross-sectional area of 1 square inch, or by a rope 1 inch in diameter, pounds;

q = pressure of pulley against 1 linear inch of belt, pounds;

r = radius of pulley or sheave, inches;

t = tension, in pounds per square inch, at any section of the belt;

t_n = tension, in pounds per square inch, on tight side of belt = working tension of belt;

t_o = tension, in pounds per square inch, on slack side of belt;

t_r = tension, in pounds per square inch, on both sides of belt when at rest or running without load;

  v = velocity of belt, feet per second;

w = weight of belt per cubic inch, pounds;

α = arc of contact between belt and pulley, degrees;

θ = .01746α = arc of contact between belt and pulley, circular measure (radians);

μ = coefficient of friction between belt and pulley;

ϵ = 2.71828 = $10^{0.434294}$ is the base of the hyperbolic, Naperian, or natural logarithms.

The general equation for the centrifugal force acting on a body moving in a curved path is

$$(\text{centrifugal force}) = \frac{(\text{mass}) \times (\text{square of velocity})}{\text{radius of curvature}}.$$

The specific form for the present case is

$$c = \frac{w}{g}\frac{v^2}{R} = \frac{w(V \div 60)^2}{g(r \div 12)} = \frac{1}{9660}\frac{wV^2}{r} = .0001036\frac{wV^2}{r}. \quad . \quad (36)$$

Fig. 71 shows a pulley with a belt passing around it, the angle of contact being θ. The nature of the forces that act along the

arc of contact, and the relation between the belt tensions t_n and t_o, when the belt is just beginning to slip, may be determined by tak-
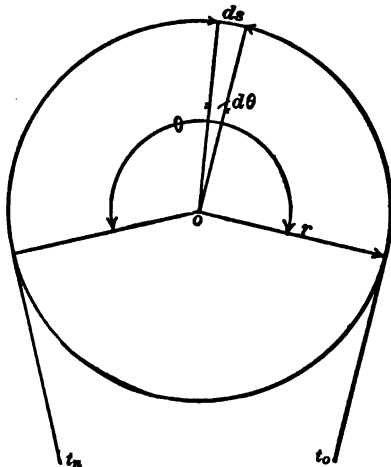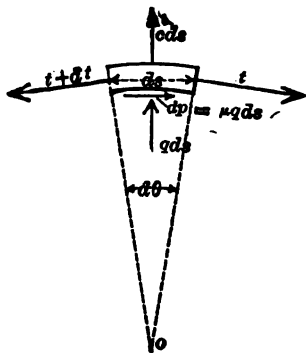
FIG. 71.   FIG. 72.

ing an elementary length of the belt $= ds$, embracing an angle $d\theta$ on any part of the arc of contact, and considering it as a free body, as shown in Fig. 72.

The forces which hold this elementary body in equilibrium are, first, t and $(t + dt)$, normal to the end sections and making an angle $d\theta$ with each other, whose resultant force acts to press the belt against the pulley; second, cds, the centrifugal force; third, qds, the pressure of the pulley against it; and fourth, $dp = \mu qds$, the frictional resistance along its surface where it is in contact with the pulley.

The resultant of t and $(t + dt)$ can be taken as normal to the surface of the belt, since dt is small in comparison with t. This resultant is counterbalanced by qds and cds, dp not having any component normal to the belt surface, since it acts along the surface. The following equation for the value of qds can now be written

$$qds + cds = t \sin \frac{d\theta}{2} + (t + dt) \sin \frac{d\theta}{2},$$

in which $d\theta$ is so small that its sine can be taken as equal to the

angle in circular measure, and $dt\left(\sin\dfrac{d\theta}{2}\right)$, being the product of two small quantities, can be neglected; the equation thus becomes

$$qds + cds = td\theta,$$

or

$$qds = td\theta - cds, \quad \ldots \ldots \ldots \ (37)$$

in which $ds = rd\theta$, and $c = \dfrac{wv^2}{gR}$ as given in equation (36). By substituting these values in equation (37) it becomes

$$qds = td\theta - \frac{wv^2}{gR}rd\theta$$

$$= td\theta - 12\frac{wv^2}{g}d\theta. \quad \ldots \ldots \ (38)$$

By equality of moments about O (Fig. 72),

$$t + dt = t + \mu qds,$$

whence

$$dt = \mu qds.$$

Substituting the value of qds as given in equation (38) in this equation, it takes the form

$$dt = \mu\left(t - 12\frac{wv^2}{g}\right)d\theta,$$

or

$$\frac{dt}{t - 12\dfrac{wv^2}{g}} = \mu d\theta.$$

By further solution

$$\int_{t_o}^{t_n} \frac{dt}{t - 12\dfrac{wv^2}{g}} = \mu\int_o^\theta d\theta,$$

whence

$$\text{Hyperbolic log.}\left(\frac{t_n - \dfrac{12wv^2}{g}}{t_o - 12\dfrac{wv^2}{g}}\right) = \mu\theta.$$

By substituting for v its equivalent value $V \div 60$, and reducing,

$$\frac{t_n - .0001036 w V^2}{t_o - .0001036 w V^2} = \epsilon^{\mu\theta} = 10^{.00758\mu a}. \quad \dots \quad (39)$$

The turning force per square inch of area is

$$p = (t_n - .0001036 w V^2) - (t_o - .0001036 w V^2)$$

$$= t_n - t_o = (t_n - .0001036 w V^2)\left(\frac{\epsilon^{\mu\theta} - 1}{\epsilon^{\mu\theta}}\right) \quad \dots \quad (40)$$

$$= (t_n - .0001036 w V^2)\left(\frac{10^{.00758\mu a} - 1}{10^{.00758\mu a}}\right) \quad \dots \quad (41)$$

From equation (39),

$$t_n = t_o \epsilon^{\mu\theta} - .0001036 w V^2(\epsilon^{\mu\theta} - 1) \quad \dots \quad (42)$$

$$= t_o 10^{.00758\mu a} - .0001036 w V^2(10^{.00758\mu a} - 1). \quad (43)$$

And from equations (40) and (41),

$$t_n = p\frac{\epsilon^{\mu\theta}}{\epsilon^{\mu\theta} - 1} + .0001036 w V^2 \quad \dots \quad (44)$$

$$= p\frac{10^{.00758\mu a}}{10^{.00758\mu a} - 1} + .0001036 w V^2. \quad \dots \quad (45)$$

TABLE XI.

ANGLE OF CONTACT α ON THE SMALLER OF A PAIR OF PULLEYS DIRECTLY CONNECTED TOGETHER BY AN OPEN BELT. NO ALLOWANCE FOR SAG OF BELT.

$D =$ diam. of large pulley ; $d =$ diam. of small pulley ; $C =$ distance between pulley-centres.

$\frac{D-d}{C}$	Angle of Contact. Degrees.	$\frac{D-d}{C}$	Angle of Contact. Degrees.	$\frac{D-d}{C}$	Angle of Contact. Degrees.
.05	177.18	.80	162.74	.55	148.07
.10	174.27	.85	159.84	.60	145.07
.15	171.87	.40	156.90	.65	142.07
.20	168.50	.45	153 67	.70	139.00
.25	165.64	.50	151.04	.75	135.84

Example.—Find the sectional area of a belt to transmit 200 H.P. when running at 4500 feet per minute, and working at 300 pounds tension per square inch on the tight side; the arc of contact on the working pulley having the smallest portion of its circumference embraced by the belt, being 160°.

The value of the turning force P which must be exerted by the belt is found by the equation

$$P = \frac{33000 \times \text{H.P.}}{V}; \quad \cdots \cdots \quad (46)$$

which becomes, by the substitution of numerical values,

$$P = \frac{33000 \times 200}{4500} = 1467 \text{ pounds.}$$

Assuming that good fulled leather is to be used, the weight per cubic inch may be taken as $w = .035$ of a pound, and the coefficient of friction may be taken as $\mu = 0.3$.

By equation (41) the turning force which a belt having 1 square inch of sectional area will transmit is

$$p = \left[300 - .0001036 \times .035(4500)^2\right]\left[\frac{10^{.00758 \times .3 \times 160} - 1}{10^{.36384}}\right]$$

$$= \left[300 - 73.4\right]\left[\frac{2.311 - 1}{2.311}\right] = 226.6 \times \frac{1.311}{2.311} = 128 \text{ pounds.}$$

The total area of belt required is therefore

$$A = P \div p = 1467 \div 128 = 11.45 \text{ sq. in.}$$

The total tensions are

$$T_n = At_n = \frac{P}{p}t_a = 11.45 \times 300 = 3435 \text{ pounds;}$$

$$T_o = T_n - P = 3435 - 1467 = 1968 \text{ pounds.}$$

If a belt .5 of an inch thick is used, it would have to be about 23 inches wide.

The turning force per inch of belt-width for a belt 23 inches wide would be $1467 \div 23 = 63.8$ pounds.

A tension of 400 pounds per square inch is sometimes used, but is higher than is advisable for a belt that works nearly or quite up

to its rated capacity continuously; from 200 to 300 pounds per square inch is more advisable.

When the belt is at rest, the tensions are practically equal in both stretches between pulleys. The sum of the tensions, $T_n + T_o$, is greater when a belt is transmitting power than when running idle or standing still. For a horizontal belt working at 300 pounds tension per square inch on the tight side, and having 2% slip (i.e., the surface of the driven pulley moving 2% slower than that of the driver) on cast-iron pulleys, the increase of the sum of the tensions may be taken, for speeds up to 1000 feet per minute, as an average of about $\frac{1}{8}$ of the value when the belt is idle. At high speeds the centrifugal action tends to reduce this increase, until, at the speed giving a centrifugal tension equal to the initial tension, they become equal on the two sides, and of the same value as when the belt is idle. It is on the safe side, so far as the stress in the belt is concerned, to allow for the full increase of tension, however, hence it will be so taken. Calling the tension in each side, when the belt is at rest, T_r, the relation between T_r and the sum of the working tensions is expressed by the equation

$$\tfrac{9}{8}(2T_r) = T_n + T_o;$$

whence

$$T_r = \tfrac{8}{9}(T_n + T_o).$$

The value of T_n has just been found above; that of T_o may be obtained by the equation

$$T_o = T_n - P.$$

Substituting this value of T_o in the last equation gives

$$T_r = \tfrac{8}{9}(2T_n - P) = \tfrac{8}{9}[(2 \times 3435) - 1467] = 2026 \text{ pounds.}$$

This is the tension that must be maintained in the belt, when at rest or running idle, in order that it shall not have more than 2% of slip when working under the full load for which it is designed. It corresponds to a tension t_r per unit area of section, when the belt is at rest, of

$$t_r = T_r \div A = 2026 \div 11.45 = 176 \text{ pounds per square inch.}$$

The required tension may be fairly well maintained by having the pulleys so that the distance between their centres can be adjusted, or by using an adjustable idle pulley as a tightener. When no means of pulley adjustment is used, the belt must be shortened at intervals in order to allow for the permanent stretch that continually occurs throughout its life of service. To allow for this stretch, the belt must, at each adjustment, be made shorter than is necessary to produce the required tension of rest. The tension, when shortening, may be weighed by a pair of belt-clamps, fitted with springs and a graduated scale, used to draw the ends together while the belt is in its working position.

If, in the example just given, μ is taken as 0.4 instead of 0.3, the following values are found:

$$T_n = 2880; \quad T_r = 1610; \quad \text{and} \quad t_r = 168.$$

41.1. Tandem Belt Drive.—When a belt drives more than one pulley, conditions additional to those for a two-pulley drive
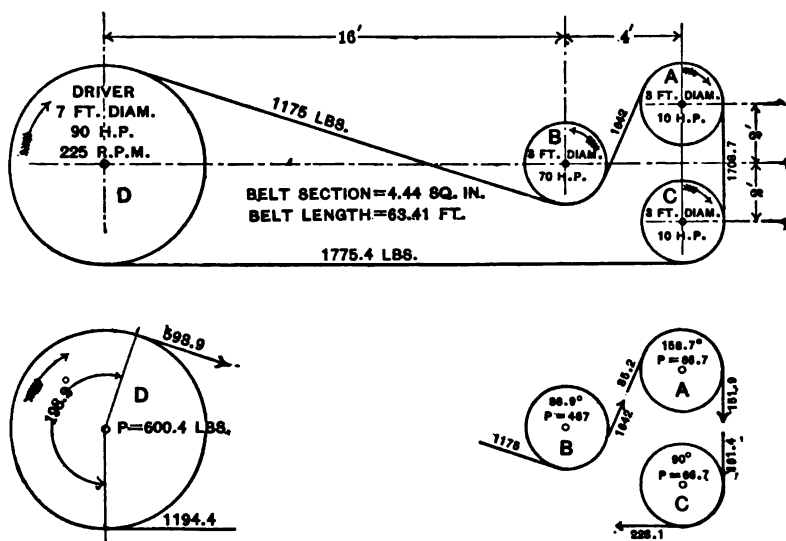
FIG. 72.1.

enter into the determination of the tensions in it. The tension that determines the size of belt may not be the highest found by

calculation for each pulley separately. It is sometimes higher than necessary for any one pulley of the group. This can be best illustrated by an example.

Fig. 72.1 shows four pulleys, three of which, A, B, and C, are to receive power from the driver D. In the upper part of the figure the arrangement, dimensions, direction of rotation, and horse-power to be delivered are given. In the lower part each pulley is shown separately, with the angle of belt wrap, turning force P, and belt tensions necessary to drive it considered apart from the others.

These tensions correspond to the assumed values

$$t_n = 400; \quad \mu = .3; \quad w = .035.$$

Inspection of the values found shows that B requires the highest tension, 1642 pounds. Since the turning force for B is 467 pounds, the tension of the slack side of B is

$$1642 - 467 = 1175 \text{ pounds.}$$

The tension on the slack side of D must therefore also be 1175 pounds since B and D are joined directly by the belt. And since the turning force for D is 600.4 pounds the tension on the tight side of D must be

$$1175 + 600.4 = 1775.4 \text{ pounds.}$$

This must therefore be the tension in the stretch of belt joining D and C. This stretch of belt is also on the tight side of C. The turning force for C is 66.7 pounds. The tension on the slack side of C is therefore

$$1775.4 - 66.7 = 1708.7 \text{ pounds.}$$

This is also the tension on the tight side of A. The turning force for A is 66.7 pounds. Therefore the tension on the slack side of A is

$$1708.7 - 66.7 = 1642 \text{ pounds,}$$

which agrees with that found for the tight side of B, as it should.

The tensions found for the pulleys collectively are either equal to or greater than those calculated for each one taken individually. They are therefore satisfactory.

It may be noted that the sum of the turning forces for the driven pulleys A, B, and C must be equal to that for the driver D. Also that these turning forces are divided among the driven pulleys in proportion to the power to be delivered to them. This would still remain true if the driven pulleys were of different diameters.

In Fig. 72.2 the only change of conditions from the above is in the way the belt passes around the pulleys. The necessary belt
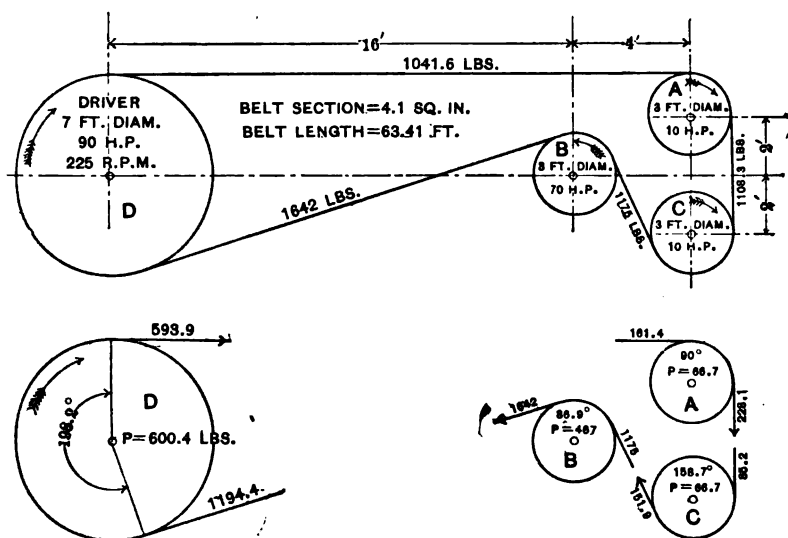
FIG. 72.2.

tensions for each pulley, considered individually, are shown in the same manner as before. The 1642 pounds for D is the highest tension found for any one pulley. The tension on the slack side of B is 1175, which must also be the tension on the tight side of C. Reducing this by the turning force 66.7 pounds gives 1108.3 pounds for the slack side of C and tight side of A. Reducing this again by 66.7 pounds gives 1041.6 for the slack side of A and also of D. Adding to this 600.4 gives 1642 pounds for the tight side of D, which agrees with that for B.

The tension in each stretch as just found and of the values shown in the upper part of the figure are either equal to, or greater

than, that calculated for any individual pulley; therefore the 1642 pounds determine the area of belt.

Fig. 72.3. The only change in the drive is in the method of wrapping the belt around the pulleys. All quantities are shown in the same manner as before. Inspection shows that the highest tension calculated for any individual pulley is 1263.5 pounds for driver D. The tension on the slack side of C is therefore 1263.5 − 66.7 = 1196.8 pounds.

This is also the tension of the tight side of B, which, reduced by the turning force of 467 pounds for B, gives 729.8 pounds for
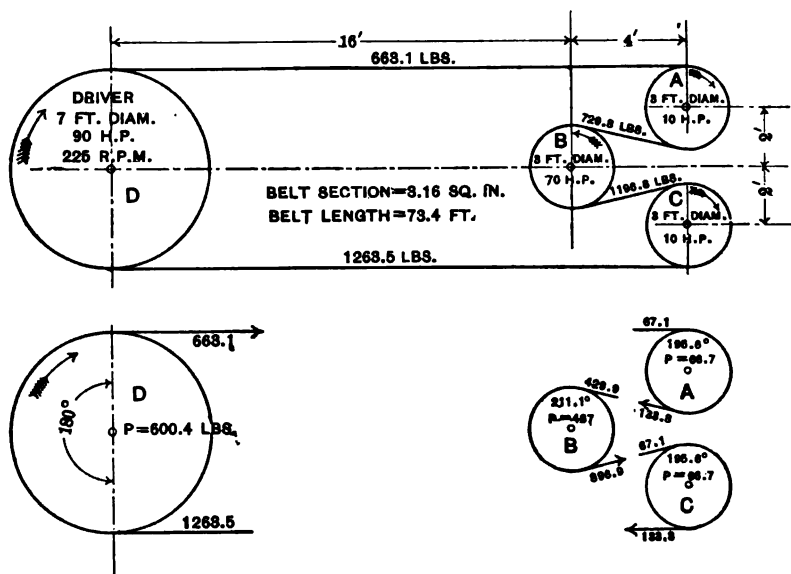
FIG. 72.3.

the slack side of B and tight side of A. Reducing this by the turning force of 66.7 pounds for A, gives 663.1 pounds for the slack side of A, which is the same as calculated for the slack side of D. The tension of 1263.5 pounds as calculated for the tight side of the driver D therefore determines the area of the belt section for this method of wrapping it around the pulleys.

The belt is more than 40% larger in cross-section in the first case than in the last. It is about 16% longer in the last case, however.
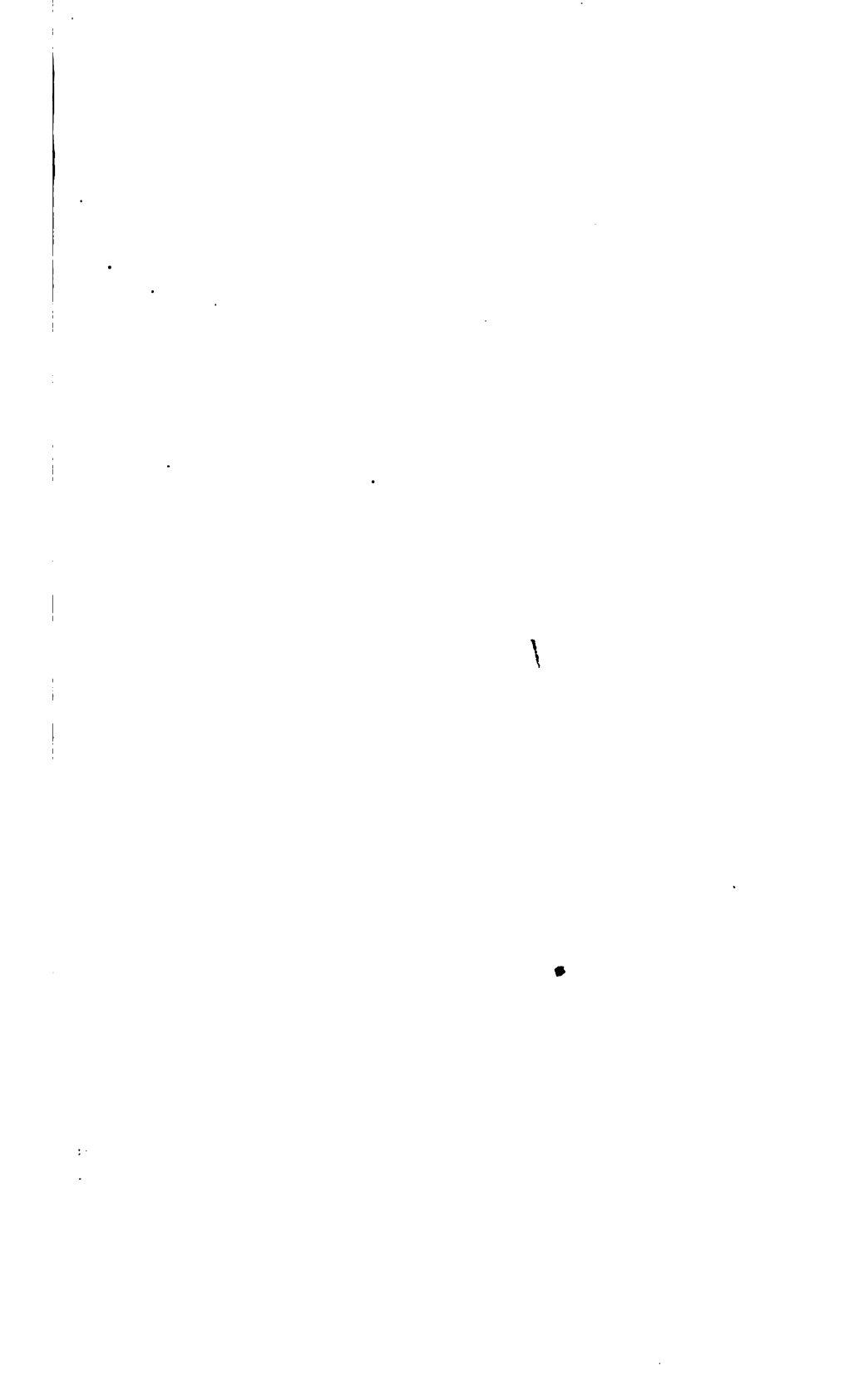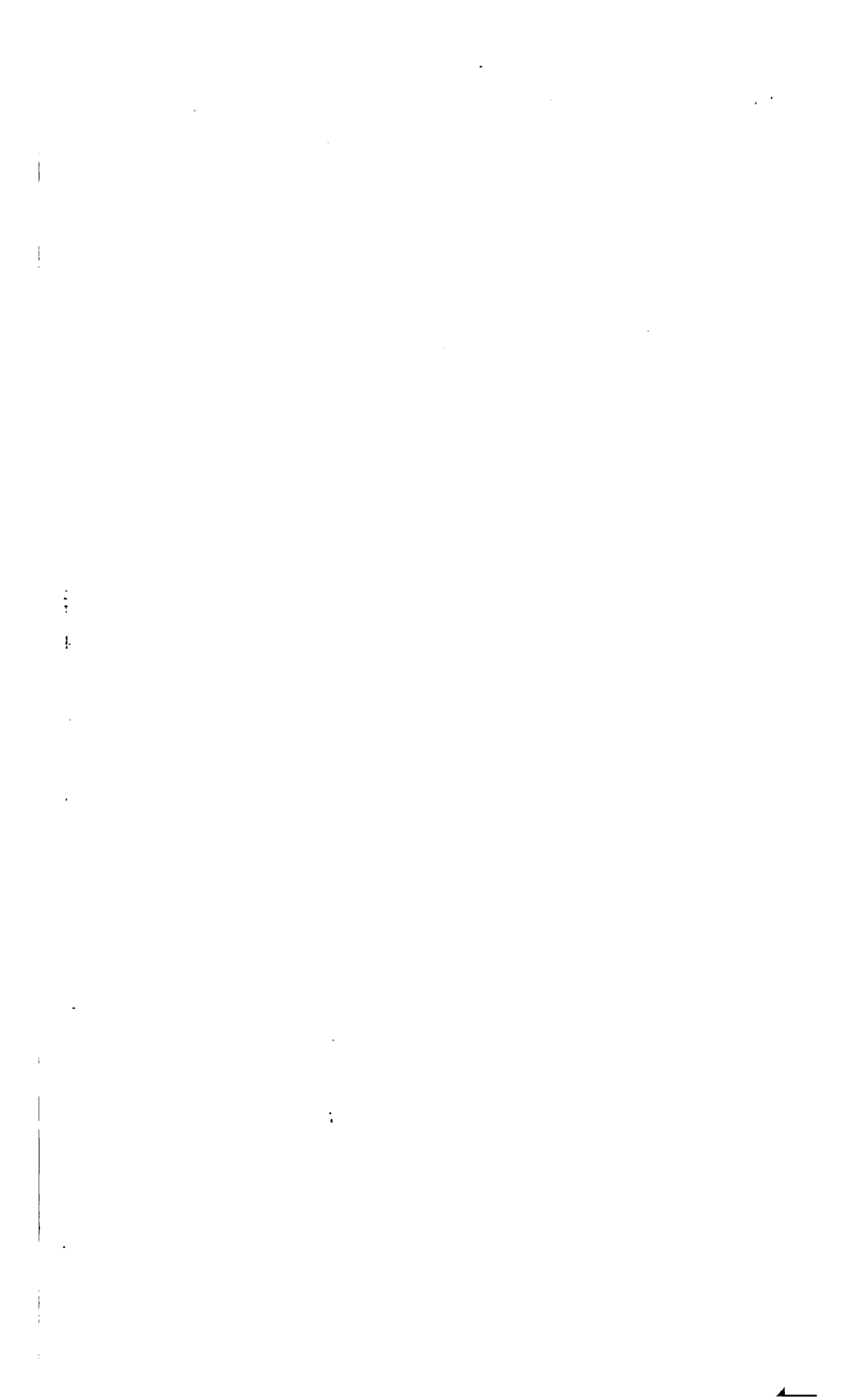
41.2. The diagrams, Figs. 72.4 and 72.5, may be used conjointly to find the sectional area of the belt in the example, § 41.

Fig. 72.4 is for belts having 180° angle of wrap, and Fig. 72.5 is for modifying the results to agree with the actual angle of wrap. For the example under consideration find, on the bottom, "velocity" scale of Fig. 72.4, a reading of 4500 feet per minute, and determine the intersection of the corresponding vertical line with the curve for $T = 300$ and $\mu = .3$; then go horizontally to the straight diagonal and, finally, vertically upward to the top scale. The reading thus found on this scale is 18.81 H.P., which is the amount of power 1 square inch of belt will transmit when making 180° of wrap, instead of 160°, as given.

By Fig. 72.5 it can be seen that a belt making 160° of wrap will transmit 7% less power, or .93 as much as one wrapping half way round the pulley.

The amount of power that 1 square inch of belt will transmit under the condition given is, therefore,

H.P. for 1 square inch section $= 18.81 \times .93 = 17.49$.

The sectional area of belt is

$$A = 200 \div 17.49 = 11.45 \text{ square inches.}$$

If the thickness of the belt is predetermined as .5 inch, then the H.P. for 1 inch of width can be found on the scale for a "belt 1 inch wide," as follows: Find as before the intersection of the 4500 feet per sectional line with the curve for $\mu = .3$ and $T = 300$. This intersection corresponds to a reading of 9.4 H.P. on the scale for a belt 1 inch wide and .5 inch thick, making 180° wrap.

Correcting for 160° wrap,

H.P. for section 1″ by .5″ $= 9.4 \times .93 = 8.74$.

And the width of belt is

Belt width $= 200 \div 8.74 = 22.9$ inches.

The diagram, Fig. 72.6, has a far wider range of usefulness than the pair just used. It can be applied to both belts and ropes of any kind of material. It is good for very much larger angles of wraps than the preceding diagram. While hardly so convenient

to use for the example just solved, it may be applied to the solution by way of illustration.

First obtain by calculation the product wv^2, in which $v = 75$ feet per second to correspond with the required 4500 feet per minute.

From this

$$wv^2 = .035 \times (75)^2 = 197.$$

Find on the left-hand scale of values of wv^2 the reading 197, go horizontally to the right to the intersection of the 197 line with the single oblique line to the left of the zero of the bottom scale and from the intersection drop down to the scale of "centrifugal tension," where the reading is 73 pounds. From the point just found go to the right a distance corresponding to 300 pounds. This gives a reading of 227 pounds on the scale of "effective tension."

The value of μa is $.3 \times 160 = 48$. From the point just found on the scale of effective tension go vertically upward to the oblique line (not drawn on diagram) for the value $\mu d = .3 \times 160 = 48$, and thence to the right to obtain a reading of the scale of "turning force exerted by belt on pulley face." The reading thus obtained gives the value of p as 128 pounds for 1 square inch of belt section. The completion of the problem is as in the solution given just after the statement of the example.

42. The coefficient of friction μ, and slip of leather belting.— The coefficient of friction is an exceedingly variable quantity, generally lying between 0.25 and 1.0 for leather in good working order running on smooth iron pulleys; its value is even greater than this in some cases. The coefficient is somewhat higher for the same belt on wooden pulleys with unvarnished faces than on iron, and leather-covered pulleys give still higher values. The varnished faces of new wooden pulleys give very high coefficients of belt friction. A special varnish is sometimes used on iron pulleys to increase μ. A change in the intensity of pressure between the belt and pulley affects the value to a slight extent, decreasing it as the pressure increases for belts that are sufficiently oiled or "dressed" to secure durability; for ordinary working conditions μ may be considered as constant between the limits of pressure that can be used without rapid wear of belting.

Corks inserted in the faces of iron or wood pulleys so that they project a small distance (.03 to .05 of an inch) above the face increase μ to some extent and reduce the slip greatly.

It has been found by several experimenters that the coefficient of friction increases as the slip of the belt over the pulley increases. The higher values of μ therefore appertain to a high rate of slipping.   If too much slipping occurs, there is danger that the heat generated will dry, or even burn, the surface of the belt, and thus not only weaken it, but at the same time reduce the coefficient of friction.

A slippage of 3% (i.e., a velocity of the driving-pulley face 3% faster than that of the driven) is probably as much as should be allowed in general practice, 2% being a good value.   The rate of slipping, at which the higher values of the coefficient are obtained, varies from 10% to as much as 20% in some cases.

Belt-driven machinery that works against a high resistance for a short interval, as is customary with punching and shearing machines, should not have a maximum reduction of speed, and consequent rate of belt-slip, greater than 20% during the working period.   Machines expending energy during a greater portion of each cycle should have a smaller variation of speed on account of the greater liability of the driving belt to slip from the pulleys.

With a belt in fair working condition, the coefficient of friction μ can safely be taken as 0.3 when the slippage is as much as 2% on iron and unvarnished wood pulleys; probably 0.4 is allowable in a majority of cases.   Fair working condition means that the surface next the pulley is not hard, dry, or cracked, or the belt stiff for the lack of belt dressing, but soft, pliable, clean, and running without excessive vibration.

When a belt becomes hard and dry, it can be softened, and the coefficient of friction increased, by the application of a suitable belt dressing, provided the belt has not been too long neglected.

43. Working strength of leather belting.—Tension causes a leather belt to elongate continuously throughout its life; upon removal of the tensile stress it will partly return to its original length, but a permanent elongation always remains.   Both the total and permanent elongations increase much more rapidly during the earlier part of its life than later, if the belt is always used under the same conditions.   During the time of its efficient service the elongation is very nearly uniformly distributed throughout the belt; but when it has elongated a certain percentage of its length, the

stretching becomes uneven, and consequently the belt soon gets crooked and unfit for service.

It may be stated almost as an axiom that the elongation is more rapid the greater the stress. The life of a belt, therefore, grows shorter as the working stress increases. The permanent stretch that a belt will endure before becoming useless is probably a nearly constant percentage of its length, which, for good belts, may safely be taken at least as great as 6% for leather.*

Mr. F. W. Taylor, in a carefully kept record of the performance of belting in continuous use for nine years in the machine-shop of the Midvale Steel Co., finds that double leather belting lasts well under a total load, t, of 240 pounds per square inch of sectional area, the annual cost of repairs, maintenance, and renewals amounting to not more than 14% of the first cost. When the stress is kept at 400 pounds per square inch of section, he finds that the annual expense for the same items becomes about 37% of the first cost. It should be kept in mind that these are the continuous working values and not, as is often the case, the maximum load under which a belt may work for a short time, the stress being much less most of the time, as, for instance, a belt driving a dynamo which carries its full load of lights or motors during only a small part of its run.

When a belt is laced together at the ends, the strength of the joint varies from 50 to 95% of that of the belt; the average efficiency of the laced joint is about 70%.† The average strength of joints made with metal fastenings is less than that for lacing. A carefully made cemented joint gives a strength about the same as that of the belt. This method of splicing, making what is commonly called an endless belt, is unquestionably the best for all cases of ordinary application, and becomes almost a necessity for high speeds, since any heavy place, such as a laced or metal-fastener joint, causes vibration. Almost any well-made leather- or wire-laced or metal-fastener belt joint is strong enough to give working tensions of 400 or even 600 pounds per square inch in the belt, and last until there is need of shortening the belt when there is no tighting device.

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* Notes on Belting, by F. W. Taylor.   Trans. Am. Soc. of Mech. Engrs., vol. xv., p. 204

Digest of Physical Tests, January, 1896, p. 46.

The ultimate strength of leather under a static load is several times the values used for transmitting power.

By inspection of equation (44) it can be seen that, for a given belt working under uniform conditions as to velocity and turning force transmitted, the total tension, T_n, decreases as the coefficient of friction, μ, increases; hence if, in designing a belt, the value of μ is taken as small as it will probably ever be for that belt, it is reasonably safe to say that T_n will never exceed its calculated value as long as the belt works against a constant load.

A long belt is more durable than a short one because it makes fewer bends about a pulley for a given number of rotations. In certain classes of work there is also another reason: If there are sudden resistances and consequent slight reductions of speed with the following recovery, a long belt, on account of its ability to stretch more in its entire free lengths, allows the slight check with less strain in it; or, if a body is to be started from rest, as when a cam strikes and lifts its follower in a stamp-mill, a long belt gives a greater length of time for acceleration and thus receives less strain than a short one.

In practice it is seldom possible to measure the tension in a belt when it is working. There is no reason, however, that the tension, practically equal in both stretches between pulleys, should not be known at the time of putting it in place or of tightening after it has become loose by service. This can be done by drawing the ends together with spring belt-clamps made to weigh the tension, as has been mentioned before. The belts used in Mr. Taylor's experiments, already cited, were adjusted in this manner. He finds that double leather belts, tightened to 240 pounds per square inch of section, or 71 pounds per inch of width, when at rest, and when made to exert a turning force on the pulley of 65 pounds per inch of width, will stretch so that the tension falls to 106 pounds per square inch, or 33 pounds per inch of width, in an average time of two and one-half months of service, their average tension during this time being 150 pounds per square inch, or 46 pounds per inch of width.

Mr. Taylor concludes that, for continuous working, a double oak-tanned and fulled-leather belt will give an effective pull of 35 pounds per inch of width on the face of the pulley when the

arc of contact is 180°; and that other types of leather belts, and 6- to 7-ply rubber belts, will give an effective pull of 30 pounds per inch of width for the same arc of contact.

In iron-working machinery, such as lathes, planers, and drill-presses, 50 pounds per inch of width is quite commonly taken as the effective turning force or pull per inch of width for 180° of contact.

44. Velocity of leather belting.—Below 2500 to 3000 feet per minute, the velocity of a belt has but slight effect upon the length of its life. The most economical speed is probably from 4000 to 5000 feet per minute. Higher velocities, up to about 6000 feet per minute, will generally increase the driving power if the belt is worked at high tension, but the life of the belt is shortened, and vibration, commonly called "flapping," is liable to occur. It may also run from side to side of the pulley, an action known as "chasing"; this is more apt to occur in a thin, pliable belt than a thick, stiff one of the same width. At higher velocities the centrifugal force becomes an objectionable feature in addition to these, and, even if the belt runs smoothly, less power can be transmitted than at slower speeds.

When $V^{2} = \dfrac{9660}{w} t_{n}$ in the equations for belting, the turning force $P = 0$, the tension in the belt due to centrifugal action being equal to the working stress. For a belt weighing .035 pounds per cubic inch, and adjusted to work at 400 pounds per square inch tension, $P = 0$ when the velocity is 10509 feet per minute; and for a working tension of 200 pounds per square inch $P = 0$ for a velocity of 7430 feet per minute. These are the velocities at which the turning force and the power transmitted become zero.

Belt speeds of 5000 feet per minute, and even more, are common in wood-working machinery having pulleys as small as 4 inches in diameter, or less. Thin leather belts are successfully used for such work.

When speeds as high as 5000 feet or more per minute are used in connection with pulleys as small as one inch in diameter, a woven linen web or tape has been found better than leather.*

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* Mr. John T. Hawkins, in Trans. Am. Soc. of Mech. Engrs., vol. VII., 1886, p. 582. Belt used for stereotyper's routing-machine.

45. Wear of leather belts.—Each time a belt is bent by passing over a pulley, the fibres farthest from the pulley are elongated, and those next to it compressed. The continuous repetition of the bending when the belt is in service has a tendency to crack the leather, particularly on the side away from the pulley. The smooth or hair side of a single belt is more easily cracked by this action than the flesh side. It is therefore advisable to run the hair side next the pulleys. Again, there is a slight wear on the surface of the belt that comes in contact with the pulleys. The flesh side is much stronger than the hair side. This is, therefore, another reason for running the hair side next the pulleys.

Double leather belts are made by cementing together the flesh sides of two thicknesses of leather, thus leaving the hair sides exposed to surface wear. Good double belts, properly cared for, are not subject to cracking on the side away from the pulley when working under proper conditions. Triple and quadruple thicknesses of leather are used for making very thick, heavy belts. It is possible, although there is no very definite proof, that a very thick belt, as the quadruple, when working up to the same stress per square inch that would ordinarily be used for lighter belts, may have so high a stress per inch of width, and consequent pressure against the pulley, that the heat generated by the slipping of the belt over the pulley face will dry and burn the surface of the leather so that it will become hard and crack on the surface. Such an action would, at the same time, reduce the coefficient of friction, and thus induce more slipping and injury to the belt.

The diameter of the pulley has a very considerable effect on the life of the belt; for, if a pulley is very small, the belt will be bent so short that there will be excessive wear between its particles, and the greater strain due to the short bend will have a greater tendency to form cracks. A common practice among engineers is to fix a minimum diameter of pulley for single, double, triple, and quadruple belts. This limitation will answer only for the ordinary thicknesses of such belts, however, and should be treated accordingly. To say that a belt is double does not fix its thickness by any means, on account of its great variation in common use. (See § 46.)

Mr. Taylor concludes, as a result of his experience, "that it is safe and advisable to use:

"A double belt on a pulley 12 inches diameter or larger;

"A triple belt on a pulley 20 inches diameter or larger;

"A quadruple belt on a pulley 30 inches diameter or larger; and that it is inadvisable to use double, triple, and quadruple belts on pulleys respectively as small as 9 inches, 15 inches, and 24 inches diameter."

He also considers it safe to say "that the life of belting is doubled by splicing and cementing the belt, instead of lacing, wiring, or using hooks of any kind."

A "quarter-turn" belt (i.e., one connecting a pair of pulleys whose axes are at right angles to each other, no intermediate or idle pulleys being used) is subjected to heavy stresses at its edges if the pulleys it connects are large in comparison with the distance between them. Belts used in this manner generally wear out rapidly, frequently becoming stretched and uneven on the edges. By giving the belt a half twist at the splice, so that in a single belt the hair side and flesh side come together so as to form a continuous surface, the edge-stretching effect is reduced to a minimum. The hair side and flesh side will alternately run next the pulley face when so spliced. It is advisable to avoid a quarter-turn. (See latter part of § 53.2.)

46. Weight and thickness of leather belting.—The weight of good leather belting varies from .03 to .04 of a pound per cubic inch when new. The various processes of tanning, and the difference between fulled and unfulled leather, are the principal causes of this very considerable variation of weight. Unfulled belts that are light when new, generally increase in weight when not allowed to become hard and dry in service; the application of belt dressing and, in many kinds of service, the accumulation of oil that gets upon them in various ways cause increased weight.

The thickness of belting cannot be given with any considerable degree of accuracy. The average thickness of single belts is about .22 of an inch, and often reaches .25 of an inch; it can readily be seen that such leather can be trimmed to any desired thickness. Double belts, such as are commonly used, vary in thickness at least from .22 to .35 of an inch, the majority lying between .30 and .35

of an inch.   Unless specified as light or heavy, the thickness may ordinarily be taken as about .33 of an inch.

47. Rawhide, semi-rawhide, rawhide with tanned-leather face, and chrome-tanned belts.—All of these classes of belts have a higher coefficient of friction than ordinary tanned leather belts, but do not seem to be so durable or satisfactory for service in dry places.

Rawhide seems to give better service in damp places than tanned leather, the moisture apparently not affecting its capacity to transmit power, or causing it to crack and distort as the tanned leather does, especially when alternately dry and wet.   The weight of rawhide is about the same as that of tanned and fulled leather.

Semi-rawhide belting is made of hide that has only a thin surface layer tanned, the inner portion retaining the qualities of rawhide.

Rawhide belts with tanned-leather faces have the tanned leather pasted on the rawhide with sizing or glue.   Under heavy service this facing is apt to come loose after a considerable time.

Chrome-tanned belts are especially adapted to damp places, as in dye-houses, etc.   They are extremely elastic, and elongate greatly during life.   They have a high coefficient of friction. The color is green.

48. Cotton belts are generally made by folding together several thicknesses of cotton canvas or ducking and fastening them by stitching or otherwise.   They are sometimes woven solid so that no folding is necessary.   Their strength is probably greater than that of any other form of belts commonly used.   The coefficient of friction is rather low when the belt is used dry without any filling, sizing, or dressing.   When properly sized or dressed, however, the coefficient of friction is equal to that of good leather belts.   A weather-proof brand which is placed on the market seems to give excellent service for outdoor work of the most trying kind.   In it the interstices seem to be completely filled with the sizing, which, on the outside at least, is water-proof.   It seems to be as durable as rubber belting, and has the advantage that there is no thin layer of rubber to rub or roll off, as is frequently the case with the latter when excessive slipping occurs.   Some kinds of cotton belts have excessive elasticity.

The weight of cotton belts depends largely upon the kind and amount of sizing that is used. Belts showing the following weights have been found in service: 0.026, 0.033, 0.037, 0.044, and 0.050 of a pound per cubic inch. The latter is the weight of the weatherproof belt mentioned above.

Cotton-leather belting is made by stitching a piece of thin leather to a cotton belt so as to make a leather facing on one side, which is used next the pulley. An unsized belt can be given a high coefficient of friction in this way. The facing is apt to tear off, especially in service where the belt must be shifted from step to step of a cone pulley.

49. Rubber belting is composed of a cotton web with a composition of rubber filling all its interstices and completely covering it generally. When of good material it is not injured by moisture, and
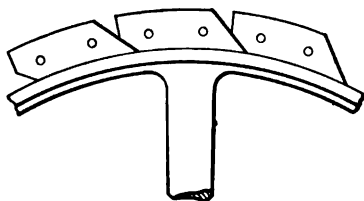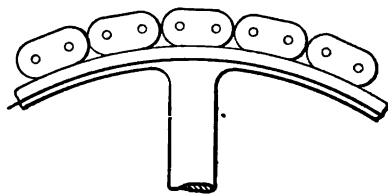
FIG. 73.   FIG. 74.

is therefore excellent for damp places and out-of-door service. The coefficient of friction of good rubber belting is high. If the rubber compound is poor, the coefficient of friction may be low and the compound will crack. When overloaded and caused to slip on the pulleys, the rubber is in danger of peeling loose and rolling up so as to tear off considerable areas on a belt that has a complete covering of rubber over the cotton, thus destroying the belt.

In some kinds of rubber belting the cotton web is not completely hidden by the rubber, but stands out distinctly on the surface.

The weight of rubber belting is about 0.045 pounds per cubic inch. The joints of rubber belting can be cemented by coating the surfaces with uncured rubber in solution and vulcanizing the splice with steam-heated clamps placed over it.

50. Leather-link belts.—Figs. 73 and 74. These are made of small pieces of leather, generally from one to two inches long and

five-eighths to seven-eighths of an inch broad, perforated at each end at right angles to the natural surfaces of the leather.   These are fastened together by pins through the holes so as to form a chain or belt of any desired width and length.   The pins are generally of iron or steel, and of such a size as will fit the holes in the links.   The edges of the links are exposed and form the broken surface which comes in contact with the pulley.   In the better makes of this kind of belts each pin extends only half-way across its width, and the two half-belts thus formed are fastened side by side, to form the complete width of belt.   By this means the belt is allowed to take the form of a "crowned" pulley whose face is made of two cone frusta placed base to base.   A round piece of leather is sometimes used for uniting the links, instead of the steel pins.   A belt thus made will adjust itself to a pulley whose crowning consists of a smooth curve, such as an arc of a circle.

The coefficient of friction of link belting seems to be about the same as that of good leather belts, possibly somewhat lower.   The weight per cubic inch generally runs, for metal pins, from 0.035 to 0.050 pounds.   On account of its great thickness it is much heavier per unit area of working surface than solid leather belting of the same capacity for transmission.

Leather-link belting is especially applicable to connecting a large and small pulley that are placed near together in the same horizontal plane, their axes being horizontal.   This is because, on account of its weight, it can be very slack, and, the slack side being above, it sags down so as to make a large arc of contact on the pulleys, thus increasing the turning force without making the belt excessively tight, as would be necessary with any of the ordinary solid belts.

When a link belt breaks, it is generally without warning, and, on account of its weight, it is capable of doing serious damage, when running at high speed, by striking whatever may be in its path.

51. **Effect of relative positions of pulleys.**—When the axes of the driving and driven pulleys are in the same horizontal plane, and the loose side of the belt is uppermost, the arc of contact on each pulley is increased, both by the sagging of the loose side as the load is applied, and by the tightening of the lower side to a more nearly

straight line on account of the increased tension in it. On account
of this increase of the arcs of contact, the tension is smaller on the
tight side of the belt than it would be if the rotation of the pulleys
were reversed, thus bringing the slack side below and decreasing
the arc of contact; hence it is always advisable, when possible, to
run the pulleys so that the slack side of the belt will be uppermost.

Again, suppose that a large pulley is driving a comparatively
small one placed vertically below it; the arc of contact on the small
pulley will be much less than on the large one, and, in addition to
this, the tension in either stretch of the belt at its point of tangency
with the small pulley will be less than at the similar point on the
large one. The difference of tension at the top and bottom of a
stretch of the belt is the same as the weight of a portion of the belt,
equal in length to the vertical distance between the points of
tangency. Now suppose the small pulley is placed above the large
one; the greater tension in the belt, still remaining at the upper
pulley, will be applied so as to counteract, in a measure, the effect
of the reduced arc of contact, while before, with the small pulley
below, it was cumulative with it. No further investigation is
necessary to show that it is advisable to place a small pulley above
a large one, especially when the belt has a considerable length.

For inclined positions of a belt connecting pulleys running on
horizontal axes, the two facts brought out above should be taken
into consideration. They indicate that the slack belt should always
be kept above, and the small pulley higher than the large one, when
they are not at the same level, and when other conditions will ad-
mit of such an arrangement.

Whatever the relative positions of a pair of pulleys, the tension,
in a clear stretch of belt, at the point of tangency with the upper
pulley, is greater than that at the similar point on the lower one,
by an amount the same as the weight of a length of the belt equal
to the vertical distance between the points of tangency.

52. A special system of flat-belt driving is illustrated in Fig.
75. It is especially applicable to close grouping of machinery.

The belt is tightened by the mechanism shown near the engine-
cylinder.

The driving capacity of this device, when used with ordinary
double belts, and with a contact on the large pulley at least as great

in linear measure as that on the small or driven pulley, can be safely taken as 1 H.P. per inch of belt width for a belt speed of 750 feet per minute.*
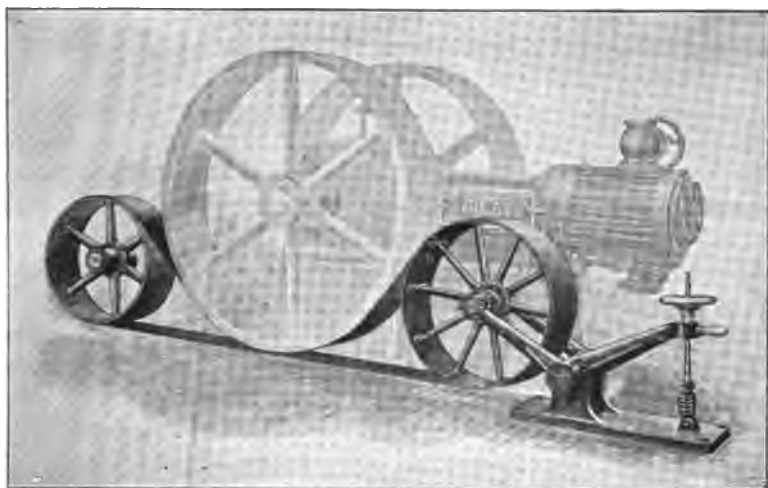
FIG. 75.

53. Efficiency of flat belting.—The power losses in flat leather belts are chiefly due to journal friction and belt slip. There is also a small loss caused by bending and straightening the belt as it runs on and leaves the pulley, but it does not seem to be great enough to need consideration. The slip is sometimes considered as made up of two elements, the creeping of the belt over the pulley, due to its elongation as it passes from the slack to the tight side, and the actual slip, which allows all parts of the belt to move at the same rate, faster than the driven pulley face or slower than the driving pulley. The effects of both are the same upon the efficiency, hence it does not seem necessary to separate them when dealing with this property.

If the journals are larger than necessary to withstand the pull of the belt, or the latter is working above or below its normal capacity, the efficiency is lowered.

* Communication from A. L. Ide & Sons.

The efficiency of flat leather belting is probably never greater than 97% in the grades of machinery that are generally used in practice. This refers to well-made machines with journal-bearings. If bearings having a very low frictional resistance, such as ball-bearings, are used, the efficiency might be increased to 98%, or possibly more. Probably 95% is a fair average for good practice when a belt is working at or near its most economical rate.

Crossed belts require less tension to transmit a given amount of power, on account of having greater arcs of contact on the pulleys, than open ones; hence, if there is no rubbing together of the two stretches between the pulleys, the efficiency should be higher. Rubbing is almost certain to occur, however, thus adding an additional power loss. The rubbing may be of small consequence if the pulleys are of the same size; but if their diameters are greatly different, and especially if they are near together, a very considerable rubbing pressure is almost certain to be caused. The power loss under such a condition is proportionately large. It is seldom, possibly never, advisable to use a crossed belt when considered with regard to power transmission and belt economy.

The efficiency of the other kinds of flat belting mentioned in the preceding paragraphs does not differ enough from that of leather to be practically appreciable.

53.1. Example showing the effect of varying the speed of belt.—Two shafts, 30 feet apart between centre lines, in the same horizontal plane, are to have 25 horse-power transmitted between them, both running at 200 revolutions per minute. Leather belting is to be used at a tension of 300 pounds per square inch on the tight side.

The coefficient of belt friction may be taken as .3 and the weight of the leather as .035 pound per cubic inch.

Since the belt wrap is 180°, the diagram, Fig. 72.4, may be used directly for finding the sectional area of the belt.

The calculations will be made for three pairs of pulleys of 3-, 6-, and 9-foot diameters. (See Figs. 75.4, 75.5, and 75.6.)

For the 3-foot pulley

$$v = 200 \times 3\pi \div 60 = 31.4 \text{ feet per second.}$$

432 LBS. TOTAL TENSION
359 " EFFECT. "

1.44 SQ. IN.
BELT SECTION

6′ DIAM.
180° ARC
200 R.P.M.
25 H.P.
P = 219 LBS.

←499○

750 LBS. TOTAL TENSION
718 " EFFECT. "

3′ DIAM.
180° ARC
←998○
200 R.P.M.
25 H.P.
P = 438 LBS.

2.5 SQ. IN. BELT SECTION
t = 300 LBS. PER SQ. IN.
ω = .035 " " CU. IN.
μ = .3

312 LBS. TOTAL TENSION
280 " EFFECT, "

Fig. 75.1.

213 LBS. TOTAL TENSION
140 " EFFECT. "

Fig. 75.2.

391 LBS. TOTAL TENSION
239 " EFFECT. "

1.3 + SQ. IN.
BELT SECTION

9′ DIAM.
180° ARC
200 R.P.M.
25 H.P.
P = 146 LBS.

←332○

245 LBS. TOTAL TENSION
93 " EFFECT. "

Fig. 75.3.

By the diagram, Fig. 72.1, a belt of 1 square inch cross-section running 31.4 feet per second will transmit 10 horse-powers. The sectional area of the belt is therefore

$$A = 25 \div 10 = 2.5 \text{ square inches.}$$

Since the pulleys are 30 feet apart,

Length of belt $=2\times30+3\pi=69.42$ feet;
Volume of the belt $=69.42\times12\times2.5=2083$ cubic inches;
Superficial area of belt 5/16 inch thick $=46.3$ square feet.

For the 6-foot pulley:

$$v=200\times6\pi\div60=62.8 \text{ feet per second.}$$

By the diagram the power transmitted by one square inch of cross-section of belt at a speed of 62.8 feet per second is 17.3 horse-powers. The sectional area of the belt is therefore

$$A=25\div17.3=1.44 \text{ square inches.}$$

Length of belt $=2\times30+6\pi=78.85$ feet;
Volume of belt $=78.85\times12\times1.44=1360$ cubic inches;
Superficial area of belt 5/16 inch thick $=30.1$ square feet

For the 9-foot pulley:

$$v=200\times9\pi\div60=94.2 \text{ feet per second.}$$

The power transmitted by one square inch of section at 94.2 feet per second is 19.2 horse-powers. The sectional area of belt is therefore

$$A=25\div19.2=1.3 \text{ square inches.}$$

Length of belt $=2\times30+9\pi=88.27$ feet;
Volume of belt $=88.27\times12\times1.3=1375$ cubic inches;
Superficial area of 5/16 inch belt $=30.6$ square feet.

The 6-foot pulleys require the least volume of belt. The 9-foot ones need about the same amount; but 53% more is necessary for the 3-foot pulleys. The drive with 6-foot pulleys is the cheapest for the distance of 30·feet between pulley centres. This might not be true for shafts very near together, for then the cost of pulley may have greater importance than that of the belt.

While there is a very considerable difference in the belt sections for the 3-foot and 6-foot pulleys, there is, as has already been pointed out, but little further reduction by using a 9-foot pulley. The reason that the reduction of the 9-foot pulley is so

slight is because, at the comparatively high belt speed with this diameter, the centrifugal tension begins to have an appreciable value. If the size of the pulley were increased to 12 feet, a larger belt than for the 6-foot pulley would be required on account of the high centrifugal tension. For a 12-foot pulley the belt would have nearly 2 inches of cross-section when working at 300 pounds per square inch tension on the tight side.

The centrifugal tension in a belt has no effect on the pulley. It causes no pressure against the pulley, and hence exerts no turning effort upon it.

The tension in the belt which is effective in turning the pulley is what remains of the total tension after the centrifugal tension has been deducted.   .

The required effective tension decreases, under the conditions for this example, in the same proportion as the diameter of pulley and belt speed increase. Therefore the pressures on the bearings due to the belt tensions also decrease in the same ratio as the pulley diameters and belt speeds increase. This will be shown in numerical values.

For any belt:

Centrifugal tension $= .0001036wV^2 = .3727wv^2$ pounds per square in.

For the 3-foot pulley:

Unit centrifugal tension $= .3727 \times .035 \times (31.4)^2 = 12.8$ pounds per square inch.

Total centrifugal tension $= 2.5 \times 12.8 = 32$ pounds.

Total tension, tight side, $= T_1 = 300 \times 2.5 = 750$ pounds.

Effective tension, tight side, $= 750 - 32 = 718$ pounds.

For the 6-foot pulley:

Unit centrifugal tension $= .3727 \times .035 \times (62.8)^2 = 51.1$ pounds per square inch.

Total centrifugal tension $= 1.44 \times 51.1 = 73$ pounds.

Total tension, tight side, $= T_1 = 300 \times 1.44 = 432$ pounds.

Effective tension, tight side, $= 432 - 73 = 359$ pounds.

For the 9-foot pulley:

Unit centrifugal tension $= .3727 \times .035 \times (94.2)^2 = 116$ pounds per square inch.

Total centrifugal tension $= 1.3 \times 116 = 152$ pounds.
Total tension, tight side, $= T_1 = 300 \times 1.3 + = 391$ pounds.
Effective tension, tight side, $= 391 - 152 = 239$ pounds.

The centrifugal tensions have the same ratio as the squares of the velocities. The latter have the ratios 1, 2, and 3. Therefore the centrifugal tensions per square inch have the ratios 1, 4, and 9.

To find the pressure on the bearings it is necessary to know the effective tension on the slack side of the belt. This is obtained by deducting the turning force P from the effective tension on the tight side.

For the 3-foot pulley:

$$P = \frac{550 \text{ H.P.}}{v} = \frac{550 \times 25}{31.4} = 438 \text{ pounds.}$$

Effective tension, slack side, $= 718 - 438 = 280$ pounds;
Bearing pressure $= 718 + 280 = 998$ pounds.

The corresponding values obtained in a similar manner for the 6-foot and 9-foot pulleys are:

For the 6-foot pulley,

$P = 219$ pounds;
Effective tension, slack side, $= 140$ pounds;
Bearing pressure $= 499$ pounds;

and for the 9-foot pulley,

$P = 146$ pounds;
Effective tension, slack side, $= 93$ pounds;

and

Bearing pressure $= 332$ pounds.

It will be seen that, by using the 6-foot pulley, the friction loss at the journals and the belt section are both reduced, even for a shaft of fixed diameter, as a line shaft. For a head shaft whose diameter may depend largely upon the effective belt tensions, the shaft itself may be made smaller for the 6-foot pulley than for the 3-foot, which is an additional advantage for the larger pulley.

53.2. Binder and guide pulleys.—The use of idle pulleys for tightening an endless (cemented-joint) belt is very frequently desirable. They can be made adjustable to take up the elongation

of the belt, to relieve its stress when not in service, and to adjust its tension according to the work it must do, thus increasing its life, and, if properly applied, the efficiency of power transmission.

To be efficient, a binder pulley should be large in diameter, placed on a slack side of the belt in a position to materially increase the angle of belt wrap, and supported on journals only as large as are necessary for the pressures on them.

When only one binder is used on a belt connecting a large pulley with a small one on parallel shafts comparatively near together, it should be placed near the small pulley so as to make its angle of belt wrap as great as on the large one.

The distance between a binder and working pulley should not be less than 18 inches for ordinary double belts. If the pulleys are closer together than this, there is apt to be noisy running on account of the stiffness of the belt joints. A stiff joint, passing from one pulley to another very near it, strikes the latter a hammer-like blow accompanied by noise, especially on iron pulleys with uncovered faces.

A binder pulley may sometimes be placed so as to press against the slack side of a belt at its point of tangency on the driving or driven pulley. This is not advisable except when it cannot be otherwise located.

In belt drives for shafts that are not parallel, one of the guide pulleys can be made adjustable as a belt tightener.

The guide pulleys may be placed so as to give large angles of belt wrap. This is always desirable. As binders only, the pulleys should be large in diameter and supported on bearings not unduly large.

Guide pulleys should be located so that the direction of motion can be reversed without displacing the belt.

The stretching of the edges of the belt is reduced to a minimum by placing the guide pulleys in this manner, and, by giving sufficient crown to the pulley faces, the edge stretch is made practically zero.

There are many excellent belt drives operating in the New England States with binder and binder-guide pulleys for transmitting amounts of power ranging from a few horse-powers to two hundred or more. Many of them are for shafts not parallel,

as for connecting a vertical-shaft water-wheel to a horizontal line shaft in a mill or factory. The belts on these drives run many years without repair or resplicing.

On the other hand, there are in operation throughout the country a far larger number of poor examples of the use of binder and guide pulleys. Small pulleys improperly placed and supported by excessively large bearings are numerous. They are inefficient, short-lived, and usually noisy. The belt is often injured by a short bend over a small pulley.

Example.—The requirements of the example, § 53.1, may be met by belt drives with binder pulleys. The results for 3-, 6-, and
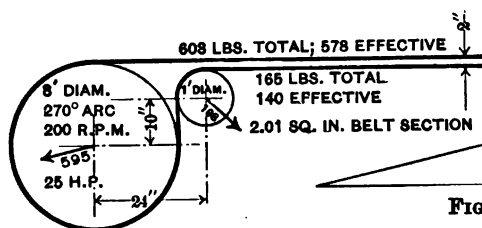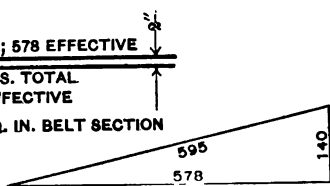
FIG. 75.4.   FIG. 75.5.   FIG. 75.6.

·9-foot pulleys with binders placed so as to give 270° of belt wrap are given below. The arrangement of the pulleys at one end of each drive are shown in Figs. 75.4, 75.5, and 75.6.

The velocities, centrifugal tensions per square inch, and the required turning force P are the same as before in each case. Other values are:

For the 3-foot pulleys: $T_1 = 603$; $T_2 = 165$; effective total tensions $= 578$ and 140; $A = 2.01$; length of belt $= 73.4$ feet; volume of belt $= 1770$ cubic inches; superficial area of belt 5/16 inch thick $= 39.3$ square feet.

For the 6-foot pulleys: $T_1 = 350$; $T_2 = 131$; effective total tensions $= 289$ and 70; $A = 1.17$; length of belt $= 87$ feet; volume of belt $= 1240$ cubic inches; superficial area of belt 5/16 inch thick $= 27.1$ square feet.

For the 9-foot pulleys: $T_1 = 315$; $T_2 = 176$; effective total volume of belt $= 1275$ cubic inches; superficial area of belt 5/16 inch thick $= 28.3$ square feet.

The amount of belt required without binders is greater than the above by about 19% for the 3-foot, 9% for the 6-foot, and 7% for the 9-foot pulleys.

There are two additional pulleys with their supports to counteract the saving of belt by the use of binders. The pulley faces are not so wide, however, and the supports may be lighter.

The bearing pressure on one of the 3-foot pulleys due to the belt tensions is the resultant of the effective tensions of 578 and 140 pounds acting at right angles to each other. This resultant, as shown in Fig. 75.5, is 595 pounds. The corresponding bearing pressure on the 1-foot-diameter binder is the resultant of the two practically equal tensions in the stretches of belt leading from it at right angles. These tensions are each 140 pounds. Their resultant, Fig. 75.6, is 198 pounds.

The bearing pressures for the 6- and 9-foot pulleys are obtained in the same way, as indicated in Figs. 75.7, 75.8, 75.9, 75.10, 75.11, and 75.12.

The sum of the bearing pressures on a working pulley and its binder is smaller than the bearing pressure for the same size of working pulley without a binder, and the latter has a smaller journal than the working pulley. The frictional loss is therefore less when the binders are used.

53.3 Chain belts. running upon sprocket-wheels which resemble in a general way the teeth of spur-gears both as to form, thickness, and distance between them, are coming into extensive use for the transmission of power. These chains have projections upon one side to fit into the spaces between the teeth of the sprocket-wheel.

In the Reynolds chain the construction is such that the chain has practically no rubbing action over the teeth of the sprocket-wheel while coming into contact with and disengaging from them. It is quiet-running. The limit of speed is that at which the lubricant is completely thrown off by centrifugal action. This limit is found to be about 1300 feet per minute. As the pitch of the chain increases with wear, it adjusts itself to a larger radius on the sprocket, thus maintaining a correct fit.

In the Morse chain the necessity of lubrication is obviated by a blunt knife-edge bearing surface between the pins joining the

350 LBS. TOTAL; 289 EFFECTIVE

181 LBS. TOTAL
70 EFFECTIVE

6' DIAM.
270° ARC
200 R.P.M.
25 H.P.

2' DIAM.

1.17 SQ. IN. BELT SECTION

FIG. 75.8.

FIG. 75.9.

FIG. 75.7.

315 LBS. TOTAL; 198 EFFECTIVE

176 LBS. TOTAL
47 EFFECTIVE

9' DIAM.
270° ARC
200 R.P.M.
25 H.P.

2' DIAM.

1.05 SQ. IN.
BELT SECTION
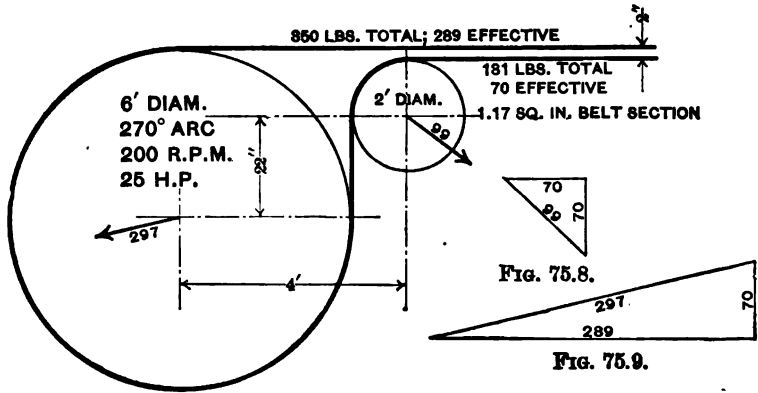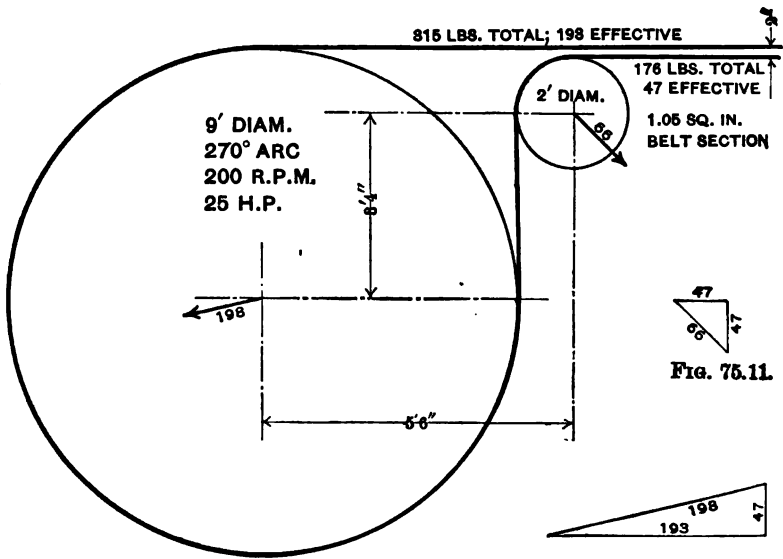
FIG. 75.11.

FIG. 75.10.

FIG. 75.12.

links. This chain runs successfully at higher speeds than one requiring lubrication.

ROPES.

54. Ropes, running on grooved pulleys or sheaves, are largely used for the transmission of power. The name "sheave" is applied to a pulley with but one groove; when there are two or more grooves the name "grooved pulley" is generally used.

While ropes are very frequently used for transmitting power between shafts that are parallel and at such a distance apart as is common for leather belts, their especial application seems to be that of connecting shafts that are not parallel, or are at a great distance apart. On account of their approximately circular form of section, they bend with equal ease in all directions, hence " quarter turns " and bends in various directions are not nearly so severe upon them as upon flat belts. Hemp, cotton, and wire ropes are the varieties almost exclusively used for power transmission, although leather, rawhide, and other materials are used to some extent, generally for light work. Manila hemp is generally called " manila," and the special make, which is largely used for power transmission, is called " Stevedore."

Fibrous Ropes.

55. Two systems of driving with fibrous ropes are in general use. They are commonly known as the Continuous or American system, and the Multiple or English system.

In the multiple system there are as many separate, endless ropes as there are grooves in each of the system of pulleys over which all the ropes run. Each rope always runs in the same groove of each pulley. The ropes are, of course, parallel to each other, practically speaking. The multiple system is generally used for transmitting very large amounts of power.

In the continuous system, shown in Fig. 76, there is a single endless rope, wound continuously over the pulleys. The winding is such that a point in the rope, starting at the groove at one end of any pulley, passes from this groove to the first groove of each of the other pulleys, then to the second groove of the first pulley and

of the other pulleys in succession, thence to the third groove, and so on, until it has passed through all the grooves; it then passes over a single-groove guide-sheave, which leads it back to the first groove of the first pulley. The guide-sheave is often made to serve the double purpose of both guide and tightener. When used as a tightener it is supported on a carriage which is free to travel back and forth on a track or guide; weights are attached to the carriage to keep the tension of the rope uniform. The tension carriage must be free to have a considerable range of travel, in order that it may adjust itself for variation in the length of the rope, which may be caused by change of load, condition of the atmosphere, the gradual lengthening of the rope with service, and other influences. The tension-carriage should always be placed on the slack side of the rope. In Fig. 76 it can be seen in the upper middle portion of the figure. The tension-weight is shown alongside the column.

There are numerous modifications of the arrangement of the tension-carriage and guide-pulleys, to conform with local conditions, one or more idler sheaves frequently being added to guide the rope, but the principle is the same in all.

The method of transmitting power by ropes to the different floors of a building is shown in Fig. 77.

A varying load on a continuous-system rope-drive causes unequal tension in the stretches of rope between the pulleys. This can be seen by assuming that a drive which is running for some time without any load other than the journal-friction of the machinery immediately appertaining to the drive, has a load equal to the full capacity of the drive suddenly thrown upon it. While running light the tension in each stretch of rope, on both sides of the pulleys, is practically equal to that caused by the tension-carriage, i.e., equal to one half the effective weight on the tension-carriage. As the full load is suddenly applied, all the stretches of rope on the slack side are slackened somewhat more by the stretch of those on the tight side; the tension-carriage, however, maintains the same tension in the stretch between it and the driven pulley. Consequently when a length of rope approximately equal to one half of what would be required for connecting both pulleys with a single band has passed from the tension-carriage to the pulley, the tension on the tight side in the first stretch after leaving the

tension-carriage is greater than in any of the others, because the
slack side of the rope running on the driven pulley is tighter for

FIG. 77.

this stretch than for any other, and consequently there is less slip
in the first groove of the driven pulley than in any of the others of

the same pulley. As the rope continues to run over the pulley, the first stretch on the slack side becomes tighter than the others on the slack side, and the second stretch on the tight side increases its tension to a higher value than that of the stretches coming later on the tight side. After a time the tension becomes uniformly distributed among the stretches on the tight side.

On account of the inequality of tension in the stretches of the rope on the tight side, when working under varying loads, which may be so marked as to very appreciably shorten its life when there are many turns of a continuous rope around a pair of pulleys, it is advisable to limit the turns to a comparatively small number; the greater the variation of load, the smaller should be the number of turns of a continuous rope.

When, in order to fulfil the requirements of power transmission, a large number of turns of rope are required, and the continuous system is adopted, it is often advisable to use two or more continuous drives, operating side by side on the same pulleys if desired; the main pulleys in such a case would be the same as for a single continuous drive; an additional tension-carriage and guide sheave or sheaves, if the latter are used, must be supplied for each continuous rope. The additional cost of the latter may be more than counterbalanced by the increased life of the rope, however.

56. The equations for ropes transmitting power are similar to those for flat belts. On account of their approximately circular sectional form, it is more convenient to take a rope of unit diameter as the basis of calculations, instead of a unit area, as is done for flat belts. The coefficient of groove friction ϕ, which is generally used, is that of the rope in the groove, and is, of course, greater than that of the same rope on a flat surface of the same material as the pulley, on account of the wedge-like action of the rope in the groove; the sharper the angle between the two sides of the groove, the higher this coefficient of friction.

The following notation is applicable to ropes only; the symbols used in the equations, but not given in this notation, are the same as for flat belts:

$W =$ weight of a rope 1 inch in diameter and 1 foot long, pounds;
$w =$ weight of a rope 1 inch in diameter and 1 inch long, pounds;

τ = working strength of a rope 1 inch in diameter, pounds;

β = angle between sides of groove, degrees;

$\phi = \mu \csc \dfrac{\beta}{2}$ = coefficient of groove friction.

The equations for ropes, deduced in the same manner as those for belts, are

$$p = (\tau - .0001036 w \, V^2)\left(\frac{\epsilon^{\phi\theta} - 1}{\epsilon^{\phi\theta}}\right)$$

$$= (\tau - .0001036 w \, V^2)\left(\frac{10^{.00758\phi a} - 1}{10^{.00758\phi a}}\right), \quad \cdots \cdots \quad (47)$$

and

$$\tau = p\frac{\epsilon^{\phi\theta}}{\epsilon^{\phi\theta} - 1} + .0001036 w \, V^2 = p\frac{10^{.00758\phi a}}{10^{.00758\phi a} - 1} + .0001036 w \, V^2. \, (48)$$

Example.—It is required to design a rope-drive to transmit 200 H.P. when running at 4500 feet per minute, and working at a tension on the tight side equivalent to 200 pounds for a rope of 1 inch diameter; the arc of contact on the working pulley having the smallest portion of its circumference embraced by the rope being 160°. Manila hemp rope to be used.

This problem is essentially the same as the example given under flat belting; the only difference being that ropes are used instead of a belt. As in that problem, the turning force

$$P = 1467 \text{ pounds.}$$

By substituting in equation (47), taking $\phi = 0.3$ and $w = .024$,

$$p = [200 - .0001036 \times .024 \times (4500)^2]\left[\frac{10^{.00758 \times .3 \times 160} - 1}{10^{.36384}}\right]$$

$$= (200 - 50)\frac{2.311 - 1}{2.311} = 85 \text{ pounds.}$$

Assuming that rope $1\frac{1}{4}$ inches in diameter will be used, and that it will work under a tension equivalent to 200 pounds for a rope 1 inch in diameter, the working strengths being taken as proportional to the squares of the diameters, gives, for the turning force of the $1\frac{1}{4}$-inch rope,

$$\frac{25}{16} \times 85 = 133 \text{ pounds.}$$

The number of 1¼-inch ropes required is, therefore,

$$1467 \div 133 = 11 \text{ (about).}$$

The total tension T_n bears the same relation to the total turning force P as τ does to p, therefore

$$T_n = \frac{\tau}{p}P = \frac{200}{85}1467 = 3460 \text{ pounds.}$$

The tension that must be put on a rope when at rest may be determined in the same manner as for a flat belt. It is not known, however, that any experiments have been made to determine whether there is the same increase of the sum of the tensions in the two sides of the rope when working, over that when at rest, as there is in flat belts, but it seems reasonable to assume that such is the condition. Upon the assumption that the increase is one third of the sum of the tensions of rest, the tension in each stretch of the rope when at rest is, as for the flat belt, taking into account the fact that there are 11 stretches of rope between the pulleys on each side,

$$\frac{T_r}{11} = \frac{3}{8} \times \frac{2T_n - P}{11} = \frac{3}{8}\frac{(2 \times 3460) - 1467}{11} = 185 \text{ pounds.}$$

If a tension-carriage is used, as in the continuous system, the effective weight for producing the total tension T_o on the slack side of the rope must be $2T_o \div 11$, since there are 11 ropes in this particular drive. Therefore the effective weight of the tension-carriage must be

$$\frac{2T_o}{11} = \frac{2(T_n - P)}{11} = \frac{2(3460 - 1467)}{11} = 360 \text{ pounds.}$$

57. The grooves for non-metallic ropes found in practice are of numerous forms. The most common angle between the sides of the groove is about 45°, however. It varies from 30° to 60° in extreme cases. It is clear that the smaller this angle, the tighter the rope will wedge into it, and the less liable will it be to slip.

More power will be required to pull it out of a sharp-angled groove, however, thus causing more power loss; it is also possible that there will be more wear of the rope. In some forms the sides are straight, as in Fig. 78, while in others they are curved, as in Fig.
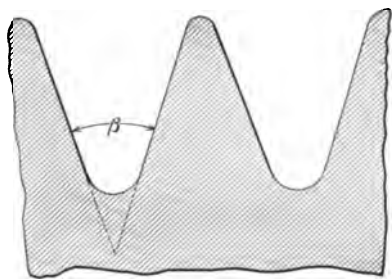
FIG. 78.

79,* where O is the centre of curvature for the left side of the groove; the right side has a similar curvature.
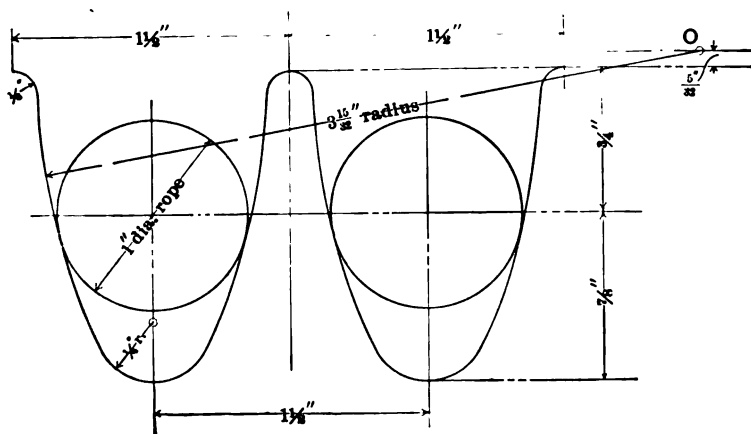
FIG. 79.

In a groove with curved sides, the angle between the sides where the rope comes in contact with them, when resting lightly in the groove, is smaller than would be used for straight sides; at the

bottom it is larger. This arrangement secures a good grip of the rope when it is at or near the top of the working part of the groove, but allows it to slip more readily when, on account of wear, it is brought nearer to the bottom. Such a provision for more ease of slipping when near the bottom of the groove is of considerable importance in the working of a rope-drive, especially of the multiple system.

In order to show the action of a multiple-rope drive, let it be assumed that one has been in use until some of the ropes have become so worn that they must be replaced by new ones, and that the pulleys are of greatly different diameters, as is frequently the case when a line-shaft or some high-speed machine is driven directly from a pulley on the main shaft of an engine. The old ropes, being smaller than the new ones of the same nominal diameter, on
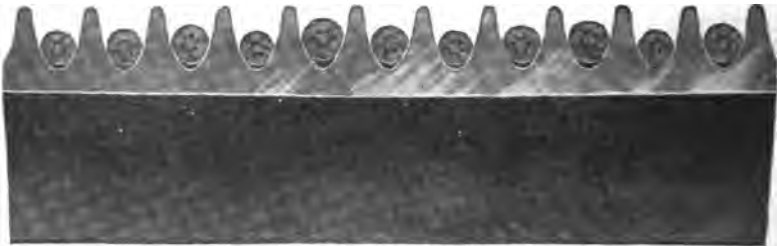
Fig. 80.

account of wear and stretching, will lie nearer the bottoms of the grooves than the new ones, and their effective radii will accordingly be less on each pulley, the whole system appearing as in Fig. 80. For convenience it may be assumed that the grooves are of the same form on both pulleys, this being very frequently the condition in practice. The reduction of the effective diameter will therefore be the same on both pulleys as the rope wears. By putting

D = effective diameter of the large pulley with new rope,
a = reduction of effective diameter of each pulley,
d = effective diameter of the small pulley with new rope,

the conditions of working can readily be expressed mathematically; it will be assumed that $D = 3d$.

If there were no slipping of the new ropes in the grooves, the small pulley would make three revolutions for one revolution of the engine; and if there were no slipping of the old ropes, the portion of length of one of them which would pass over the large pulley during one of its revolutions is $\pi(D - a)$. This length of rope, by passing over the small pulley at an effective diameter of $d - a$, would turn it through

$$\frac{\pi(D - a)}{\pi(d - a)} = \frac{\pi(3d - a)}{\pi(d - a)} = 3 + \frac{2a}{d - a} \text{ revolutions.}$$

This shows clearly that there is a tendency for the smaller, or old, ropes to drive the small pulley faster than the new ones. As a result, the entire load will be carried by the old ropes unless they slip in their grooves. A form of groove which will let a rope slip more easily as it becomes smaller with wear is better than one which does not allow such slippage.

If, instead of a large pulley driving a small one, the small one drives the large, there will be a tendency for the load to be thrown upon the larger or new ropes. The increased tendency to draw these ropes down into the grooves acts as a corrective, however, and thus prevents the excessive loading of the larger ropes.

In order to show the action of the ropes in a drive having curved grooves similar to that of Fig. 79, let it be assumed that an engine is driving a line-shaft through a rope-drive, the pulley on the engine being larger in diameter than the one on the line-shaft. It may be further assumed that the drive has been in service for some time, and that half the ropes have become so worn that they must be replaced by new ones. The old ropes left in place will, on account of wear and stretch, be smaller than the new ones of nominally the same diameter, and will therefore lie nearer the bottom of the groove. The effective diameter at which the new ropes work on each pulley will therefore be greater than that of the old ones. The difference of the effective pulley diameters, for the new and old ropes, is less than for straight-side grooves, provided the groove angle where the new rope comes in contact with the groove is the same as used for straight sides; at the bottom the angle is larger. The grip on the rope is therefore not as great when it approaches the bottom of the groove as it would be if the sides of the latter were straight. The

curved sides of the grooves thus afford a means of preventing the excessive tension which would be thrown on the rope if it were running in V grooves. The life of the ropes is therefore lengthened.

A groove whose angle grows more flat at the bottom strains the rope less when an excessive load is thrown on the system than does one with straight sides making an angle which is the mean of that of a curved-side groove.

Another method of preventing unequal loading of the ropes in a multiple drive, when pulleys of different diameters are used, is to make the groove of the larger one with a smaller angle than those of the smaller. The decrease of the effective diameter of the larger pulley is thus made more rapid than for the smaller, as the rope becomes smaller. A case is cited where the engine driving pulley was about three times the diameter of the driven pulley. By making the angle of the groove 30° on the larger pulley, and 45° on the smaller, unequal pulling was obviated.*

There seems to be a difference of opinion among engineers as to what form of groove should be used for idler- or guide-pulleys. On the one hand it is maintained that a groove of circular section at the bottom, and large enough to let the rope run in it without wedging against the sides, is the best on account of causing no frictional loss as the rope enters and leaves the groove; the contrary argument is that the rope slips in the round-bottom groove, and thus causes as much loss as the V groove, together with more rapid wear of the rope. It would seem that the round-bottom groove would be at its best when the rope is in contact with a considerable portion of the circumference of the pulley, and the V groove when the arc of contact is small, the round-bottom groove being the more apt to permit slipping with a small arc of contact; but whether the round bottom is better, even for large arcs of contact, does not seem to be positively settled.

Cast iron and hard wood, such as maple, are materials which are almost universally used for the rope to run on; the grooves are generally turned in the rim of the pulley, no lining being used. Cast-iron and wooden grooves therefore correspond to pulley-rims of these materials.

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* Kent's "Mechanical Engineers' Pocket-book."

The sides of the groove should be perfectly smooth in order to prevent rapid external wear of the rope.

58. Coefficient of friction of non-metallic ropes.—There seem to be no experiments showing the coefficient of friction of ropes running in grooved pulleys. The experiments made on ropes running over flat pulleys are scarcely applicable to modern practice in rope-driving, for there is little probability that the ropes used in the two cases were in even approximately the same condition with regard to lubrication. The coefficients given below are not experimentally determined, but based upon modern practice. It is believed, however, that they are sufficiently correct for all practical applications.

The value of μ for well-lubricated manila ropes running in polished grooves generally lies between 0.1 and 0.3; 0.12 to 0.15 are safe values to use in designing. These latter two values give for 45° groove angles the corresponding values of the coefficient of groove friction

$$\phi = 0.12 \csc \frac{45°}{2} = 0.32; \text{ and } \phi = 0.15 \csc \frac{45°}{2} = 0.40,$$

and for 30° grooves,

$$\phi = 0.12 \csc \frac{30°}{2} = 0.46; \text{ and } \phi = 0.15 \csc \frac{30°}{2} = 0.58.$$

When the rope is dry, or lubricated with a somewhat sticky substance, the coefficient of friction is higher.

Other qualities of hemp rope have practically the same coefficient of friction as manila, when lubricated in the same manner.

Cotton rope has a higher coefficient of friction than hemp. This may be due to the fact that it does not require so much lubrication, on account of its containing a natural lubricant, as well as its softer texture. It is doubtless safe to take μ as high as 0.2 in designing; this gives for 45° grooves $\phi = 0.52$, and for 30° angles $\phi = 0.77$.

Rawhide rope does not seem to have been used extensively enough in large sizes to give a very definite knowledge of its frictional qualities. It is much used in small sizes cut flat from the hide and twisted into a round rope. Its length can be varied and

tension adjusted by slightly twisting or untwisting it.   Several small rawhide ropes used in parallel are more satisfactory for oily places, as automatic screw machines, than a single flat belt.   It is doubtless safe to take μ as high as for cotton ropes when the rawhide is kept in good condition and not allowed to get moist; if there is any liability to moisture, μ may be taken the same as for well-lubricated hemp ropes.

59. **Working strength of non-metallic ropes.**—When manila ropes are used for power transmission they have been found to be durable when working under a stress of $200d^2$ pounds on the taut side, where $d =$ diameter of rope in inches.   Cotton ropes have given good service under the same tension.   Rawhide ropes can be successfully operated at a tension at least one quarter higher than that of hemp and cotton, i.e., $250d^2$ pounds or more.

The above values are for economical working and reasonable durability.   As with flat belts, ropes can be operated at much higher stresses, for a short time, than those just given; double the above values, or even more, may be used for a short time.

60. **The velocity of ropes for power transmission** is limited by the action of centrifugal force in the same manner as for flat belts. Non-metallic ropes run much more steadily at high speeds, however, the flapping and chasing common to flat belts being practically absent.   On account of this latter advantageous quality they are commonly run at higher speeds than are found satisfactory for belts; a speed of 5000 feet per minute, or more, is frequently used.   The tension due to centrifugal force in a rope weighing 0.32 of a pound per foot, and running at 8493 feet per minute, is equal to 200 pounds; this is practically the weight and working strength of a 1-inch manila rope.   Roughly speaking, therefore, no power can be transmitted by a well-lubricated hemp rope working at $200d^2$ pounds tension when the velocity reaches 8500 feet per minute.   The speed at which maximum power can be transmitted with such a rope, working under $200d^2$ pounds tension, is about 5500 feet per minute for an arc of contact $\theta = 160°$ to $180°$.   Cotton rope, being somewhat lighter, has its speeds of maximum power transmission and of theoretically no power transmission both somewhat higher.

The most economical speed for manila rope does not vary much from 4500 feet per minute, although there is no great change in economy for variations of 1000 feet per minute on either side of this value.   The speed that is most economical, for the rope alone, is that which gives the minimum cost of rope per horse-power transmitted, which cost includes first cost and maintenance of the rope only.

61. Wear and lubrication of non-metallic ropes.—The wear of vegetable-fibre rope belts is of two kinds, external and internal. External wear is caused by the slipping of the rope in the groove, and the rubbing against the side of the groove as it winds on and is unwound from the pulley.   The outer strands are gradually worn away by external wear, and the rope weakened accordingly.   In order to prevent any considerable weakening of the rope by external wear, a covering, of some material weaker and cheaper than that of the body of the rope, is sometimes placed over it, thus forming a rope which maintains a nearly uniform strength until the covering is worn through, provided the rope is well lubricated internally.

Internal wear requires serious consideration, for, unless some suitable lubricant is used to reduce the friction between the strands and fibres, the life of the rope is apt to be short.   When an unlubricated or improperly lubricated rope has been in service for some time, a fine dust is formed in it by the particles worn off by internal friction.   This dust can be easily seen by opening the strands.   A reverse bend in a rope is a cause of greatly increased internal wear.   For this reason all sheaves and pulleys should be placed, as far as practicable, so that the belt will bend in the same direction in passing over them.

A large number of the lubricants used for rope are made of graphite mixed with some substance such as molasses, beeswax, or tallow.   The best way of applying a lubricant is to saturate the strands as they are being laid up to form the rope.   Hemp rope for power transmission is generally so treated during its manufacture. A lubricant that is applied externally must be of such a nature that it will penetrate the rope and act upon all the strands to reduce their frictional resistance to rubbing against each other.

Cotton fibres are covered with an oleaginous wax in their natural

condition; this coating serves as a lubricant for the rope, and eliminates the necessity of artificial lubrication.

When a vegetable-fibre rope is exposed to the weather while in service, a dressing which forms a water-proof coating should be applied to it, care being taken first to have the interior lubricated. A mixture of beeswax and graphite is frequently used for water-proofing. Substances of an adhesive nature, such as tar, while answering excellently as a preservative for a rope that does not run over pulleys or sheaves, is not suitable for those used for power transmission. Such a substance cements the fibres of the rope together so that they cannot slip over each other freely when the rope is bent around the pulley, thus causing them to tear or break each other. It may also cause the rope to adhere to the pulleys so that the fibres will be picked off by the pulley, which, of course, causes rapid deterioration.

Roughness of the grooves is certain to cause rapid wear, and, as has already been stated in § 57, care should be taken to have them perfectly smooth and without flaws.

The diameter of the pulleys over which a rope runs has much to do with the wear upon it; the smaller the pulley, the more rapid the wear on account of the sharper bend. A diameter of the pulley equal to about 40 times the diameter of the rope represents a fair average of the size found in practice. A somewhat smaller size than is represented by this may be used for the smaller diameters of rope up to ¾ inch; but for ropes as large as 2 inches diameter the pulley may be a few inches larger than this indicates. At high velocities it is believed to be advantageous to use somewhat larger pulleys than are suitable for low speeds.

62. Weight of hemp and cotton ropes for power transmission.— The weight of new hemp transmission rope as made by two manufacturers is given in Table XIA.

TABLE XIA.
WEIGHT OF HEMP TRANSMISSION ROPE.

Diameter of Rope, Inches.	Pounds per 100 Feet.		Diameter of Rope, Inches.	Pounds per 100 Feet.	
	Plymouth Cordage Co., 3- and 4- strand.	C. W. Hunt Co., "Stevedore" 4-strand.		Plymouth Cordage Co., 3- and 4- strand.	C. W. Hunt Co., "Stevedore" 4-strand.
½	9½	12	1¼	50	54
9/16	12½	....	1⅜	65	69
⅝	16	15	1½	70	79
¾	20	21	1⅝	....	94
⅞	30	31	1¾	112	114
1	34	34	2	130	135
1⅛	42	48	2¼	....	175
1 3/16	46				

The average weight of cotton transmission rope that has been in service for some time is about $0.26d^2$ pounds per foot.*

The splice for power transmission must be much longer in a rope than for most other purposes. Table XIB gives lengths for good practice.

TABLE XIB.
LENGTH OF SPLICE IN HEMP TRANSMISSION ROPE.

Diam. of rope, inches...	½	⅝	¾	⅞	1	1⅛	1¼	1⅜	1½	1⅝	1¾	2	2¼
Length of splice, feet..	10	10	10	10	12	12	12	12	14	16	18	20	22

63. The diameter of ropes used for power transmission depends very largely upon the size of the sheave that can be conveniently used. A rope 2 inches in diameter is the largest that is found in practice to any considerable extent. From 1½ to 2 inches in diameter are the limits of the sizes generally used where there is a large amount of power to be transmitted, and the pulleys can be made large without inconvenience, as in rope-drives connecting the main pulley of a large engine with a line-shaft, electric generator, or cable-drums of a cable railway. Smaller sizes are used for general transmission in buildings using power for manufacturing purposes.

64. The effect of the relative position of pulleys upon ropes used for power transmission is of the same nature as for flat belts. In practice, however, it is demonstrated that hemp and cotton ropes

* Rope Transmission of Power, by John J. Flather.

work more satisfactorily than flat leather belts when one pulley is at a considerable distance above the other.   This is doubtless largely due to the fact that a tension-carriage is used to take up the slack in the rope, while the belt, ordinarily operated without any device to make adjustment for stretch, soon becomes loose on the lower pulley.   The weight of a leather belt for such a drive is generally greater than that of a hemp or cotton rope, which is disadvantageous for the belt.

Hemp and cotton ropes are often used successfully, in factories, for directly connecting sheaves or pulleys three or four stories apart, one being nearly or quite vertically above the other.

65. The efficiency of rope belting seems to be about the same as that of flat belts when the distance of transmission is short, so that only a single stretch is necessary between the driving and the driven pulleys.   For long distances of transmission the rope is more efficient than flat leather belting; this is partly owing to the fact that much longer stretches of rope can be used than of flat belting.

<div align="center">WIRE ROPES.</div>

66. Wire ropes made of iron or steel, and generally with a hemp core, have been extensively used for transmitting power through long distances.   They are being replaced by electrical transmission machinery for the greater distances, however, and by non-metallic ropes for the lesser.   The large diameter of the sheaves, generally from 100 to 140 times that of the rope, is an objectionable feature on account of the space required for them, their great weight and comparatively high cost.   The rope itself is much more expensive than non-metallic ones for transmitting the same amount of power.

The sheaves for wire ropes are generally made of cast iron. The grooves are made much wider than the diameter of the rope, which runs against the bottom only; the flanges act only as guides to prevent it from leaving the sheave in case of excessive swaying. The bottom of the groove is lined with wood, leather, gutta percha, or some similar material, soft as compared with the iron.

Wire rope sometimes has a strong tendency to sway when running.   One of the chief causes of this is lack of roundness in the

sheave. After the lining is placed in the groove, it should be accurately turned to run true; a small depression, sufficient to fit possibly one quarter of the circumference of the rope, will hold it in the centre of a sheave rotating on a horizontal axis, when the rope is running without swaying.

The Richmond Manufacturing Company of Lockport, New York, have doubtless had the most extensive experience with wire-rope power transmission of any concern in the United States. As a result of an experience of thirty years, they have arrived at the following conclusions:

" First. The best motion for the pulleys is not less than 100 or more than 140 revolutions per minute. The power will often do well at a less or greater motion, but generally the result is not so satisfactory.

" Second. If a cable is run over level ground for a long distance a support will be needed every 400 or 500 feet, but where both ends of the cable are high enough so as to allow plenty of room for the sag in the middle, a support is not required for less distance than 1000 feet. Where supports are required, a stout post set firmly in the ground and extending about 20 feet above it, with two small pulleys upon the top, answers the purpose.

" Third. Where the cable is short, some method of tightening it should be provided, either by arranging the driving pulleys so that they can be pushed farther apart, or by using an extra pulley as an idler; where the cable is long, this is not required, as the weight of the cable itself will then prevent any slipping on the pulleys.

" Fourth. Large cables must never be used upon small pulleys, as the continual bending of coarse wires around too small pulleys would soon break them."

Table XII has been adopted by this company for its own practice, and is recommended for general work.

TABLE XII.

SHOWING THE NUMBER OF HORSE-POWER EACH SIZE OF CABLE WILL
SAFELY TRANSMIT ; THE DIAMETER OF DRIVING AND DRIVEN
LEATHER-PACKED GROOVED PULLEYS NECESSARY TO BE USED
WITH EACH SIZE ; THE NUMBER OF REVOLUTIONS THEY
SHOULD MAKE, ETC.

Diameter of Cable. Inches.	Diameter of Pulley. Feet.	Revolutions per Minute.	Horse-power Transmitted.
½ {	3	100	2
{	3	140	8
⅝ {	4	100	4
{	4	140	6
7⁄16 {	5	100	9
{	5	140	18
¾ {	6	100	14
{	6	140	20
9⁄16 {	7	100	28
{	7	140	82
⅝ {	8	100	88
{	8	140	42

Their experience has also shown that 100 feet is the shortest
distance between the driving and driven pulleys that will give satis-
factory operation.   They find that a cable working according to
the conditions given above will last from five to six years when
working ten hours a day.*

In the use of wire rope on cable railways, and where power is
transmitted over a long distance by a single endless rope, it is
desirable to have the tension on the tight side of the rope several
times that on the slack.   This is accomplished by using a pair of
winding drums at the power plant of a railway, and at both ends of
the line when power is transmitted between two points.   The two
drums of a pair are placed near together, with their axes parallel;
the tight side of the rope winds in the groove at one end of one
drum, passing half-way around it, and then goes to the correspond-
ing groove on the other, then back to the second groove of the first
drum, and so on, winding consecutively in all the grooves of both
drums.   This device is most effective when the drums are geared

* Data and information kindly furnished by the Hon. William Richmond,
president of the Richmond Manufacturing Co.

together so that both act as drivers, or, if they are at the driven end of the line, power can be taken from both.

The wear on such a pair of winding drums, used as drivers, is most rapid in the groove where the rope first winds on, and gradually decreases in each successive groove. When the first groove has become worn to a smaller effective diameter than the last, there is a tendency toward unequal winding, and, as a consequence, heavy strains are produced in the rope on and between the two drums, which can only be relieved, in a measure, by the rope's slipping in the groove.

In order to prevent the rapid destruction of the rope and drums by this action, Mr. John Walker * has designed a differential pulley. A section of the rim of this pulley is shown in Fig. 81. Each groove is cut in a ring, separate from the rest of the pulley, and a number of the rings are placed side by side on what corresponds to a flanged pulley. The rings have a free-running fit on the cylindrical part of this pulley, but their resistance to turning is regulated by one of the flanges, which is separate from the pulley and held in place by bolts that can be adjusted to
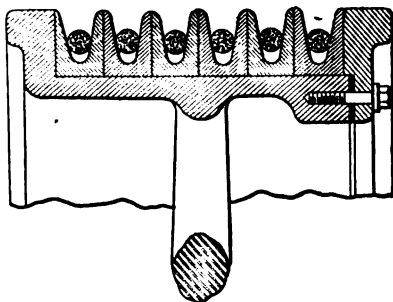
FIG. 81.

give the desired pressure against the sides of the rings. A cushion of some elastic material is placed between the loose flange and the main part of the pulley. The loose flange should be adjusted so that the rings will turn on the pulley a little more easily than the rope will slip in the grooves.

The differential pulley can be equally well applied to any system of rope-driving, using any kind of rope, where there are two or more grooves in a pulley. †

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* Formerly of the Walker Co. of Cleveland, Ohio; now Consulting Engineer, located in Chicago.

† A large amount of data on power transmission by wire ropes is given in Kent's "Mechanical Engineers' Pocket-book."

CHAPTER IV.

67. In some classes of machines, screws are used for applying a great force acting through a small distance. Examples of such application may be seen in testing-machines for determining the strength of materials, and in presses for copying, baling cotton, etc. Again, in such machines as those for planing the edges of boiler-plates, a proportionately smaller force, exerted by the screw against the tool-carriage, acts through a greater distance.

The thread used on such screws is generally square, and in all cases, unless some special requirements make it necessary to have some other form, the surface of the driving side of the thread should be such as is generated by a line always remaining perpendicular to the axis of the thread. In other words, the working side should be the same as that of a square thread.

The following notation for screws will be used:

A = sectional area of screw at bottom of thread, square inches;

D = mean diameter of collar- or step-bearing, inches;

E = efficiency of screw and collar for forward motion;

E' = efficiency of screw and collar for backward motion or over-
    hauling;

F = turning force applied to the screw for raising the load,
    pounds;

F' = turning force applied to the screw for lowering the load,
    pounds;

F'' = turning force exerted by screw when overhauling, pounds;

I_p = polar moment of inertia of the section of the screw, biquad-
    ratic inches;

T = tension or compression in screw, pounds;

d = mean diameter of thread, inches;

d_1 = diameter of top of thread, inches;

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e = efficiency of screw-thread alone for forward motion;

e' = efficiency of screw-thread alone for backward motion or overhauling;

l = length of lever-arm of the turning force, inches;

p = pitch of thread, inches;

$r_{_1}$ = radius of screw at bottom of thread, inches;

s = shearing stress per unit area caused by the screw-thread resistance, pounds per square inch;

t = tensile or compressive stress per unit area caused by the axial force, pounds per square inch;

β = angle between surface of thread and a normal to the axis of the screw, degrees;

θ = angular pitch of thread, which is the angle between the mean helix and a plane perpendicular to axis of screw, degrees;

μ = tan ϕ = coefficient of friction between threads;

μ' = tan ϕ' = coefficient of friction between collar and supporting surface;

ϕ = angle of friction between threads, degrees;

ϕ' = angle of friction between collar and supporting surface, degrees.

The pitch p is the distance, parallel to the screw axis, between similar points of adjacent turns of the thread on a single-thread screw; in a screw having more than one thread, p is the axial distance between similar points on successive turns of the same thread. The mean thread-diameter d is taken as that of a helix lying midway between the top and bottom of the thread, and, for convenience, all the pressure of the nut against the thread is considered as acting along this mean helix. Theoretically the diameter of this helix is slightly greater than this mean value, but the difference is so small as to be far within the necessary limits of accuracy for any practical requirements. The same is true of D. The mean angular pitch θ is found by laying out a right triangle, making one side equal p, and the other equal πd, θ being the angle between the latter and the hypothenuse. This gives

$$\tan \theta = \frac{p}{\pi d} \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (49)$$

Square-thread Screws.

68. Relation between the turning moment and axial force in a square-thread screw.—When a screw is used for power transmission, as described above, it is often desirable to know the turning moment which must be applied to produce a given axial force, or *vice versa;* the axial force may be either tension or compression.

Fig. 82 may be taken to represent a portion of such a screw. As a convenient method of dealing with the problem, it may be assumed that a load is suspended from it by means of a nut (not shown) which fits on the thread; also, that the screw is supported by a bearing on which the collar near the top rests.

It is evident that friction must be considered. Since the nut and supporting surface under the collar may be of different materials, or differently lubricated, the coefficient of friction between the screw and nut, and that between the collar and the material against which it bears, may have different values; to make the case general, they will be considered as different.

If there were no friction between the threads of the nut and screw, the pressure between them would be normal to their working surfaces and make an angle θ with the axis of the screw. On account of friction, however, the direction of the force acting between them at any point makes the angle of friction ϕ with a normal to the thread at that point. When the screw is turning to raise the load, the direction of pressure between the threads is at an angle $(\theta + \phi)$ with the axis of the screw.

An elementary portion ab (see figure), of the tension T, is therefore held in equilibrium by the forces ac and cb; $cb = ab \tan(\theta + \phi)$. The latter is the external force, normal to the axis of the screw, which must be applied to turn the screw against the resistance due to the elementary axial force ab. The nut resistance to the turning of the screw, acting as it does at a distance $\frac{d}{2}$ from the screw's axis, is of a value $(cb) \times \frac{d}{2} = ab \tan(\theta + \phi) \times \frac{d}{2}$. The total nut resistance, for the total load T, is the sum of the resistances for all the elementary forces, each equal to ab, and is expressed by the equation

$$\text{Nut resistance} = T \tan(\theta + \phi) \times \frac{d}{2}.$$
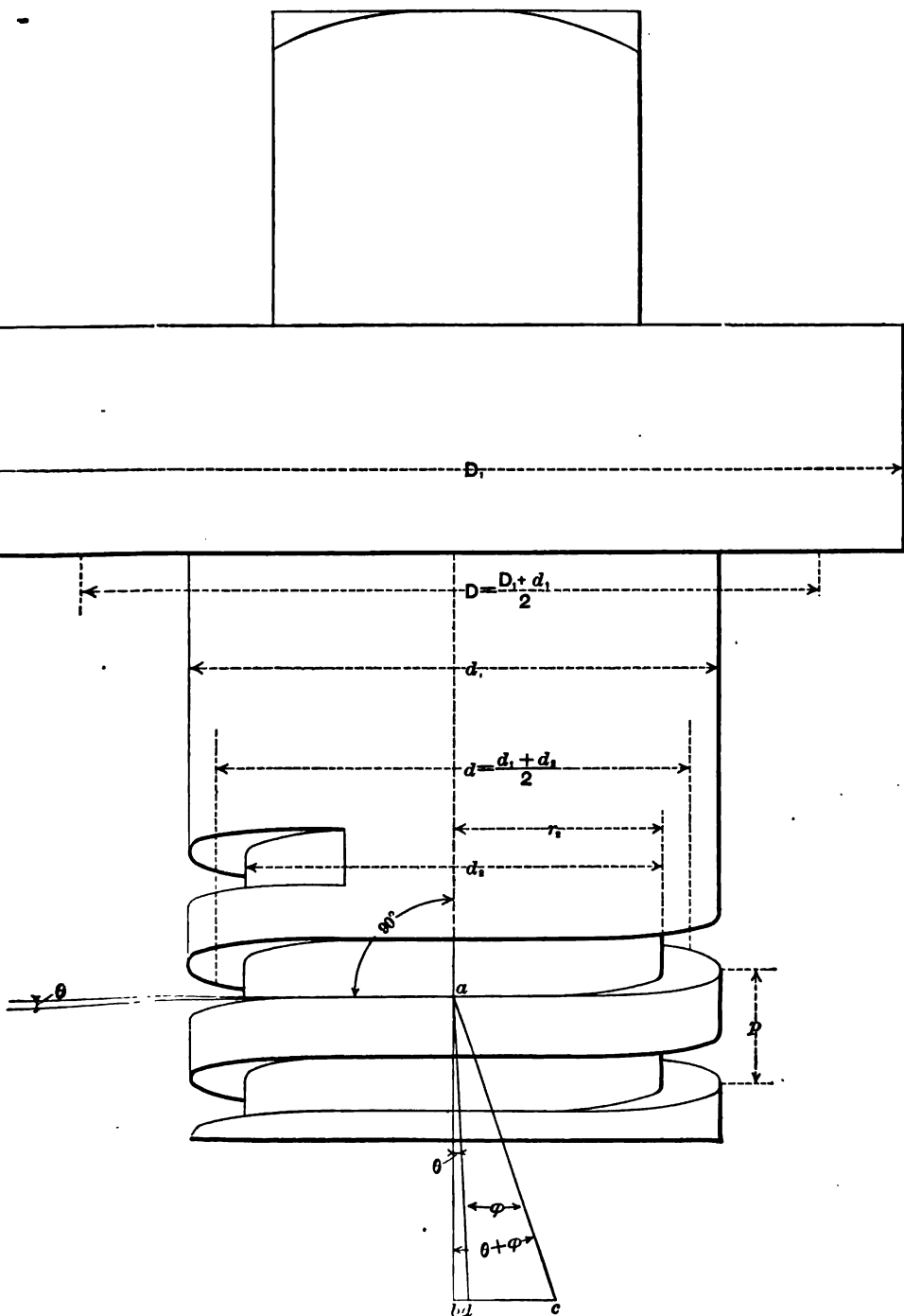
FIG. 82.

This represents the value of the turning moment which must be applied to overcome the resistance in the nut.

By the same method, the turning moment necessary to overcome the friction of the pressure T against the collar, is shown by the equation

$$\text{Collar friction} = T \tan \phi' \frac{D}{2} = T\mu' \frac{D}{2}.$$

In order to prevent side pressure of the body of the screw against the supporting frame, the torsional moment acting to turn the screw may be applied as a couple, each force having a lever-arm equal to $\frac{l}{2}$ about the axis of the screw, thus making the arm of the couple equal to l. Calling each of the external turning forces F, the driving torsional moment becomes Fl.
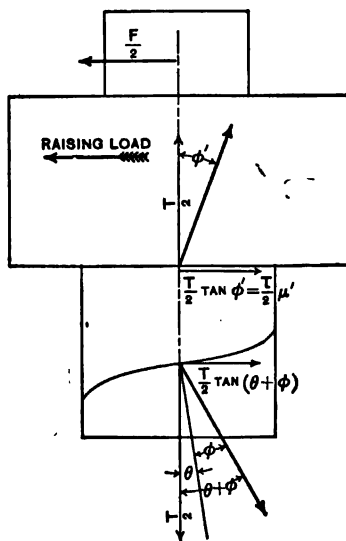
FIG. 82.1.

The following equation of turning and resisting moments can now be written for a square-thread screw with thrust-collar when lifting a load (Fig. 82.1):

Screw Resistance.   Collar Friction.

$$Fl = T\tan(\theta + \phi)\frac{d}{2} \qquad + T\mu'\frac{D}{2} \quad . \quad . \quad . \quad (50)$$

$$= T\frac{\tan\theta + \tan\phi}{1 - \tan\theta\tan\phi} \times \frac{d}{2} + T\mu'\frac{D}{2}$$

$$= T\frac{p \div \pi d + \mu}{1 - \mu(p \div \pi d)} \times \frac{d}{2} + T\mu'\frac{D}{2}$$

$$= T\frac{p + \mu\pi d}{\pi d - \mu p} \times \frac{d}{2} \qquad + T\mu'\frac{D}{2} \quad . \quad . \quad . \quad (51)$$

In equations (50) and (51), as well as those between them, the screw resistance is expressed by the first part of the right-hand members, and the collar friction by the last part of the right-hand members.
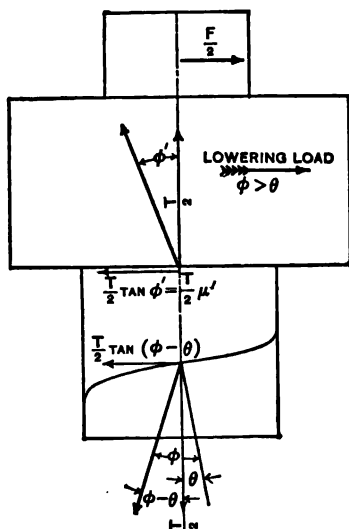
FIG. 82.2.

For lowering the load by turning the screw backward, the angle of friction ϕ must be taken on the opposite side of the

normal *ad* to the thread from that shown in the figure, because the frictional resistance acts in a direction opposite that when the load is lifted. The equation of turning moment and tension then becomes, for lowering the load, when ϕ is greater than θ (Fig. 82.2),

<div align="center">Thread Resistance.   Collar Friction.</div>

$$F'l = T \tan (\phi - \theta) \frac{d}{2} + T\mu' \frac{D}{2}, \quad \ldots \quad (52)$$

or

$$F'l = T \frac{\mu \pi d - p}{\pi d + \mu p} \times \frac{d}{2} + T\mu' \frac{D}{2} \ldots \ldots (53)$$
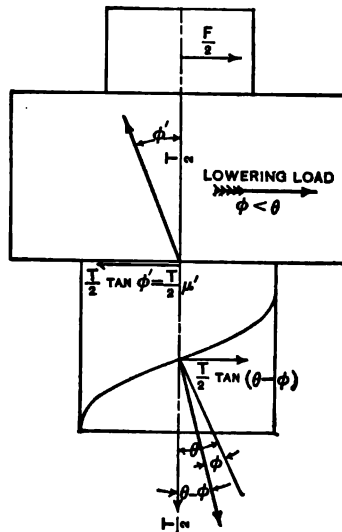
<div align="center">FIG. 82.8</div>

And when θ is greater than ϕ (Fig. 82.3),

$$F''l = T\mu' \frac{D}{2} - T \tan (\theta - \phi) \frac{d}{2}, \quad \cdots \cdots \quad (54)$$

where the first term represents **Collar Friction** and the second term **Driving Moment of Screw-thread.**

or

$$F''l = T\mu' \frac{D}{2} - T \frac{p - \mu \pi d}{\pi d + \mu p} \times \frac{d}{2}. \quad \cdots \cdots \quad (55)$$

The value of θ that will just hold the load at rest, or allow it to descend uniformly, when no external turning force is applied, is given by the equation

$$\tan (\theta - \phi) = \mu' \frac{D}{d}. \quad \cdots \cdots \quad (56)$$

Any larger value of θ than obtained by this equation will require a resisting moment applied to the screw to hold the load.

A screw that will allow a load to descend by its own weight is said to overhaul.

The turning moment $F''l$ that a screw will exert when overhauling is

$$F''l = T \tan (\theta - \phi) \frac{d}{2} - T\mu' \frac{D}{2}, \quad \cdots \cdots \quad (57)$$

where the first term represents **Driving Moment of Screw-thread** and the second term **Collar Friction.**

or

$$F''l = T \frac{p - \mu \pi d}{\pi d + \mu p} \times \frac{d}{2} - T\mu' \frac{D}{2}. \quad \cdots \cdots \quad (58)$$

If there were no collar friction, overhauling would occur for any value of θ greater than ϕ.

The principle of overhauling is applied to advantage in some mechanisms. Probably the most familiar example is the screw-driver having a handle which contains a nut that engages with a screw-thread of rapid pitch on the stock for holding the bit or blade: by forcing the handle down over the screw, the blade is rotated.

69. Efficiency of a square-thread screw and collar.—For forward motion, when the end thrust of the screw is resisted by a collar, the efficiency may be found by dividing the useful work accomplished during one revolution of the screw, $= Tp$, by the work, $F(2\pi l)$, applied to turn the screw through one revolution; the efficiency for forward motion is therefore expressed by the equation

$$E = \frac{Tp}{F(2\pi l)}. \quad \cdots \quad \cdots \quad (59)$$

By substituting in this equation the value of p, as expressed in equation (49), and that of Fl as given in equation (50), it becomes

$$E = \frac{T\pi d \tan \theta}{T\pi d \tan (\theta + \phi) + T\pi D\mu'} = \frac{\tan \theta}{\tan (\theta + \phi) + \dfrac{D}{d}\mu'}. \quad (60)$$

If the coefficients of collar friction and thread friction are equal, which corresponds to $\mu' = \mu = \tan \phi$, the maximum value of E is obtained when

$$\tan \theta = \frac{\sqrt{1 + \dfrac{D}{d}}}{\sec \phi + \tan \phi \sqrt{1 + \dfrac{D}{d}}}. \quad \cdots \quad \cdots \quad (61)$$

The efficiency e of the screw-thread alone is obtained by dropping from the denominator of equation (60) the expression $T\pi D\mu'$ for the work done in overcoming collar friction, which gives

$$e = \frac{\tan \theta}{\tan (\theta + \phi)}. \quad \cdots \quad \cdots \quad \cdots \quad (62)$$

The maximum value of e is obtained when

$$\tan \theta = \sec \phi - \tan \phi = \sqrt{1 + \mu^2} - \mu. \quad \cdots \quad (63)$$

The value of the thread angle θ is less for maximum efficiency when there is no collar friction than when there is.

When a screw is overhauling, the points of application of the driving force and resistance are exchanged; the axial force or load becomes the driving force, and the external resistance is applied at the same point as the driving force when lifting the load.

The efficiency E' for overhauling is therefore expressed by the equation

$$E' = \frac{2\pi F''l}{Tp}. \quad\quad\quad\quad (64)$$

The value of $F''l$, in this expression, is given in equations (57) and (58); that in (57) will be substituted. Therefore,

$$E' = \frac{T\pi d \tan(\theta - \phi) - T\pi D\mu'}{T\pi d \tan\theta} = \frac{d \tan(\theta - \phi) - D\mu'}{d \tan\theta}. \quad (65)$$

Dropping the collar friction $T\pi D\mu'$ from this equation gives for the efficiency e' of the screw-thread alone, when overhauling,

$$e' = \frac{\tan(\theta - \phi)}{\tan\theta}. \quad\quad\quad\quad (66)$$

When a driving force F' must be applied to a screw when lowering its load, there is no efficiency. The efficiency is zero when the load runs down uniformly of its own accord.

70. Coefficient of friction μ for square-thread screws.—This quantity has been very carefully determined by Prof. Albert Kingsbury for different materials and lubricants.[*] The dimensions of all the bolts and nuts tested were as follows:

Outside diameter of screw .... 1.426 inches
Inside diameter of nut .... 1.273 "
Mean diameter of thread .... 1.352 "
Pitch of thread..... .... $\frac{1}{3}$ "
Depth or height of nut (effective)...... $1\frac{1}{16}$ "
Area of rubbing surface of thread...... 1 sq. in. (about)

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[*] Trans. Amer. Soc. Mech. Engrs., vol. XVII., 1896, p. 96.

The nuts fit the screws loosely in order that there might be no friction other than that between the working surfaces. " The threads were cut carefully in the lathe and had been worn to good condition by trials previous to those here recorded.   Screw No. 5 was not quite so smooth as the others."   The lubricant was applied when the nut and screw were put together, and no more was added during the taking of the series of cards for them.   The speed was slow; about one revolution in two minutes.

The results of the tests are given in Table XIII.

TABLE XIII.

COEFFICIENTS OF FRICTION FOR SQUARE-THREAD SCREWS.

Working surface of thread approximately 1 square inch in area ; pressure per square inch of thread approximately the same as the total tension in screw-bolts.

Screws.	Coefficients of Friction μ. Average Values of Four Readings.				Value of μ for a Single Card.		Pressure and Lubricant.
	Nut 6, Mild Steel.	Nut 7, Wrought Iron.	Nut 8, Cast Iron.	Nut 9, Cast Brass.	Highest.	Lowest.	
1. Mild steel	.141	.16	.136	.136			
2. Wrought iron.	.139	.14	.188	.147	Screw 5	Screw 3	Pressure 10000
3. Cast iron	.125	.189	.119	.171	Nut 9	Nut 8	lbs. per sq. inch.
4. Cast bronze...	.124	.135	.172	.132	μ = .20	μ = .11	Machinery-oil.
5. Mild steel, case-hardened.	.133	.143	.13	.193			
1.........	.12	.105	.10	.11			
2.........	.1125	.1075	.10	.12	Screw 4	Screw 3	Pressure 10000
3.........	.10	.10	.095	.11	Nut 9	Nut 8	lbs. per sq. inch.
4.........	.1150	.10	.11	.1325	μ = .25	μ = .09	Lard-oil.
5.........	.1175	.0975	.105	.1375			
1.........	.111	.0675	.065	.04			Pressure 10000
2.........	.089	.07	.075	.055	Screw 5	Screw 5	lbs. per sq. inch.
3.........	.1075	.071	.105	.059	Nut 6	Nut 9	Machinery-oil
4.........	.071	.045	.044	.086	μ = .15	μ = .08	and graphite.
5.........	.1275	.055	.07	.035			
1.........	.147	.156	.132	.127			
2.........	.15	.16	.15	.117	Screw 5	Screw 2	Pressure 3000
3.........	.15	.157	.14	.12	Nut 7	Nut 9	lbs. per sq. inch.
4.........	.127	.13	.13	.14	μ = .19	μ = .11	Machinery-oil.
5.........	.155	.1775	.1675	.1325			

The coefficient of friction was low enough, in some cases, to allow the screw to " overhaul "; i.e., a pressure against the screw, parallel to its axis, caused it to rotate and pass through the nut.

Following out the fairly well established fact that, in oil-lubricated journals, the coefficient of friction decreases rapidly as the velocity of rubbing increases from slightly above zero to 10 or 100 feet per minute, according to whether the pressure is high or low, it would seem that lower values of the coefficients than given in the table might be found in screws running at the higher speeds that are frequently used for power transmission.

71. Problem.—Design a screw to lift 7 tons = 14000 pounds; end thrust of screw to be taken by collar-bearing; screw must not overhaul; driving gear attached to screw to be 18 inches in diameter.   Screw-thread to be "square."

For finding the pitch angle to prevent overhauling, the lowest values of the screw and collar friction that may occur with the materials used should be adopted.   It will be assumed that the screw is to be of mild steel; the nut and collar-bearing may be of cast brass.   The lowest values of the coefficients of friction may be taken as

$$\mu = .03 = \tan \phi, \text{ whence } \phi = 1° \ 43'; \text{ and } \mu' = .025.$$

The ratio of the mean diameter of the collar to that of the thread will be taken as $1.6 = D \div d$.

The value of the pitch angle θ is found by equation (56), by which

$$\tan (\theta - \phi) = \mu' \frac{D}{d} = .025 \times 1.6 = .04,$$

whence

$$\theta - \phi = 2° \ 18';$$
$$\theta - 1° \ 43' = 2° \ 18';$$
$$\theta = 4° \quad 1'.$$

Since the pitch must generally be such that the screw can be cut in an ordinary lathe, it will be assumed that $p = .5$ inch, and the mean diameter d of the screw, to give the required angle θ, calculated.

By equation (49),

$$\tan \theta = \frac{p}{\pi d},$$

whence

$$d = \frac{p}{\pi \tan \theta} = \frac{.5}{\pi \tan 4^\circ} = \frac{.5}{.06993\pi} = 2.27.$$

Taking the outside diameter of the screw as $0.45p$ greater than the mean diameter d for a single-thread screw gives

Outside diameter of screw $= 2.27 + .45 \times .5 = 2.5''.$

The mean diameter of the collar is

$$D = 1.6d = 1.6 \times 2.27 = 3.63 \text{ inches.}$$

The turning moment Fl which is necessary to raise the load is found by equation (51). The greatest values of the coefficients of friction, μ and μ', that are probable to occur at any time when the lubrication is poor, should be used in this equation in order that the maximum value of the turning moment may be found.

The values $\mu = .15$ and $\mu' = 0.1$ are probably the highest that occur in well-made machines operated with reasonable care and cleanliness.

Applying equation (51),

$$Fl = T\frac{p + \mu\pi d}{\pi d - \mu p} \times \frac{d}{2} + T\mu'\frac{D}{2}$$

$$= 14000 \;\frac{.5 + .15\pi 2.27}{\pi 2.27 - .15 \times .5} \times \frac{2.27}{2} + 14000 \times 0.1\frac{3.63}{2}$$

$$\begin{array}{ccc} \text{Screw Resistance.} & \text{Collar Friction.} & \\ = \quad\quad 3532 & + \quad\quad 2541 & = 6073 \text{ inch-lbs.} \end{array}$$

And the force F acting tangent to the pitch circle of a gear whose pitch radius is 9 inches is

$$F = 6073 \div 9 = 675 \text{ pounds.}$$

No allowance has been made for journal friction in the above calculations.   This makes the value of F greater than 675 pounds. The pressure between the teeth of the driving gears is greater than F on account of their obliquity of action.   Assuming that the actual pressure between the teeth, taking into account both journal friction and obliquity of action, is 800 pounds, and that the coefficient of journal friction is .06, then the corresponding value of the tangential force is

$$F = 675 + (.06 \times 800)\frac{1.25}{9} = 682.$$

The efficiency of the screw when working with the highest assumed coefficients of friction, and including journal friction, is, by equation (59),

$$E = \frac{Tp}{F(2\pi l)} = \frac{14000 \times 0.5}{682 \times 2\pi \times 9} = .18 = 18\%.$$

72. Maximum stress in a screw.—A screw that is turning and lifting a load is subjected to an axial tensile stress equal to the weight of the load, together with a torsional moment equal to the turning moment that must be applied to overcome the resistance of the screw-thread.   The combination of these two stresses produces what may be called a maximum tensile stress and a maximum shearing stress.   The values of these maximum stresses should not exceed the working strengths, tensile and shearing, of the material.

If the axial force acting on the screw is compression instead of tension, and the distance between the nut and collar is small enough to allow the part in compression to be considered as a short column, not liable to bend or buckle, the maximum compressive stress will be equal to the maximum tensile stress for the same load, and the maximum shear will be the same in both cases.

The formulas for maximum stress in a rod of circular section, as given in works on the " mechanics of materials," are:

$$\text{Maximum tension or compression} = \frac{t}{2} + \sqrt{\frac{t^2}{4} + s^2}; \quad (67)$$

$$\text{Maximum shear} \qquad\qquad = \sqrt{\frac{t^2}{4} + s^2}. \quad (68)$$

In dealing with the strength of a screw of ordinary construction, the thread may be neglected, since it adds but little to the strength. The screw may therefore be considered as a cylindrical bar of a diameter the same as that of the bottom of the thread. Under this assumption the value of t is

$$t = T \div A = T \div \pi r_2^2,$$

and that of s, for square-thread screws, is

$$s = \frac{Fl}{\frac{1}{2}p \div r_2} = \frac{T \tan (\theta + \phi) \frac{d}{2}}{\frac{1}{2}\pi r_2^4 \div r_2} = \frac{T \tan (\theta + \phi)d}{\pi r_2^3}.$$

By substituting these values of t and s in equations (67) and (68), the following equations are obtained for square-thread screws:

$$\text{Max. tension or compression per sq. in.} = \frac{T}{2A} + \sqrt{\frac{T^2}{4A^2} + \frac{T^2 \tan^2 (\theta + \phi)d^2}{\pi^2 r_2^6}},$$

which reduces to

$$\text{Max. tension or compression per sq. in.} = \frac{1}{2}\frac{T}{A}\left[1 + \sqrt{1 + 4 \tan^2 (\theta + \phi)\left(\frac{d}{r_2}\right)^2}\right]. \quad (69)$$

In the same manner,

$$\text{Maximum shear per sq. in.} = \frac{1}{2}\frac{T}{A}\sqrt{1 + 4 \tan^2 (\theta + \phi)\left(\frac{d}{r_2}\right)^2}. \quad (70)$$

Example of application of formulas for maximum stress in a screw.—In the problem solved in § 71 the following data were assumed, or else determined for the highest coefficients of friction: $T = 14000$ pounds; $\tan \phi = .15$; $\phi = 8° 31' 10''$; $\theta = 4° 1'$; $\theta + \phi = 12° 32' 10''$; and $d = 2.27$ inches. In addition to these data, the radius at the bottom of the thread may be taken as $r_2 = 1$ inch; whence $A = 3.1416$ square inches.

By the substitution of these quantities in equation (69),

$$\text{Maximum tension} = \frac{1}{2} \frac{T}{A}\left[1 + \sqrt{1 + 4 \tan^2 (12° \ 32' \ 10'')\left(\frac{2.27}{1}\right)^2}\right]$$

$$= \frac{1}{2} \frac{T}{A}[1 + \sqrt{1 + 4(0.22236)^2(2.27)^2}]$$

$$= \frac{1}{2} \frac{14000}{3.1416}(1 + 1.4211) = 5390 \text{ lbs. per sq. in.;}$$

and, by equation (70),

$$\text{Maximum shear} = \frac{1}{2} \frac{14000}{3.1416} \times 1.4211 = 3180 \text{ lbs. per sq. in.}$$

Angular-thread Screws.

73. **Relation between turning moment and axial force in an angular-thread screw.**—When the working surface of a screw thread makes an angle β with a normal to the axis of the screw, as in Fig. 83, the pressure between the threads of the nut and screw is
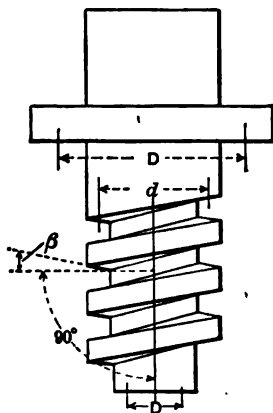
FIG. 83.

increased by the wedge-like action of the thread.

The following equations apply to an angular-thread screw when lifting a load.*   (Notation given in § 67.)

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* The development of the equation for thread resistance is given in § A of the Appendix.

<center>Thread Resistance.   Collar Friction.</center>

$$Fl = T\frac{\tan\,\theta + \tan\,\phi\,\sec\,\beta}{1 - \tan\,\theta\,\tan\,\phi\,\cos\,\beta} \times \frac{d}{2} + T\mu'\frac{D}{2} \quad . \quad . \quad (71)$$

$$= T\frac{(p \div \pi d) + \mu\,\sec\,\beta}{1 - (p \div \pi d)\mu\,\cos\,\beta} \times \frac{d}{2} + T\mu'\frac{D}{2}$$

$$= T\frac{p + \mu\pi d}{\pi d - \mu p}\frac{\sec\,\beta}{\cos\,\beta} \times \frac{d}{2} + T\mu'\frac{D}{2}; \quad . \quad . \quad . \quad . \quad (72)$$

$$E = \frac{Tp}{F(2\pi l)} = \frac{Td\,\tan\,\theta}{2Fl}; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (73)$$

$$e = \frac{\tan\,\theta\,(1 - \tan\,\theta\,\tan\,\phi\,\cos\,\beta)}{\tan\,\theta + \tan\,\phi\,\sec\,\beta}. \quad . \quad . \quad . \quad . \quad (74)$$

For the backward motion of an angular-thread screw when a driving force is applied to lower the load:

<center>Thread Resistance.   Collar Friction.</center>

$$F'l = T\frac{\tan\,\phi\,\sec\,\beta - \tan\,\theta}{1 + \tan\,\theta\,\tan\,\phi\,\cos\,\beta} \times \frac{d}{2} + T\mu'\frac{D}{2}. \quad . \quad (75)$$

If $\tan\,\theta$ is greater than $\tan\,\phi\,\sec\,\beta$ in equation (75), then the thread resistance is negative, and must be subtracted from the collar friction.

The thread resistance for backward motion becomes zero when

$$\tan\,\theta = \tan\,\phi\,\sec\,\beta,$$

or

$$\tan\,\phi = \tan\,\theta\,\cos\,\beta.$$

The last two equations indicate the values of θ and ϕ which would just allow the screw to overhaul if there were no collar friction.

When overhauling, the driving moment, exerted by the nut, is

<center>Driving Moment of Screw-thread.   Collar Friction.</center>

$$F''l = T\frac{\tan\,\theta - \tan\,\phi\,\sec\,\beta}{1 + \tan\,\theta\,\tan\,\phi\,\cos\,\beta} \times \frac{d}{2} - T\mu'\frac{D}{2}. \quad . \quad (76)$$

Experimentally determined efficiency of a_V-thread screw-bolt. —A series of experiments upon a screw-bolt were made by James McBride to determine its efficiency.* An ordinary V-thread screw-bolt was purchased on the market, and was not specially prepared for the experiments; the nut fitted the thread freely, so that it could be run along the thread easily with the hand; the flat end of the nut rested on a washer which formed the bearing-surface; both nut and washer were of malleable iron with bearing-surfaces left rough or unfaced.   Lard-oil was used as a lubricant.   The bolt was suspended vertically by the nut, which rested on a support 4 feet 6 inches high; a load was hung on the lower end of the bolt.   The torsional moment was applied by different men pulling on a horizontal, single-ended wrench which spanned the nut.   The following are the more important data:

Diameter of bolt = 2 inches;
Pitch of screw   = 0.22 of an inch = 4.5 threads per inch;
Standard V thread;
Load suspended = 7500 pounds.

The highest efficiency found was 12.29%, the lowest 9.71%, the average being 10.19%.

At the average efficiency of 10.19% a torsional moment of 1 inch-pound, applied to the wrench, would produce an axial stress $T = (2\pi \div 0.22)0.1019 = 2.91$ pounds in the bolt.

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* Trans. Amer. Soc. Mech. Engrs., vol. XII., 1891, p. 781.

CHAPTER V.

74. **The most common form of screw-gearing that is used, when** there is to be any considerable amount of power transmitted, is the worm and worm-wheel; and, of this mechanism, that having the axes of the worm and wheel at right angles is most generally adopted, doubtless because it is only when the shafts are at a right angle that the face of the wheel can be concaved to embrace a portion of the worm. In the worm and worm-wheel there is generally a great difference of angular velocities, the worm making many revolutions to give one turn to the wheel. The diameter of the wheel is ordinarily much greater than that of the worm.

When the axes are at any other angle than 90°, the face of the wheel cannot be concaved, and its teeth, instead of being curved so as to secure line contact with the worm, must be portions of a many-threaded screw, or else similar to those of a spur-gear, both of which give only point contact theoretically.

When the worm and wheel do not have greatly different angular velocities, and the teeth on both are short lengths of the threads of a many-threaded screw, the name "screw-gears" is commonly applied. This term will be used to designate all such mechanisms in which the wheel does not have a concave face. The axes may be at any angle.

WORM AND WORM-WHEELS.*

75. **The strength of a worm and worm-wheel** seldom needs consideration, for, under ordinary conditions, the allowable pressure

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* The coefficient of friction for worm-gearing is probably about the same as that of screw-gears (Table XXI) or square-thread screws (Table XIII), according to the velocity of rubbing.

between them, that will not produce abrasion, is much less than would be required to break the teeth when made of any of the materials common to engineering.

Since the teeth are cut on a wheel concaved to fit the worm, they are much stronger than those of the same proportions that are straight. Their strength cannot be satisfactorily calculated, and it is scarcely desirable to make such a calculation.

The strength of the worm-thread cannot be calculated, but when of the same material and thickness at the pitch surface as the teeth on the wheel, it is the stronger of the two.

76. Equations for turning force and efficiency of worm and worm-wheel.—The equations of the relation between the turning moment and pressure against the teeth in a direction parallel to the axis of the worm are essentially the same for a worm-wheel as those of an angular-thread screw, given in § 73. This assumes that the thread is angular, not square, for it is not known that a square-thread worm is ever used when it must perform a service that demands even moderately high speeds and pressures.

If a worm works in engagement with two wheels that are tangent to its pitch surface at diametrically opposite points, and whose axes are parallel to each other and at right angles to that of the worm, the equations for an angular-thread screw apply to it without modification. For the more common mechanism, in which the worm engages with a single wheel, there is a side pressure on the supporting journal-bearings which may materially increase the turning moment required to rotate the worm against a given pressure of the wheel against it. This is especially true if the journal is large or has a high coefficient of friction.

The thrust-bearing of a worm can be obviated by using two worms, one right-hand and the other left-hand, on the same shaft, each engaging with a suitable worm-wheel; the shafts of the worm-wheels must be geared together so as to give them both the same rate of rotation in opposite directions, provided each worm and wheel forms a part of the mechanism differing from the other only in the direction of the thread. It can readily be seen that the thrust of one worm will be annulled by that of the other, thus eliminating the need of a thrust-bearing.

For convenience of reference the notation used in the equations

for worm-gearing will be given in full, although most of the symbols are the same as for screws:

D' = mean diameter of thrust-bearing, inches;

D'' = mean diameter of journal, inches;

E = efficiency of mechanism for forward motion;

E' = efficiency of mechanism for overhauling;

F = force acting to rotate the worm, pounds;

F'' = turning force exerted by the worm when overhauling, pounds;

P = turning force acting on driven wheel, pounds;

T = thrust of worm or screw-gear parallel to its axis, pounds;

d = mean diameter of worm-thread, inches;

e = efficiency of worm-thread alone, journal and thrust-bearing friction not included;

e' = efficiency of worm-thread alone when overhauling;

l = length of lever-arm of turning force, inches;

p = pitch of worm-thread, inches;

β = angle between an element of the thread surface and a plane normal to the axis of the screw;

θ = angular pitch of screw-thread;

μ = tan ϕ = coefficient of friction between worm and wheel;

μ' = tan ϕ' = coefficient of thrust-bearing friction;

μ'' = coefficient of journal-friction;

ϕ = angle of friction between worm and wheel;

ϕ' = angle of friction for thrust-bearing.

In the following discussion it is assumed that the axes of the worm and worm-wheel are at right angles.

The pressure, due to the forces acting between the teeth, against the journals supporting a worm that engages with but one worm-wheel, assuming that there is a journal at each end of the worm, is equal to the resultant of two forces, one approximately equal to the force resisting the rotation of the worm when driving the wheel, acting parallel to the axis of the wheel, and at a distance $d/2$ from the axis of the worm, and the other equal to the component, normal to and intersecting the axis of the worm, of the pressure T tangent to the mean diameter of the wheel and parallel to the axis of the worm.*

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* Strictly the force T causes some additional pressure on the journal since it acts with a lever arm $d + 2$. This pressure is so small as to be negligible in any ordinary design.

The first of these forces, acting parallel to the axis of the wheel, is approximately $T \dfrac{\tan\theta + \tan\phi \sec\beta}{1 - \tan\theta \tan\phi \cos\beta}$*; the second, normal to and intersecting the axis of the worm, is $T \tan\beta$. Since these two forces act at right angles, their resultant, which is the

$$\left.\begin{array}{c}\text{Pressure on}\\ \text{worm-journals}\end{array}\right\} = T \sqrt{\left(\dfrac{\tan\theta + \tan\phi \sec\beta}{1 - \tan\theta \tan\phi \cos\beta}\right)^2 + \tan^2\beta}.$$

The journal friction, assuming that both journals are of the same diameter D'' and have the same coefficient of friction μ'', is

$$\left.\begin{array}{c}\text{Journal fric-}\\ \text{tion of worm-}\\ \text{shaft}\end{array}\right\} = \mu'' T \dfrac{D''}{2} \sqrt{\left(\dfrac{\tan\theta + \tan\phi \sec\beta}{1 - \tan\theta \tan\phi \cos\beta}\right)^2 + \tan^2\beta}. \quad (77)$$

If the journals supporting the worm are of different diameters or have different coefficients of friction, the pressure on each journal may be found and the friction of each determined. The total pressure is divided between the two journals in amounts inversely proportional to their distances from the pitch point of the worm and wheel. Such a refinement as this would seldom, if ever, be worth applying in practice.

There is an end thrust on the worm-wheel shaft approximately equal to $T \dfrac{\tan\theta + \tan\phi \sec\beta}{1 - \tan\theta \tan\phi \cos\beta}$; also a journal pressure having a value $T \sqrt{1 + \tan^2\beta}$.

The following equations may now be written by referring to those for an angular-thread screw. In them the frictional losses in the bearings supporting the worm-wheel are not taken into account as producing part of the thrusting force T. The equations of efficiency, therefore, do not exactly represent the efficiency of a complete mechanism, on account of the friction of the worm-wheel bearings being neglected, but the speed of the worm-wheel is so slow that the friction losses in its bearings are comparatively small ordinarily. The efficiencies given by the following equations are

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* See Appendix, § A.

far more accurate than the assumptions that can be made for the coefficients of friction.

The value of the " journal friction " in the following equations may be obtained by equation (77).

When the worm drives the wheel:

<div align="center">Thread Resistance.</div>

<div align="center">Thrust-bearing Friction.</div>

$$Fl = T\frac{\tan\theta + \tan\phi\sec\beta}{1 - \tan\theta\tan\phi\cos\beta} \times \frac{d}{2} + T\mu'\frac{D'}{2} + \text{Journal friction,} \quad (78)$$

or

$$Fl = T\frac{p + \mu\pi d\sec\beta}{\pi d - \mu p\cos\beta} \times \frac{d}{2} \quad + T\mu'\frac{D'}{2} + \text{Journal friction,} \quad (79)$$

and

$$E = \frac{Tp}{F(2\pi l)} = \frac{Td\tan\theta}{2Fl}; \quad \cdots \quad \cdots \quad (80)$$

$$e = \frac{\tan\theta(1 - \tan\theta\tan\phi\cos\beta)}{\tan\theta + \tan\phi\sec\beta}. \quad \cdots \quad (81)$$

If it is desired to neglect the effect of the angle β, then, for the approximate turning moment,

<div align="center">Approximate Thread Resistance.</div>

<div align="center">Thrust-bearing Friction.</div>

$$Fl = T\tan(\theta + \phi)\frac{d}{2} + T\mu'\frac{D'}{2} + \text{Journal friction,} \quad \cdots \quad (82)$$

or

$$Fl = T\frac{p + \mu\pi d}{\pi d - \mu p} \times \frac{d}{2} + T\mu'\frac{D'}{2} + \text{Journal friction,} \quad \cdots \quad (83)$$

And, for the approximate worm-thread efficiency, neglecting β,

$$e = \frac{\tan\theta}{\tan(\theta + \phi)} \text{ (approximately).} \quad \cdots \quad (84)$$

When the wheel drives the worm, which action corresponds to overhauling in a screw:

<div align="center">Turning Force due to Pressure of Wheel against Worm.</div>

<div align="center">Thrust-bearing Friction.</div>

$$F''l = T\frac{\tan\theta - \tan\phi\sec\beta}{1 + \tan\theta\tan\phi\cos\beta} \times \frac{d}{2} - T\mu'\frac{D'}{2} - \text{Journal friction,} \quad (85)$$

or

$$F''l = T\frac{p - \mu\pi d \sec \beta}{\pi d + \mu p \cos \beta} \times \frac{d}{2} \qquad - T\mu'\frac{D'}{2} - \begin{array}{l}\text{Journal} \\ \text{friction,}\end{array} \quad (86)$$

and

$$E' = \frac{F''2\pi l}{Tp} = \frac{2F''l}{Td \tan \theta}; \quad \cdots \cdots \cdots \quad (87)$$

$$e' = \frac{\tan \theta - \tan \phi \sec \beta}{\tan \theta (1 + \tan \theta \tan \phi \cos \beta)}. \quad \cdots \cdots \quad (88)$$

As is indicated by the above equations, the efficiency of worm-gearing increases with the angular pitch up to a certain limiting value, which depends upon the coefficient of friction of the worm and the frictional resistance of the thrust-bearing. This limiting angle is greater than has apparently ever been found satisfactory in practice. There seems to be a general tendency, however, to use a greater angular pitch than formerly, when the primary function of the worm-gearing is to transmit power in considerable amounts. A pitch angle of 20°, or even more, is quite commonly used for worm-driven machine-tools.

77. **Tests of worm-gearing.**—In 1885 Wilfred Lewis published the results of an extensive series of tests made by Wm. Sellers & Co. on worm and spiral gearing. The result of each individual test on a particular mechanism, at different speeds and pressures, was plotted on a diagram, which also had a curve of the mean efficiency of the mechanism for the entire range of speed covered. From these curves of mean efficiencies the readings given in Table XIV were taken. The teeth were approximately of the involute system.

In the experiments it was found that abrasion, or cutting, began between the surfaces of the worm-thread and wheel-teeth at certain limiting pressures and speeds. The mechanism could still be run after abrading, but the efficiency was materially reduced. This is shown very clearly in the third and fourth columns of Table XIV, which give the efficiency both before and after cutting ; the drop in efficiency in the last column, at 100 revolutions, also shows the effect of cutting.

TABLE XIV.

EFFICIENCY OF CAST-IRON WORM-GEARING.*

Efficiency = ratio of power delivered by worm-wheel shaft to that applied to worm-shaft.

All worms 4″ pitch diameter; all worm-wheels 18.62″ pitch diameter, 39 teeth, 1½-inch pitch. Worm ran in oil-bath. When two values are given for the efficiency in the same column, the lower one was obtained after abrasion began. The drop from 69 to 65 in the last column is due to abrasion.

Revolutions per minute, approximately equals the velocity of sliding in feet per minute.	Efficiency, per cent.			
	2 threads, 8 in. pitch. Angular pitch 13° 51'. Cast thread and teeth.		1 thread, 1½ in. pitch. Angular pitch 6° 49'. Step-bearing 1.6 in. mean diam.	
	Collar thrust-bearing 4″ mean diam. Pressure 200 to 6000 lbs.	Step-bearing 1.6″ mean diam. Pressure 1200 to 5500 lbs.	Cast thread and teeth. Pressure 450 to 5600 lbs.	Machine-cut thread and teeth. Pressure 1200 to 5600 lbs.
3	55	49	....	42
5	59	52	....	45
7	62	54	46	48
10	65	57	49	51
15	68	59	52	55
20	70	61	53	57
30	72	64	57	61
40	74	66	59	63
60	75	68	64 and 46	67
80	76	70 and 59	67 and 48	69
100	....	71 and 60	69 and 50	65
120	....	72 and 61	71 and 52	67
150	....	73 and 62	73 and 53	68
200	....	74 and 63	54	70
300	....	65	56	....
400	....	66	57	....
500	....	67	57.5	....
600	....	....	58	....
900	....	....	59	....

(In the "Step-bearing" column the vertical note reads "After abrasion." In the "Cast thread and teeth" column the vertical note reads "After abrasion." In the "Machine-cut" column the vertical note reads "Abraded.")

* Readings of efficiency taken from diagrams by Wilfred Lewis in Trans. Amer. Soc. Mech. Eng., vol. VII., p. 273.

The second column gives the efficiency of a worm whose thrust was taken by a collar-bearing formed by turning down the end of the thread to a flat surface ; this did not form a complete ring of metal to bear against the bearing, about half the material being removed on account of the space between the threads. In all the other tests the thrust was carried by a step-bearing made of two hardened-steel disks, carefully ground, with a hard-brass washer

interposed. The step-bearing was 2⅛ inches outside diameter. The shaft at one end of the worm was 2⅟₁₆ inches diameter, and 1⅟₁₆ inches at the other.

An examination of the table shows that the efficiency increased with the speed, in all cases, until abrasion began. The greater efficiency of the double-thread worm may be seen clearly for the lower speeds, but is not so decided as the speed increases.

A series of experiments were also made on the single-thread worm with cast teeth, whose efficiency is given in the fourth column of Table XIV, to determine the speeds and pressures liable to produce cutting during a ten-minute run. The results of these experiments are given in Table XV, taken from Mr. Lewis's paper.

TABLE XV.

SPEEDS AND PRESSURES LIABLE TO PRODUCE CUTTING IN CAST-IRON WORM-GEARING.

Velocity of sliding, feet per minute.	Pressure on teeth, pounds.	Temperature.		Efficiency.		Duration of run, minutes.	Ft. lbs. per min. consumed in friction before cutting began.
		Initial.	Final.	Initial.	Final.		
800	1785	106°	140°	.609	.387	6	117,600
880	1780	118	132	.607	.462	8	129,300
880	1205	137	150	.575	.360	8	97,000
800	448	118	138	.594	.445	10	29,400
480	2822	144	167	.591	.450	7	117.800
400	3481	170	180	.689	.415	8	98,300
360	4837	138	166	.641	.473	6	122,400
*306	5558	163	186	.677	.677	10	102,000

* No cutting at 306 feet per minute.

In the last of these experiments, at a speed of rubbing of 306 feet per minute, and a thrust of 5558 pounds on the worm, no abrasion occurred. The temperatures given are those of the oil-bath in which the worm ran ; the initial efficiency was obtained before abrasion, and the final after cutting began.

In 1883–84 Dr. R. H. Thurston made a series of experiments on worm-gearing for the Yale & Towne Mfg. Co. The results are given by Henry R. Towne in the Transactions of the American Society of Mechanical Engineers.† The data show that the worm

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† Vol. VII., 1886, p. 300.

and wheel were both of cast iron, with machine-cut threads and teeth ; the worm was $6\frac{1}{8}$ inches pitch diameter, double-threaded, and 4 inches long on the thread ; the worm-wheel was $15\frac{1}{8}$ inches pitch diameter, with 50 teeth and 2-inch face. The pitch, calculated from these data, is 2 inches linear, or 5° 59′ angular.

During the experiments three forms of thrust-bearings for taking the thrust of the worm were used. They were : First, a collar thrust-bearing having a collar 1 inch wide and $2\frac{1}{2}$ inches mean diameter. The rubbing surfaces were the faced ends of the worm-hub and of the cast-iron supporting frame of the mechanism. Second, a button thrust-bearing, or step, made by capping the end of the worm-shaft with a piece of hardened steel, having its exposed face slightly convex, and letting it run against the hardened end of an adjusting set-screw. Third, a roller thrust-bearing, consisting "of 12 chilled cast-iron coned rollers of $\frac{9}{16}$ inch mean diameter, contained within a brass cage having a separate pocket for each cone, the cones travelling at a mean radius of $1\frac{3}{8}$ inches from the axis of the shaft, between two steel collars or rings, one bearing against the hub of the worm, and the other against the face of the frame-bearing, the faces of these collars being coned to the shape of the rollers. The centrifugal thrust of the cones was resisted by a wrought-iron ring surrounding the cage, the ends of the cones being convex."

Table XVI shows a comparison between the button-bearing and

TABLE XVI.

COMPARATIVE EFFICIENCIES OF THE SAME WORM AND WHEEL
WITH BUTTON THRUST-BEARING AND ROLLER STEP-BEARING.

Horse-power per 100 revolutions of worm per minute.	Efficiency, per cent.		Horse-power per 100 revolutions of worm per minute.	Efficiency, per cent.	
	Button thrust-bearing.	Roller thrust-bearing.		Button thrust-bearing.	Roller thrust-bearing.
.25	6	9	4.0	38	43
.50	10	12	5.0	44	47
.75	13	18	6.0	49	50
1.00	16	22	7.0	54	52
1.5	21	27	8.0	59	54
2.0	25	31	8.28 *	60	....
2.5	29	35	9.00	....	56
3.0	32	38	10.5 †	....	57

* Highest power given for button-bearing.　† Highest power given for roller-bearing.

roller-bearing for different rates of working, and Table XVII between the collar-bearing and roller-bearing. The readings in the tables are taken from diagrams showing the results of the experiments.

TABLE XVII.

COMPARATIVE EFFICIENCIES OF THE SAME WORM AND WHEEL
WITH COLLAR THRUST-BEARING AND WITH ROLLER STEP-BEARING.

Revolutions of driver per minute.	Efficiency, per cent.		Revolutions of driver per minute.	Efficiency, per cent.	
	Collar thrust-bearing.	Roller thrust-bearing.		Collar thrust-bearing.	Roller thrust-bearing.
60	38	48	· 250	40	60
100	42	52	300 *	38	63
150	48	55	350	. . . .	65
200	42	58	400 †	. . . .	67

* Highest speed for collar-bearing.   † Highest speed given for roller-bearing.

The difference in the efficiencies obtained with these three forms of thrust-bearings shows clearly how much power may be lost in this part of the mechanism. It should be noted that while the efficiency of the roller-bearing is higher than that of the button-bearing for rates of working up to 6 horse-power, it becomes lower at 7 horse-power. The original diagram shows equal efficiencies at about 6.5 horse-power.

The results of a number of experiments on three tool-steel worms, hardened, running against cast-iron worm-wheels, are given by Bertram P. Flint.* Two of the worms were of the same diameter, but one was of practically twice as great lead or pitch as the other. The results of the experiments are given in Table XVIII. That the limit of pressure is lower at the highest speeds of rubbing is clearly shown.

The double worm-and-wheel mechanism shown in Fig. 83.1 requires no thrust-bearing for the worms. Two worms, one right- and the other left-hand, are placed on the same shaft. The worm-wheels are geared together by a pair of spur-gears. In this particular design, used for elevator-service, a worm-wheel and spur-

* *Engineering News,* April 9, 1892, p. 348.
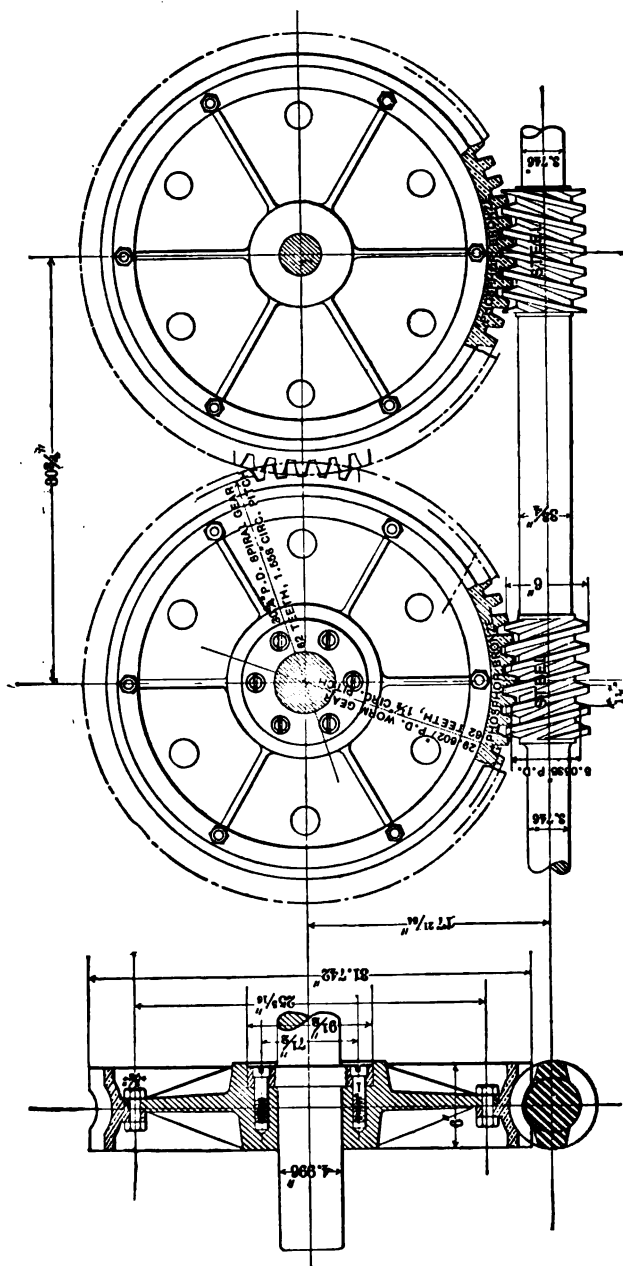
FIG. 88.1.

gear are made together as a unit. The spur-gear has helical or spiral teeth cut at an angle with its axis equal to the pitch angle of the worm thread. The middle of the gear is hobbed out to form a true worm-wheel. In some designs where a spur-gear and worm-wheel are a unit, the spur-gear has teeth running straight across its face parallel to its axis. This necessitates cutting a pair of circumferential grooves around the wheel so as to separate the worm-wheel teeth from those of the spur-gear on each side of it. The worm-wheel and spur-gear can, of course, be made entirely separate and fastened to the same shaft. In the design shown, a winding drum for the elevator cables is placed upon one worm-wheel shaft so that all the power is finally transmitted to the drum shaft by one worm-wheel.

The worm-and-wheel drive, Fig. 83.1, with bronze worm-wheels and steel worms, submerged in castor-oil or high-grade cylinder oil, will operate satisfactorily at 500 revolutions per ' minute of the worm when exerting a turning force of 6000 pounds on each worm-wheel tangent to its pitch-circle. This corresponds to 6000 pounds tension or compression in the worm-shaft between the worms. The rubbing velocity is 793 feet per minute at the pitch-circle of the worm.

The tangential pressure between the teeth of the spur-gears is slightly less than 6000 pounds on account of the somewhat larger pitch diameter of the spur-gears. This pressure is

$$\frac{29.6027}{30.75} \times 6000 = 5780 \text{ pounds.}$$

A smaller, similar mechanism, with worms $3\frac{1}{2}$ inches outside diameter, 3.0226 inches pitch diameter and $\frac{3}{4}$ inch pitch, engaging with worm-wheels having 80 teeth each on a pitch radius of 9.5493 inches operates successfully at 750 revolutions per minute of the worms and 4400 pounds tension or compression in the part of the worm shaft between the worms. This corresponds to a rubbing speed of 710 feet per minute and a tangential pressure 4400 pounds on each worm-wheel. The spiral spur-gears are 20 inches pitch diameter with 80 teeth of 4 diametral pitch.

TABLE XVIII.

EFFICIENCY OF WORM-GEARING.*

Efficiency = ratio of power delivered by worm-wheel shaft to that received by the worm-shaft.

Worm of tool-steel, hardened ; wheel of cast-iron ; axes of worm and wheel both horizontal ; worm on top of wheel ; bottom of wheel dipped in oil-bath. Thrust of worm taken by phosphor-bronze plate against which the end of the shaft turned.   The limit of pressure indicates the highest pressure that did not cause abrasion.

Thrust of Worm. Pounds.	Efficiency, per cent.							
	Single-thread Worm. Angular Pitch 6° 20'.			Double-thread Worm. Angular Pitch 12° 30'.			Double-thread Worm. Angular Pitch 10°.	
	198 rev. per min. 96 ft. per min. Sliding velocity.	201 rev. per min. 151 ft. per min. Sliding velocity.	272 rev. per min. 204 ft. per min. Sliding velocity.	198 rev. per min. 96 ft. per min. Sliding velocity.	201 rev. per min. 151 ft. per min. Sliding velocity.	272 rev. per min. 204 ft. per min. Sliding velocity.	200 rev. per min. 226 ft. per min. Sliding velocity.	272 rev. per min. 330 ft. per min. Sliding velocity.
200	....	37	80	48	49	47.5		
300	89.5	39	32	47	53	49.5	50	53
400	uni-	41	33	51	56	51	53	54
500		43	34.5	53	57	51.5	54	54
600	formly	44	36	54	54	51	54	52
700	in-	44.5	37	54	52	49	53	51
800		44	37	53	51.5	48	52	
1000	creases	43	36.8	51	49.5	47	51	
1200		42	36	50	48			
1400	to	39	......	48.5				
1600	41							
Limit of pressure, pounds.	1400	1225	1470	1260	1060	1275	710	

*Readings of efficiency taken from diagram by Bertram P. Flint, *Engineering News*, April 9, 1892, p. 348.

† The efficiencies shown in Table XIX were obtained with a pair of Hindley worms in an elevator drive arranged in general as shown in Fig. 83.1.   Two winding drums, one on each worm-wheel shaft, were used.   The experiments were conducted by driving the worms with an electric motor whose armature was direct-connected to the worm-shaft, and hoisting a weight correspond-

† *American Machinist*, Jan. 21, 1897, p. 45.

TABLE XIX.

EFFICIENCY OF HINDLEY WORM AND WORM-WHEEL DRIVEN ELEVATOR.*

$$\text{Efficiency} = \frac{\text{Load lifted} \times \text{distance traversed.}}{\text{Electrical energy received by motor.}}$$

Two Hindley worms, right- and left-hand, on same shaft. No thrust-bearing or step. Pitch diam. of worm, 5.47 inches ; pitch diam. of gear, 26.89 inches ; double-thread worm ; lead or pitch of each thread, 2.86 inches nearly; angular pitch at pitch line 9° 39′; velocity ratio 29½ to 1; speed of worm, 500 rev. per min.; velocity of rubbing at pitch line, 718 ft. per min.; speed of travel of worm-wheel at pitch line, 112 ft. per min.

Pressure in Pounds on		Efficiency, per cent.	Pressure in Pounds on		Efficiency, per cent.
Both Worms.	One Worm.		Both Worms.	One Worm.	
1000	500	44	3000	1500	68
1500	750	54	4000	2000	72
2000	1000	60	6000	3000	76
2500	1250	65	8500	....	78

* Readings taken from diagram in *AmericanMachinist* of Jan. 21, 1897, p. 45.

ing to an elevator-cage, which was lifted by means of a rope running from the winding-drums on the worm-wheel shafts. Measurements of the electrical energy received by the motor, and of the work done upon the weight, were taken. The ratio of the latter to the former corresponds to the efficiency given in the table. This, of course, does not represent the efficiency of the worm-gearing alone, as has been given in the preceding tables, since the motor losses, as well as those of the machinery between the worm-wheels and weight lifted, are included. The experimenters estimate that the efficiency of the worm-gearing alone "can scarcely be less than 90%." Attention is also called to the fact that the percentage motor loss increases as the load becomes lighter; hence the efficiency-curve drops more rapidly than the real efficiency of worm-gearing.

A parallel series of tests on a pair of worms of the ordinary form, and working under the same conditions as the Hindley, gave an efficiency 2% lower at 2000 pounds pressure on both worms, which gradually decreased to 10% lower at 6000 pounds.

The requirement of the Hindley worm for accurate adjustment should be kept in mind when applying it where the service is heavy.

78. Screw-gears seem to be rapidly increasing in favor for use where the service is light. Numerous examples are found in the feed mechanism of both light and heavy machine tools. In a great many cases they are now used where heretofore bevel-gears were almost universally applied. The objection against them, that the teeth cannot be accurately cut in a milling-machine, but must be finished by hand, is not sufficiently strong to prohibit their use.

79. Strength of screw-gear teeth.—When the velocity ratio is great, and the teeth of the gear corresponding to a worm-wheel run nearly or quite straight across its face, they may be dealt with for strength as those of a spur-gear having pressure applied at one point. While the point of application may be at the top of the tooth, it should never come at the end of its length across the gear face; therefore it is doubtless safe to consider the effective width of the gear face for resisting fracture at least as great as 1.5 times the circular pitch. When the teeth run at a considerable angle across the face of the gear, they are somewhat stronger than those of a spur-gear of the same diameter and thickness. When the angle between the axes of the gears is not so greatly different from 90° as to prevent cutting the teeth of only such a length, in the middle of the gear face, as is necessary to allow the engagement of other gears, the teeth are much stronger on account of the supporting material at their ends; such a gear corresponds in a way to a worm-wheel or one that is shrouded.

80. Equations for turning force and efficiency of screw-gears.—When the axes of the gears are at right angles, the equations for worm-gearing apply equally well to screw-gears. If the angle between the axes does not differ greatly from 90°, the same equations will give results more accurate than our knowledge of the co-efficients of friction entering into them.

The slight practical use to which equations for smaller angles between the gear-axes can be put does not seem to warrant their presentation.

It is worth noting, however, that, as the angle between the axes decreases, the amount of sliding between the teeth of the gears also decreases, and in consequence of this, together with the

reduction of thrust on the spiral pinion, the efficiency increases uniformly, reaching that of spur-gears when the axes become parallel.

In the tests already referred to as being made by Mr. Lewis, he found the screw-gear efficiencies given in Table XX. The larger

TABLE XX.

EFFICIENCY OF CAST-IRON SCREW-GEARS.*

Efficiency = ratio of power delivered by spur-gear shaft to that applied to spiral-pinion shaft ; or, ratio of output to input.

All spiral pinions 4" pitch diameter ; all spur-wheels 18.62" pitch diameter, 89 teeth, 1½ inch pitch. Spiral pinion ran in oil-bath.

Cycloidal teeth, accurately cut.

Revolutions of Screw-pinion per minute.	Efficiency, per cent.			
	1 thread. 1.511" pitch. 6° 51" angular pitch.	2 threads. 3.086" pitch. 13° 49' angular pitch.	4 threads. 6.828" pitch. 28° 31' angular pitch.	6 threads. 12.894" pitch. 45° 44' angular pitch.
3	....	60	70	81
5	....	63	73	88
7	45	66	75	84.5
10	46	68.5	78	86
15	48	72	80	87.5
20	49	74	82	89
30	51	77.5	84.5	91
40	53	80	86	92
60	65.5 and 55.5	83	89	94
80	68 and 57.5	85	90	94.8
100	70 and 59	86	91	95.8
120	71.5 and 60.5	87	92	96
150	73.5 and 62	88.5	92.3	96.5
200	75.5	89	92.3	96.6
275	77.5	....	92	96.4

* Readings of efficiency taken from diagrams by Wilfred Lewis in Trans. Amer. Soc. Mech. Eng., vol. vii., p. 273.

wheel in each test was a spur-gear with accurately cut cycloidal teeth; the screw-pinion was also accurately cut to fit the gear. The angle between the gear-axes was 90° − θ, the pinion-shaft being set at the pitch angle θ with a normal to the axis of the spur-gear, in order to obtain accurate intermeshing of the gears. The two efficiencies of the single-thread pinion, at speeds from 60 to 150 revolutions, are due to different conditions of the rubbing sur-

faces; the higher efficiencies correspond to an improvement of
these surfaces, secured by running the mechanism for some time
under a light load.

81. Coefficient of friction of screw-gears.—By determining the
journal friction as accurately as possible, and allowing for it, Mr.
Lewis found, by calculation, the coefficient of friction μ for the
two-, four-, and six-thread pinions given in Table XX. These
coefficients are given in Table XXI, together with their average

TABLE XXI.

COEFFICIENT OF FRICTION OF SCREW-GEARS.

For Three of the Gears given in Table XX.

Revolutions of Screw-pinion per minute.	Coefficient of Friction, μ.			
	2 threads 13° 49', angular pitch.	4 threads 28° 31', angular pitch.	6 threads 45° 44', angular pitch.	Average Values.
3	.086	.105	.094	.095
5	.078	.097	.089	.088
10	.064	.081	.076	.074
20	.050	.065	.061	.059
50	.035	.043	.038	.038
100	.025	.030	.024	.026
200	.018	.026	.015	.020

values for various speeds.   In view of the low values of μ for
square-thread screws, given in Table XIII, when lubricated with
graphite and oil, it would seem that such a lubricant might be
excellent for worm- and screw-gears.   In fact it is recommended
by a leading concern building large worm-driven metal-working
planers.

Valuable data could doubtless be obtained by a series of experi-
ments on worm- and screw-gears of different materials and with
different lubricants.

CHAPTER VI.

SCREW-FASTENINGS.

82. In machine construction it is frequently desirable to fasten parts together in such a manner that they can readily be separated and put together again without injuring the fastening or destroying its usefulness. In order to accomplish this, fastenings having screw-threads cut upon them are used.
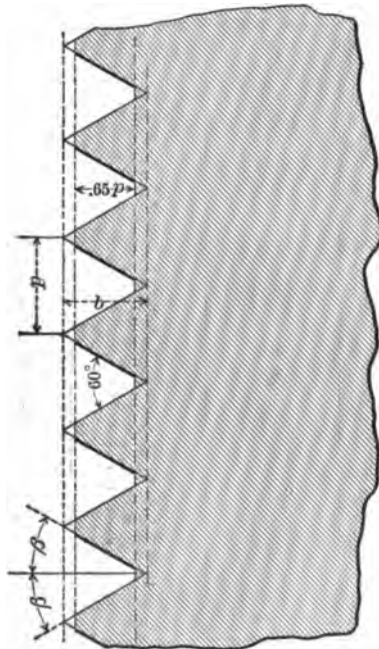
SELLERS UNITED STATES
STANDARD THREAD
FIG. 84.

The Sellers United States Standard thread, shown in sectional outline in Fig. 84, has the sides inclined at an angle $= 60° = 2\beta$.

The proportions are obtained by extending the lines of the sides until they intersect, then cutting off one eighth of the distance b from the top, and filling in to the same extent at the bottom, of the angles thus formed; this makes the height of the thread $= h = 0.65$ of the pitch.

Table XXII gives the proportions of U. S. Standard threads.

TABLE XXII.

U. S. OR SELLERS SYSTEM OF SCREW-THREADS.

Diameter of Bolt, Inches.	Threads per Inch.	Diameter of Root of Thread, Inches.	Width of Flat, Inches.	Area of Bolt-body in Square Inches.	Area at Root of Thread in Square Inches.
1/4	20	.185	.0062	.049	.027
5/16	18	.240	.0074	.077	.045
3/8	16	.294	.0078	.110	.068
7/16	14	.344	.0089	.150	.093
1/2	13	.400	.0096	.196	.126
9/16	12	.454	.0104	.249	.162
5/8	11	.507	.0113	.307	.202
3/4	10	.620	.0125	.442	.302
7/8	9	.731	.0138	.601	.420
1	8	.837	.0156	.785	.550
1⅛	7	.940	.0178	.994	.694
1¼	7	1.065	.0178	1.227	.893
1⅜	6	1.160	.0208	1.485	1.057
1½	6	1.284	.0208	1.767	1.295
1⅝	5½	1.389	.0227	2.074	1.515
1¾	5	1.491	.0250	2.405	1.746
1⅞	5	1.616	.0250	2.761	2.051
2	4½	1.712	.0277	3.142	2.302
2⅛	4½	1.962	.0277	3.976	3.023
2¼	4	2.176	.0312	4.909	3.719
2½	4	2.426	.0312	5.940	4.620
3	3½	2.629	.0357	7.069	5.428
3¼	3½	2.879	.0357	8.297	6.510
3½	3¼	3.100	.0384	9.621	7.548
3¾	3	3.317	.0413	11.045	8.641
4	3	3.567	.0413	12.566	9.963
4¼	2⅞	3.798	.0435	14.186	11.329
4½	2¾	4.028	.0454	15.904	12.753
4¾	2⅝	4.256	.0476	17.721	14.226
5	2½	4.480	.0500	19.635	15.763
5¼	2½	4.730	.0500	21.648	17.572
5½	2⅜	4.953	.0526	23.758	19.267
5¾	2⅜	5.203	.0526	25.967	21.262
6	2¼	5.428	.0555	28.274	23.098

The proportions of heads and nuts, as made by the leading manufacturers, do not generally conform with the U. S. Standard, and not always with each other. It is advisable to consult the catalogue of the manufacturer whose bolts are to be used.

Table XXIII gives the proportions of bolt-heads and nuts adopted by some of the leading makers.

TABLE XXIII.*

PROPORTIONS OF BOLT-HEADS ADOPTED BY DIFFERENT
MANUFACTURERS.

The dimensions are the same whether finished or rough.

Diam. of Screw.	Dimensions of Square and Hexagon Bolt-heads.†							
	Hoopes & Townsend.		Rhode Island Tool Co.		Wm. H. Haskell & Co.		J. H. Sternbergh & Son.	
	Width.	Thick-ness.	Width.	Thick-ness.	Width.	Thick-ness.	Width.	Thick-ness.
1/4	7/16	3/16	3/8	3/16	7/16	3/16	7/16	3/16
5/16	1/2	1/4	15/32	1/4	1/2	1/4	17/32	1/4
3/8	19/32	9/32	9/16	5/16	5/8	9/32	5/8	5/16
7/16	11/16	3/8	21/32	3/8	23/32	11/32	23/32	3/8
1/2	3/4	7/16	3/4	7/16	13/16	3/8	13/16	13/32
9/16	27/32	1/2	27/32	1/2	15/16	7/16	29/32	15/32
5/8	15/16	17/32	15/16	17/32	1	1/2	1	1/2
3/4	1⅛	5/8	1⅛	5/8	1 1/16	5/8	1 1/16	9/16
7/8	1 1/16	3/4	1 1/16	3/4	1⅛	3/4	1⅛	11/16
1	1¼	7/8	1¼	7/8	1 3/16	13/16	1 3/16	3/4
1⅛	1⅜	1	....	....	....	....	1⅜	7/8
1¼	1½	1⅛	....	....	....	....	1 11/16	1

* *Machinery*, April 1897, page 242.

† The width of the nut is the same as that of the head; its thickness is commonly equal to the diameter of the thread.

Fig. 85.1 shows the thread section recommended by an international committee for use on the European continent. It differs from the U. S. standard thread only in having clearance between the top and bottom of the threads. The clearance space is rounded at the bottom and has a depth equal to 1/16 of what would be the

total height of the thread if it ran to sharp angles. A bolt made with this thread will fit correctly into a nut of the U. S. standard, or a nut of the international standard thread will fit over a U. S. standard bolt thread, provided of course that each has the same number of threads per inch.

WHITWORTH ENGLISH
STANDARD THREAD
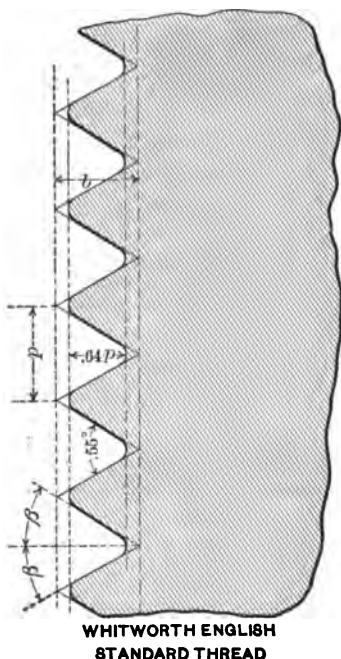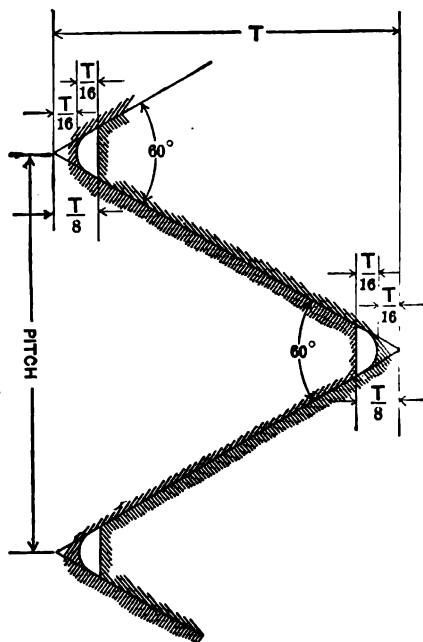
FIG. 85.   FIG. 85.1.

The Whitworth English Standard thread, Fig. 85, has an angle of 55° between the sides, and the top and bottom are rounded for a distance equal to one-sixth of the depth of the corresponding V thread; this leaves the height of the thread $= h = 0.64$ of the pitch.

By cutting off the sharp points of the older forms of V threads,

so as to form the standard threads just mentioned, they become less subject to bruising by accidental blows, and the taps, dies, and turning tools used to form them are more durable than those with sharp threads.

The square thread, Fig. 86, is commonly used when there is considerable wear due to relative motion of the threaded parts engaging together; it has the advantage of presenting
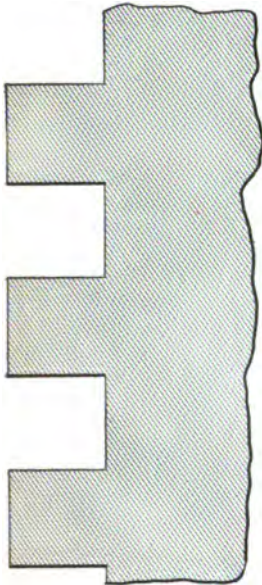
SQUARE THREAD

Fig. 86.
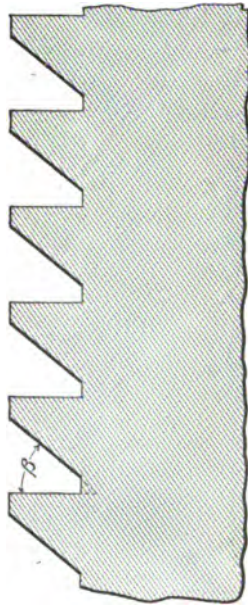
BUTTRESS THREAD

Fig. 87.

a surface almost at right angles to the line of pressure, which is ordinarily parallel to the axis of the thread. For fastenings the pitch is usually twice as great as for the U. S. standard thread.

The buttress thread, Fig. 87, is sometimes useful when the pressure against the thread is all, or nearly all, in one direction.

The surface taking the thrust is made perpendicular to the axis of the thread, the other having any convenient angle with it.   By this means a strong thread is obtained.

The forms of screw-fastenings are almost infinite in number; a few of the more common ones, together with their ordinary functions, are given below, chiefly in order that they may be referred to with a clear understanding of what each is.

A through bolt consists of a bar of metal, forming the body of the bolt, having a thread and nut at one end, and a head, forming an integral part with the body, at the other.   Ordinarily it is used to clamp together parts of machinery by passing through them so that the inner surfaces of the head and nut press against the pieces held together by the bolt, as shown in Fig. 88.   A washer is frequently placed between the nut and the clamped piece, to prevent marring the latter, and to give a better bearing surface for the nut.

A cap-screw, Fig. 89, is of the same form as a bolt with the nut removed.   It is commonly used to fasten parts together by screwing into a threaded hole in one of them.   The depth of the threaded · hole depends largely upon the material; in all cases where the parts are drawn tightly together, its depth should be as great as the diameter of the screw.   When the material tapped into is much weaker than that of the screw, as when a machine-steel cap-screw is screwed into cast iron, the threaded depth of the hole should be made considerably greater; twice the diameter of the screw, or more, is advisable under such conditions.

A stud, Fig. 90, consists of a bar threaded at both ends.   One end is screwed into a tapped hole in one of the pieces to be held together, and the other receives a nut which presses against the clamped piece.   A stud performs the same service as a cap-screw. It is useful where the parts are to be separated frequently.   If a cap-screw is drawn down hard and removed several times from cast iron, there is danger that the threads of the hole will break and crumble away, unless the material is close-grained and of good quality.   By using a stud this is obviated, for it can be screwed tightly into the hole and left there.   The nut and stud can both be

made of a strong, durable material, and in case of injury to the threads, can be replaced readily and at small expense. A stud does
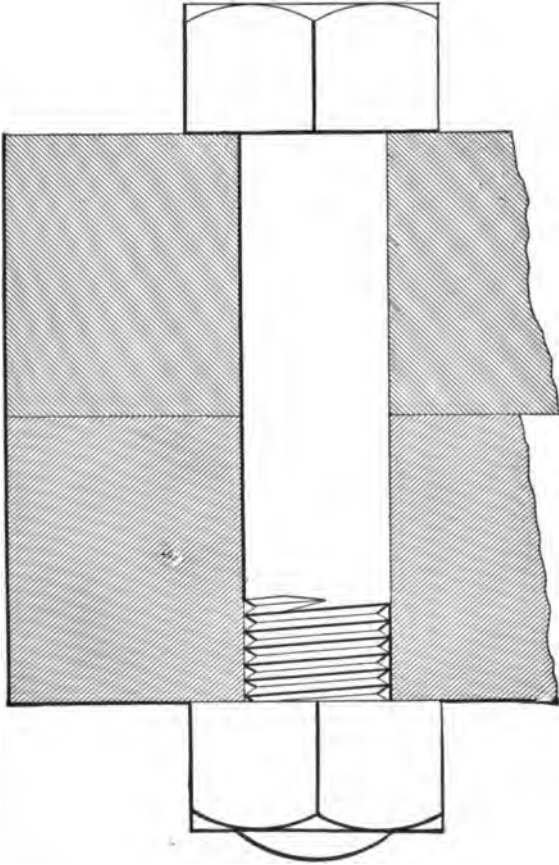
FIG. 88.

not need as long a thread in the tapped hole as a cap-screw, for the reason that it does not turn when under stress.

Fig. 91 shows a stud that can be used, when only a shallow hole can be made in a weak material, by screwing the larger end into the threaded hole and putting the nut on the smaller. Fig. 92 shows another form that can be used in the same way, or it can be used

when the stud is screwed into a thin piece, through which the threaded hole passes. For such a purpose the small end is screwed in until the shoulder prevents it from entering farther; the nut is, of course, placed on the large end in such a case.
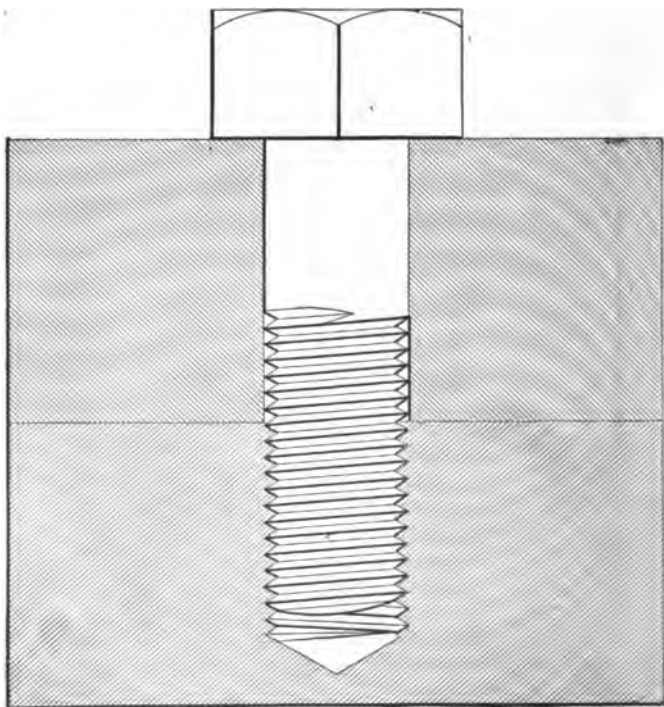
FIG. 89.

A set-screw is used to prevent sliding or rotation between parts that fit together; an ordinary application is that of fastening pulleys in place upon shafting. A common form of set-screw is that of Fig. 93; the point shown is round; other points, largely used, are the cup, cone, and pivot, shown in Figs. 94, 95, 96, and 97.

The method of using a set-screw is shown in Fig. 98, which represents a collar or pulley-hub held in place on a shaft by the set-screw. When the piece through which the set-screw passes must be held very firmly in position, so as to resist forces of con-

siderable magnitude, tending to displace it, a recess, generally
conical, is drilled in the shaft to receive the end of the screw; for
light work, the screw is tightened against the smooth shaft so that
the point is set into it far enough to hold.
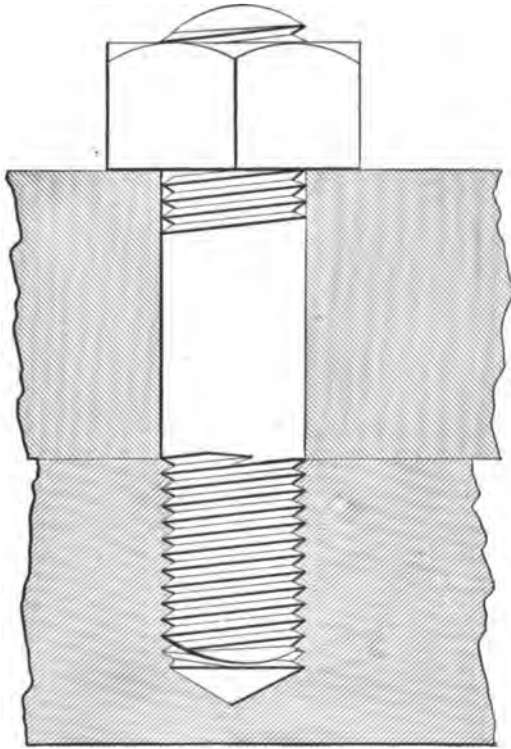
Fig. 90.

The conical and cup points hold more firmly than the round
point when set lightly against the smooth shaft, but at the same
time they mar it to a greater extent, thus making it more difficult
to separate the parts.

Prof. Gaetano Lanza gives the results of a series of tests on the
holding power of set-screws made on screws having different kinds
of points.* The experiments were made on a pulley held in place

* Trans. Amer. Soc. Mech. Eng., vol. x., p. 280.

on a 1¼-inch shaft by two ⅝-inch set-screws having ten threads to the inch. The screws were tightened with a force of 75 pounds applied to the end of a 10-inch wrench. All screws were of wrought

FIG. 91.   FIG. 92.   FIG. 93.
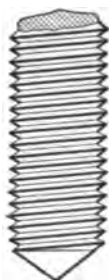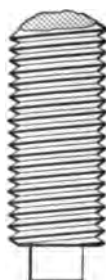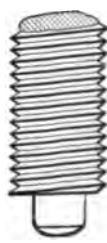
FIG. 94.   FIG. 95.   FIG. 96.   FIG. 97.

iron, and only one was case-hardened at the point. The shaft was of rather hard steel, and the set-screws made but little impression upon it. Six tests were made on each of the points A, B, and D; four on C. A summary of the results is given in Table XXIV.

TABLE XXIV.

HOLDING POWER OF SET-SCREWS.

	A Flat ends ⅝″ diam.	B End rounded to ⅛″ radius.	C End rounded to ¼″ radius.	D End cupped and case- hardened.
Lowest holding power, pounds..	1412	2747	1902	1962
Highest " " "	2294	8079	8079	2958
Mean " " "	2064	2912	2578	2470
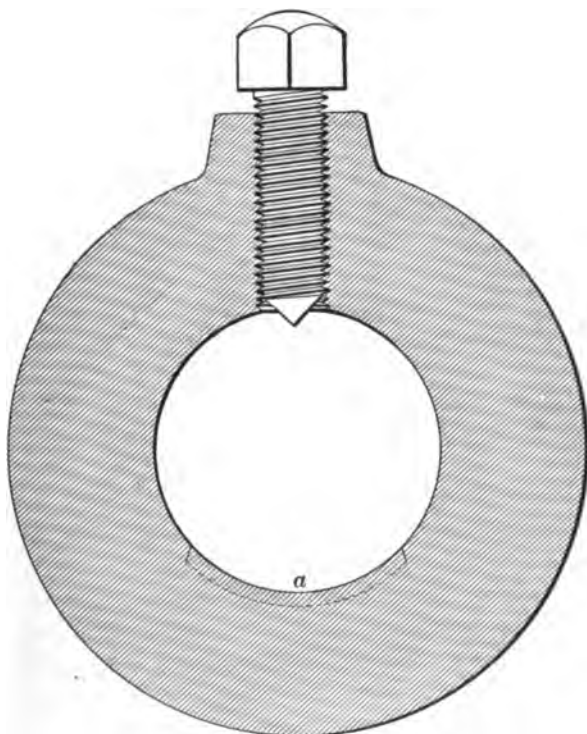
FIG. 98.

The following remarks are made by Professor Lanza regarding the screws:

"A. The set-screws were not entirely normal to the shaft;

hence they bore less in the earlier trials before they had become flattened by wear.

"B. The ends of these set-screws, after the first two trials, were found to be flattened, the flattened area having a diameter of about ¼ of an inch.

"C. The ends were found, after the first two trials, to be flattened as in B.

"D. The first test held well because the edges were sharp, then the holding power fell off till they had become flattened in a manner similar to B, when the holding power increased again."

A pulley that does not fit its shaft is difficult to hold in place by a single set-screw, or even by two or more placed on the same side of the shaft. In either case, the screw-points, pressing against the shaft on one side, cause contact between the hub and shaft on the opposite side, as at a, Fig. 98, while the remaining parts of the bore and shaft do not touch. If a torsional moment acts, tending to rotate the pulley clockwise around the shaft, the set-screw, holding firmly to the shaft, will cause the hub to slide toward the left over the shaft at a; a reversal of the turning force will allow the surfaces at a to slide back over each other. Numerous repetitions of this action will cause the point of the set-screw to enlarge the indentation in the shaft, and, finally, the point may wear away, or the screw work backward in the hub, so that the parts can rotate relatively to each other. Such a repeated application and removal of the turning force or resistance is of common occurrence in practice.

There are two methods of obtaining a more reliable set-screw fastening than the above, when there is not a close fit between the hub and shaft. One is to place two set-screws at an angle of about 90° with each other, in the same cross-sectional plane of the shaft and hub, both screws being radial; the other is to cut away a portion of the hub, as shown by the dotted line below a, Fig. 98, still retaining the single set-screw, or more, as the case may be, on one side of the shaft. Both these methods give three-point contact in a plane normal to the axis of the shaft, and passing through the axis of the screw. This prevents slipping between the hub and shaft, thus giving a firmer fastening.

In all the above fastenings the form of the head or nut, whether

designed for a wrench or a screw-driver, does not change the name
of the fastening. Very small screws, with heads slotted for a
screw-driver, are, however, commonly called machine screws.

83. Locking devices for nuts and screws.—Since the constant
jarring to which many kinds of machinery are subjected, frequently
causes nuts and screws to work loose, and even to fall from their
places, the necessity of securing them by some safety locking
appliance has brought forward numerous devices for this purpose.
A few will be described.

One of the most common is the lock- or jam-nut shown in Fig.
99. The ordinary proportions are given. They are locked together
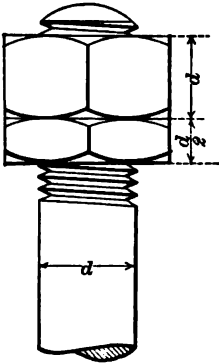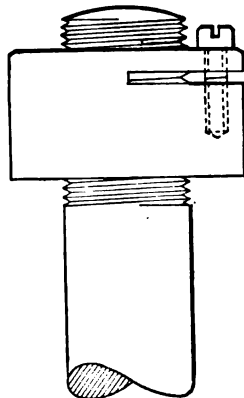
FIG. 99.   FIG. 99.1.

by screwing them tightly against each other. When they are also
tightened against the piece held in place, a greater stress is brought
upon the nut nearest the end of the bolt, which therefore should
be thicker than is necessary for the one next the clamped piece.
In practice the thin nut is frequently placed on top, but this is
wrong. It is probably due to the fact that ordinary wrenches are
not thin enough to turn the thin nut, when it is under, without
catching against the top one.

Fig. 99.1, in which the small screw draws the split part together,
is a useful device.

Coiled-spring nut-locks, Figs. 100 and 101, are used by placing
one just under the nut in the place ordinarily occupied by a washer;
those in the figures are intended for bolts with right-hand threads.
When the nut is tightened by turning it clockwise, its bearing sur-

face slips freely over the nut-lock, and presses it down against the clamped piece; as soon as the nut is turned backward, however, the sharp corners of the lock cut into the surfaces of both the nut and the piece under it, thus preventing the loosening of the former. It can easily be seen that such a nut-lock can be used only where the nut is not to be removed, or where the cutting of the material by the lock is not objectionable.

FIG. 100.
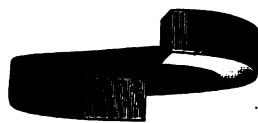
FIG. 101.

Another device, in which the thread is made so as to lock the nut, is shown in Fig. 102. The thread of the bolt is undercut so

FIG. 102.

FIG. 103.

that its working side makes about five degrees less than a right angle with the axis of the bolt, and the apex of the thread is cut to a knife-edge. The nut has a thread cut so that its working side makes about five degrees more than a right angle with the axis of

the nut. This leaves a cavity of about ten degrees between the bolt- and nut-threads, so that when the nut is tightened the thin edges of the bolt-thread are forced out into the nut, thus locking them together. It is claimed that this device is serviceable even where the nut is to be removed as often as a dozen times during its use.

Fig. 103 illustrates a bolt with a cold-rolled or " undercut " thread as made by J. H. Sternbergh & Son, of Reading, Penn.*

84. Strength of screw-bolts.—In many cases a bolt must have sufficient tensile strength to resist a known or estimated stress, as, for example, when used to attach cylinder-heads to engines, pumps, etc.; to fasten together flanged pipes carrying some substance under pressure; or to support a suspended load.

If, as is ordinarily the case, the nut is to be screwed up tightly enough to prevent even the slightest separation of the clamped parts when under stress, the initial tension in the bolt, due to the tightening of the nut, must be at least as great as the service tension, in order to prevent lengthening of the bolt under the service tension, and the consequent separation of the parts clamped together by it. This assumes that the parts are rigid and large in comparison with the bolt.

For example, suppose that, in Fig. 104, the part B is to be clamped against A by the bolt, so that they will not separate when a given load T is suspended from B, the line of action of T being coincident with the axis of the bolt. The initial tension in the bolt must therefore be at least as great as T. In order to allow for a factor of safety, assume the working strength of the material of the bolt as 6000 pounds per square inch. The load T may be taken as 5000 pounds. The required area of the bolt to resist tension only, in a section across the roots of the thread, is therefore

$$\frac{5000}{6000} = 0.833 \text{ sq. in.}$$

The sectional area at the roots of a U. S. Standard thread, on a bolt having a body $1\frac{1}{4}$ inches in diameter, is 0.89 of a square inch,

---

* Other designs of nut-locks are illustrated in Rose's " Modern Machine-shop Practice," vol. I., pp. 118–121; Unwin's " Elements of Machine Design," Part I., pp. 159–163; Reuleaux's " Constructor," p. 56; Klein's " Elements of Machine Design," p. 9 and Plate II.

the root diameter being 1.065 inches. Since this is the nearest standard size having an area as large as that required, it would be used in practice.

The turning moment necessary to give the tension of 5000
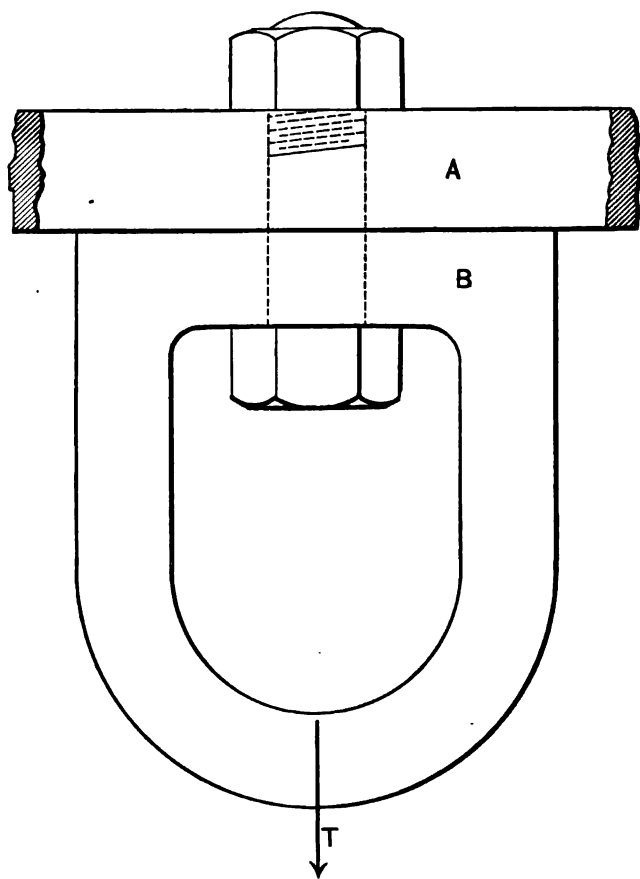
FIG. 104.

pounds can be found by equation (71)* or those following it, in which, for this case: $p = \frac{1}{4}$ inch; $d = \dfrac{1.25 + 1.065}{2} = 1.16$ inches; $D = 1.7$ inches, about; $\beta = 30°$; and, since the coefficient of friction

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* Page 164, notation p. 150.

is uncertain and should be taken large enough to cover poor lubrication and rough bearing-surfaces, it is admissible to take $\mu = 0.15$, and $\mu' = 0.12$.

Substituting in equation (72),

$$Fl = 5000\frac{\frac{1}{7} + .15 \times \pi \times 1.16 \times 1.15}{\pi\,1.16 - .15 \times \frac{1}{7} \times .866}\frac{1.16}{2} + 5000 \times .12 \times \frac{1.7}{2}$$

$$= 650 + 510 = 1160 \text{ inch-pounds.}$$

Assuming $l = 16d = 16 \times 1.25 = 20$ inches, this being about the value used in practice for a solid wrench, gives

$$F = \frac{1160}{20} = 58 \text{ pounds,}$$

which is 1 pound of torsional force for every $5000 \div 58 = 86$ pounds tension in the bolt.

The value of Fl can also be obtained by assuming an efficiency E for the bolt, and substituting in equation (59), which can be applied to angular- as well as square-thread screws. Taking $E = 0.1$, and substituting (see § 79),

$$0.1 = \frac{5000 \times 1/7}{F(2\pi l)}; \quad \text{or} \quad Fl = \frac{10 \times 5000}{2\pi \times 7} = 1140 \text{ inch-pounds.}$$

The maximum tensile and shearing stresses can be found by equations (67) and (68); the values of t and s to be used are $t = T \div A = T \div \pi r_2^2$, and $s = Fl \div \frac{1}{2}\pi r_2^2$. These maximum stresses do not need consideration, however, unless the nut is tightened while under the stress of the load.

If the material through which the bolt passes in Fig. 104 were totally without elasticity, and the bolt tightened to produce a tension T in it, then, for any suspended load not exceeding T, the tension in the bolt will always be equal to T; and when the load equals T there would be no pressure between the surfaces clamped together, for the bolt would elongate enough to relieve the pressure.

If a spring were interposed between the surfaces before tightening the bolt, and put in compression, without clamping it solid, by screwing down the nut till a tension T is produced, then a suspended load would make the tension in the bolt equal to T plus

the load. This assumes that the elongation in the bolt is so slight as to be inappreciable compared with the capacity of the spring for elongation.

Since all material is elastic, there is always something of the spring action on the bolt. This, of course, is exceedingly slight when heavy parts of rigid material are bolted together; when short bolts are used to clamp together pieces separated by a springy substance, such as rubber packing, the spring action may be great enough to require consideration.

Experiments show that grooves used for standard bolt-threads very materially increase the tensile strength per unit area, although the total strength may be made less than that of the original bar. This is due to the fact that fracture is made to occur at the groove, which, on account of having a large amount of resisting material on each side, requires a greater force to fracture it than a bar of the same diameter as a section at the bottom of the groove.

The effect of cutting a V thread upon a bolt is nearly the same as for a single circumferential groove of the same form as the thread-groove. It has been found by experimental investigation that, on account of this strengthening effect of the thread, with fairly good rubbing surfaces reasonably well lubricated, the axial tensile stress per square inch which will cause rupture in a U. S. Standard screw-bolt, while tightening the nut, is practically the same as the breaking strength per square inch of the body of the bolt.* This means that the twisting effect of the nut may be neglected.

The sectional area of U. S. Standard screw-bolts at the bottom of the thread is roughly 0.7 of the area of the bar on which the thread is cut, for diameters up to 2 inches. It therefore seems safe to say that the axial tensile stress which will rupture such a screw-bolt, when tightening the nut, is 0.7 as great as will fracture the body of the bolt, it being assumed that the body and the top of the thread are of the same diameter.

Major Wm. R. King found that by doubling the number of threads per inch on a screw-bolt, the total tensile strength was increased about 20%, and the resilience or shock-resisting power to a much greater extent. The gain was somewhat greater when the

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* Zeitschrift des Vereines deutscher Ingenieure, April 27, 1895, p. 505.

number of threads per inch was tripled.* The average results of several tests were as follows:

Threads per inch	6	12	18
Relative tensile strength	1.	1.21	1.23
Elongation	.025	.06	.08
Relative work or resilience	.025	.0726	.0984

The stress was applied at the nut and head. Stripping of the thread did not occur in any of the experiments, but the reduction of the diameter of the screw by elongation was so great as to let a portion of the threads of the nut and bolt slip past each other.

It has been shown by numerous tests upon screw-bolts having the thread made by the cold-rolling process, which forces the metal up so that the top of the thread is somewhat larger in diameter than the body of the bolt, that, when subjected to tensile stress applied to the working faces of the head and nut, the bolt invariably fractures in the body.† These tests show very clearly the strengthening effects of the thread, for the sectional diameter across the bottom of the thread is, of course, considerably less than that of the body of the bolt.

85. Endurance of screw-bolts.—Repeated stresses in a screw-bolt, such as occur in the fastenings of a trip-hammer, steam or pneumatic rock-drill, connecting-rod ends of pumps, engines, etc., frequently cause bolts to fracture across the threads between the nut and body. This is caused by the slight temporary elongation of the bolt every time the shock or stress occurs. Such an elongation may occur without allowing the surface of the parts held together to separate even to the slightest extent; for the elasticity of any two parts clamped together with a bolt will, when an additional stress is applied to separate them, especially if it is a shock, cause them to increase their dimensions in the direction of the length of the bolt, and thus elongate it.

By reducing the sectional area of the body of the bolt so that it is not greater than that across the bottom of the thread, the bolt is made more durable. This is due to the fact that, by reducing the

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* Trans. Amer. Inst. Mining. Engrs., 1885, p. 90.
† Catalogue of J. H. Sternbergh & Co.

sectional area of the body, the bolt elongates more readily, the
slight elongation that occurs with each shock distributing itself
nearly uniformly throughout the length of the bolt, thus reducing
the actual stress which occurs in it with each shock.   When the
body of the bolt is of the same diameter as the top of the thread,
the slight elongation which occurs with each repetition of stress is
largely localized at the bottom of the thread between the nut and
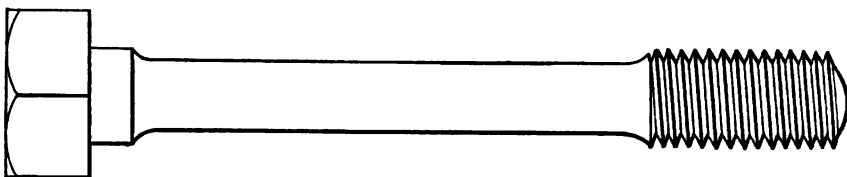
.FIG. 105.
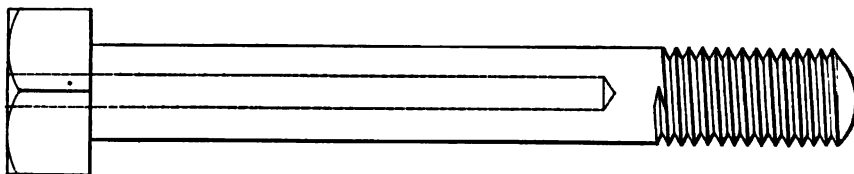
FIG. 106.

body of the bolt, and causes the metal to gradually give way by
fatigue.
    The reduction of the sectional area of the body of a bolt may be
accomplished either by reducing its diameter, as in Fig. 105, or by
drilling a hole in the centre, as in Fig. 106.   The hollow bolt is the
stronger torsionally, and will therefore withstand a greater tensile
stress while the nut is being screwed on.   The increased endurance
of bolts so made has been demonstrated in practice.

CHAPTER VII.

MACHINE KEYS, PINS, FORCED AND SHRINKAGE FITS.

MACHINE KEYS AND PINS.

86. The principal function of machine keys is to prevent rotary motion of one part about another, as of a pulley about a shaft on which it fits; less frequently they are used to prevent lateral motion also, when the tendency to such motion is comparatively small. They are in a general way divided into three classes, commonly called flat, square, and feather or sliding keys.

There are no accepted standards of keys and key-way proportions. Tables XXV, XXVI, and XXVII, by John Richards,[*] give the average proportions of general practice, however.

The **flat key**, Fig. 107, is most suitable for heavy machinery, such as is used for mill-work, when both the relative rotation of the parts and the lateral motion incidental to the vibration of the machinery, and, in some cases, the weight of the parts, are to be resisted. A flat key should completely fill the key-way; the pressure against the sides should be greater than at the top and bottom, where a light pressure is all that is necessary. A heavy pressure against the top
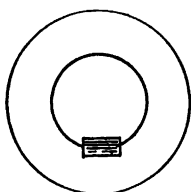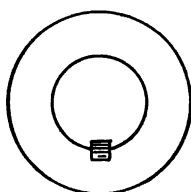
FIG. 107.   FIG. 108.

and bottom of the grooves has a tendency to spring the parts out of true and fracture the hub. The top of a key is the side remaining

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[*] Richards' "Manual of Machine Construction." *Cassier's Magazine*, April, 1893, p. 416.

completely exposed after the key has been put in position in one of the parts to be keyed.

"Square" keys, Fig. 108, which are generally only approximately square in section, are generally used for the lighter classes of work, and especially for machine-tool construction. They should fit tightly on the sides, only merely touching at the top and bottom so as to prevent their tipping over under a heavy load, or not touching at all at the top.

TABLE XXV.*

DIMENSIONS OF FLAT MACHINE KEYS, INCHES.

Diam. of shaft.	1	1¼	1½	1¾	2	2½	3	3½	4	5	6	7	8
Breadth of keys	¼	5/16	⅜	7/16	½	⅝	¾	7/8	1	1¼	1⅜	1¾	1¼
Depth of keys..	1/8	5/16	¼	9/32	5/16	⅜	7/16	½	⅝	11/16	13/16	⅞	1

TABLE XXVI.*

DIMENSIONS OF APPROXIMATELY SQUARE MACHINE KEYS, INCHES.

Diameter of shaft............ .	1	1¼	1½	1¾	2	2½	3	3½	4
Breadth of keys................	5/16	3/8	7/16	11/16	11/16	11/16	¾	13/16	11/16
Depth of keys.................	7/16	¼	5/16	⅜	7/16	½	⅝	⅝	¾

TABLE XXVII.*

DIMENSIONS OF FEATHER OR SLIDING MACHINE KEYS, INCHES.

Diameter of shaft...........	1¼	1½	1¾	2	2½	2½	3	3½	4	4½
Breadth of keys........ ..	¼	¼	5/16	5/16	⅜	⅜	½	9/16	9/16	⅝
Depth of keys.............	⅜	⅜	7/16	7/16	½	½	⅝	¾	¾	⅝

* Taken from *Cassier's Magazine*, April, 1898, p. 416.

If only a small portion of the power that the shaft is able to transmit is taken off through the key, the latter may be made smaller than given in the tables. The side pressure on the key is practically inversely as the diameter of the shaft.

A feather or sliding key is adapted to a service requiring lateral motion of the parts over each other, but not relative rotation. On account of the wear on the sides, due to the lateral motion, and in

order to give the key a firm hold upon the part to which it is
attached, its radial height is greater than for "square" keys.
Such a key is fastened to one of the parts, either tightly or loosely,
as is most suitable for other requirements, and moves laterally with
regard to the other. Probably the simplest method of fastening
such a key to a shaft is to dovetail it slightly on the sides and ends
near the bottom, then force it into place and rivet the edge of the
key-way down against it with a key-set. Another method of
attaching a short feather to a shaft is shown in Fig. 109. The key
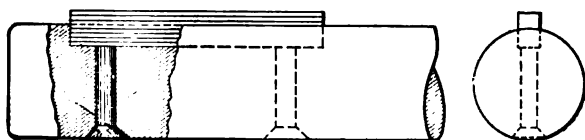
FIG. 109.

is made with a couple of round lugs or pins which pass through
holes drilled in the shaft and are riveted on the side opposite the
key. The same device is applied to a feather attached to a hub in
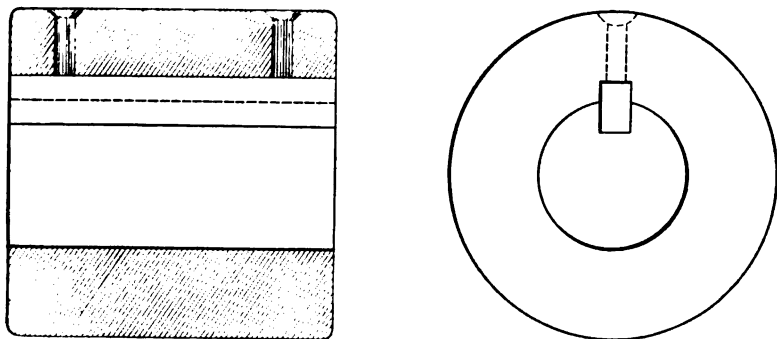
FIG. 110.

Fig. 110. Another method is shown in Fig. 111; the feather may
be made a loose fit in this form, and can be removed by slipping the
hub off the end of the shaft.

A method of using a square key which is not very generally
applied in practice, but which appears to possess an advantage in
the ease with which it can be fitted, is shown in Fig. 112; the key

is square, and is placed so that one diagonal is radial.   A heavy load would exert a strong bursting pressure on the hub.   For this reason the key is hardly suitable for heavy service.
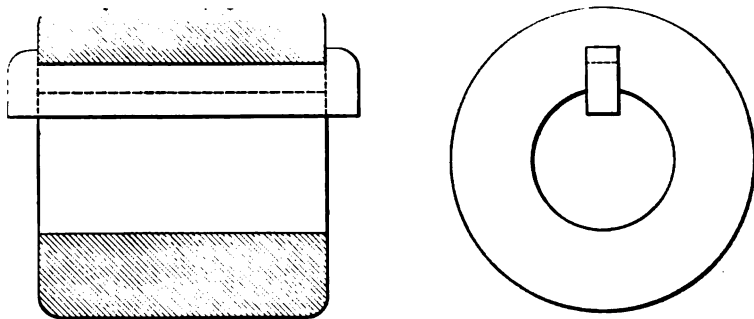
FIG. 111.
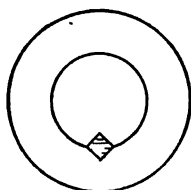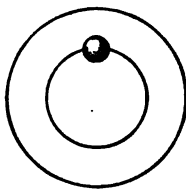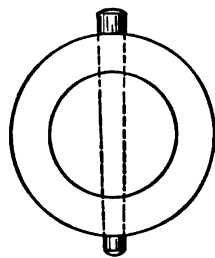
FIG. 112.   FIG. 113.   FIG. 114.

A cylindrical pin can be used, as in Fig. 113, when the shaft and hub are of the same material, or near enough alike to make it practicable to drill a hole for the pin as shown.   Such a key or pin is hardly suitable for heavy machinery.

A taper pin, Fig. 114, can be used to advantage when there is considerable end pressure to be resisted, as well as turning force.   In practice the smallest size of such a pin is ordinarily determined by the smallest taper reamer that is long enough to ream out the hole. Morse standard taper pins are generally used.   The torsional moment, which a pin that is not large enough to weaken the shaft unduly will resist, is much less than for a key of the proportions given in the tables, if the length of the key is as great as the diameter of the shaft.

87. Roller keys are used to some extent for fastening small pulleys, etc., to shafting. The hub of the pulley is bored a short distance at each end to fit the shaft; the centre is bored eccentric with the ends, and larger in diameter by something more than the diameter of the roller key to be used. A hardened cylindrical key of about the same length as the larger part of the bore is placed in it, and the shaft slipped through the hub. A slight rotation of the latter causes the key to bind between it and the shaft so as to prevent further rotation in that direction; a slight rotation in the opposite direction loosens the connection. It is self-evident that this device can be used only when the tendency to turn the parts over each other is always in the same direction.

88. Eccentric keys or **fastenings** of the form shown in Fig. 115,
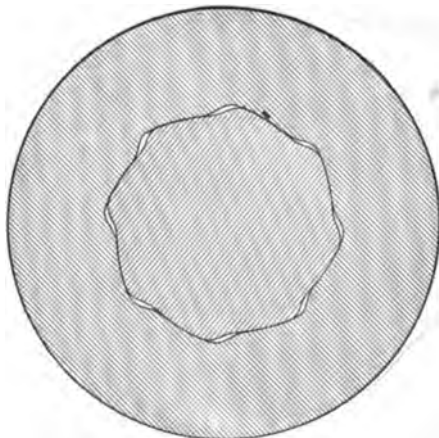
Fig. 115.

known as Blanton Patent Fastenings, are used to good advantage in certain classes of machinery. The surface of the shaft is turned into a series of corrugations as shown, and the hub is bored to the same form, being made large enough to slip along the shaft freely when the parts are put in the loose position. A slight angular rotation of one causes it to lock with the other and drive it. By turning them in the opposite direction relatively to each other they are loosened.

This fastening has been found especially applicable to the lifting-cams of ore stamp-mills, largely on account of the ease with which the cams can be removed and new ones substituted. When

there is danger that the fastening may become loose on account of the momentum of the parts, slight variations of speed, reversals of motion, etc., a key is applied, as in Fig. 116, to prevent the
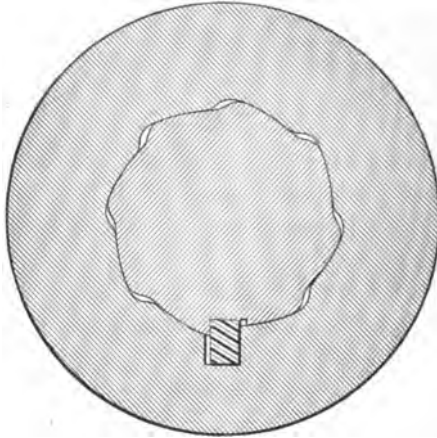
FIG. 116.
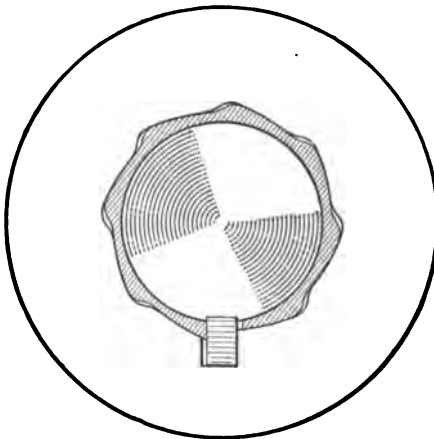
FIG. 117.

loosening.   The fastening, although thus modified, is still adapted to driving in only one direction.

The Blanton fastening is adapted to round shafts as shown in Fig. 117.   The turning moment is largely resisted by the friction

between the shaft and corrugated sleeve, thus relieving the rectangular key of much of the pressure that would come upon it if preventing rotation between two round parts.*

SHRINKAGE AND FORCED FITS.

89. Shrinkage and forced fits are adopted frequently when it is desired to have parts fit together very tightly; the parts are almost invariably either cylindrical or slightly conical on the surfaces in contact.   For a shrinkage fit, the outer member, of two parts that are to be fastened together, is finished to a diameter slightly smaller than the inner.   It is then heated so as to expand it enough to pass over the inner member, put into place and cooled.   The contraction of cooling causes it to grip the inner part firmly.

For a forced fit the parts are prepared in the same manner as for shrinkage fits, but are forced together cold instead of the outer one being expanded by heat and then shrunk into place.

90. Tension in and pressure against a thin ring fitted by shrinking or forcing.—The tension in a ring that is thin radially in comparison with its diameter, as well as the pressure between the ring and body it encircles, can be calculated if the modulus of elasticity E of the material is known.   This assumes that the ring is either forced into place, or is heated to a uniform temperature throughout and all parts cooled at the same rate.   While such heating and cooling can probably never be attained in practice, they can be nearly enough approximated to warrant the above assumption. If cooled unevenly, the greatest tension will be at the place cooled last.

The following notation will be used for shrinkage and forced fits:

A = sectional area of ring, square inches;
D = diameter of ring before heating, inches;
D_1 = diameter of ring after putting into place, inches;
E = tensile modulus of elasticity of material of ring, pounds per square inch;

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* The Blanton Patent Fastening is used in the United States by Fraser & Chalmers of Chicago, who control its use on stamp-mills. (Statement from pamphlet issued by the Blanton Patent Syndicate, Ltd., London.)

$F =$ force required to slip the ring from place, pounds;

$P =$ total pressure between surfaces of ring and inner part, pounds;

$T =$ total tension in ring, pounds;

$h =$ radial thickness of ring, inches;

$p =$ unit pressure between surfaces of ring and inner member, pounds per square inch;

$r =$ radius of ring, inches;

$t =$ unit tension in ring, pounds per square inch;

$w =$ width of bearing surface of ring, measured parallel to axis of bore, inches;

$\mu =$ coefficient of friction between surfaces bearing together.

Remembering that $E =$ (stress in pounds per square inch) $\div$ (elongation per inch of length), the two following equations may be written:

$$t = E\frac{D_1 - D}{D}; \quad \ldots \ldots \quad (89)$$

$$T = At = AE\frac{D_1 - D}{D}. \quad \ldots \ldots \quad (90)$$

The equation for the unit pressure p between the surfaces may be obtained by assuming that the ring is cut in two on a diametral
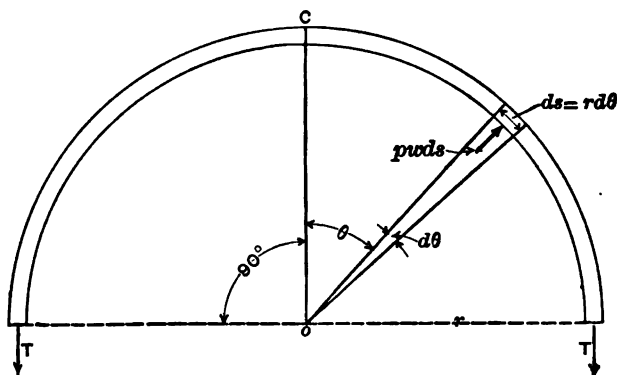
FIG. 118.

plane, and one half considered as a free body with the forces necessary to hold it in equilibrium acting upon it, as shown in Fig. 118.

These forces consist, first, of the two tensions T and T which the other half of the ring exerted when in place, and, second, of the elementary forces normal to the inner surface of the ring and uniformly distributed over it. T and T are both normal to the plane of the section of the ring. The forces acting normal to the inner cylindrical surface of the ring may be taken as having a unit value of p pounds per square inch.

Take an elementary portion ds of the circumference of the ring. The corresponding inner surface of the ring is wds. The total pressure acting on this elementary area is $pwds$. This pressure may be resolved into two components, parallel and perpendicular to T and T. The one which is normal to T and T is annulled by the equal and opposite component of the pressure acting on an equal area symmetrically situated with regard to the median line OC. This component may therefore be neglected. The other component, parallel to T and T, and acting in the opposite direction from them, must be resisted by a portion of the tensile forces T and T acting at the ends of the half-ring. The value of this component is $pw(ds) \cos \theta$, in which θ is the angle between the median line OC and a radial line drawn to the elementary area $w(ds)$. The sum of the components parallel to T and T for all the elementary areas of the half-ring must equal $2T$, since T and T are the forces which hold these components in equilibrium. Therefore

$$\int pw(ds) \cos \theta = 2T$$

Since r is the radius of the ring, and $d\theta$ is the angle between the two radial lines drawn to the ends of the elementary length ds of the circumference, $ds = rd\theta$. The last equation may therefore be written

$$pwr \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos \theta \, d\theta = 2T,$$

which gives

$$2pwr = 2T.$$

Transposing and substituting the value of T, as given in equation (90), gives

$$p = \frac{T}{wr} = \frac{At}{wr} = AE\frac{D_1 - D}{Dwr}. \quad . \quad . \quad . \quad (91)$$

The total pressure P between the surfaces bearing together is

$$P = pw\pi D = \frac{T\pi D}{r} = 2T\pi = 2At\pi. \quad . \quad (92)$$

The force F necessary to slip the ring laterally along the surface against which it presses is

$$F = \mu P = 2\mu T\pi = 2\mu At\pi. \quad . \quad . \quad . \quad (93)$$

Example.—If a steel ring 4 inches wide, 0.7 of an inch thick radially, and 60 inches in diameter before putting in place, is shrunk or forced on another part which increases its diameter .06 of an inch, the tension per square inch is, by equation (89), putting $E = 30,000,000$,

$$t = 30,000,000\frac{.06}{60} = 30000 \text{ lbs. per sq. in.}$$

The total tension is, by equation (90),

$$T = At = (4 \times 0.7)\,30000 = 84000 \text{ lbs.}$$

The pressure per square inch between the cylindrical surfaces is, by equation (91),

$$p = \frac{T}{wr} = \frac{84000}{4 \times 30} = 700 \text{ lbs. per sq. in.}$$

The total pressure between the bearing surfaces, determined by equation (92), is

$$P = 2T\pi = 2 \times 84000\pi = 527780 \text{ pounds.}$$

The force required to slip the ring laterally, taking $\mu = .15$, is, by equation (93),

$$F = .15 \times 527780 = 79170 \text{ pounds}$$
$$= 19.6 \text{ tons (about).}$$

91. Shrinkage and forced fits for thick rings and heavy parts. —When the ring or outer member is very thick or heavy, the

material in it is not subjected to even an approximately uniform stress when two parts are shrunk or forced together. The layers near the smaller diameter of the outer member are put into greater tension than those more remote, on account of the elasticity of the material and the supporting action of each successive annular layer against the one inside of it. On account of this uneven distribution of stress, it is not probable that any mathematical formula can be deduced that is of sufficient practical value for very thick pieces to warrant its adoption. This applies to such a member as a crank shrunk or forced on its shaft, or having a pin forced or shrunk into it.

Again, when a heavy part is shrunk or forced over another, the great pressure exerted on the inner piece decreases the diameter of the latter on account of its elasticity. This has been perceptibly shown when the steel tires are shrunk on locomotive drive-wheel centres. The increase of diameter of the tire, due to putting it in place, is less than the difference between the bore of the tire and the outer diameter of the wheel centre before putting the tire on. The extent of the reduction of the inner part depends upon the form and strength of the parts. It is therefore a quantity that cannot well be introduced into a formula, except empirically.

92. Allowance for shrinkage and forced fits.—For locomotive drive-wheels and steel-tired car-wheels, the bore of the tire is quite commonly made .001 of the diameter smaller than the wheel centre. The Midvale Steel Co. makes a greater difference of diameter than this, their rule being to bore the tire $\frac{1}{1000}$ of the nominal diameter smaller than the wheel centre.

The collection of data, Tables XXVIII, XXIX, XXX, and XXXI,* shows the practice of several machine-builders in allowing for shrinkage and forced fits. The force necessary to press the parts together is given in nearly all cases.

It is the practice of some leading concerns to slightly taper the parts that are forced together. This obviates the necessity of slowly forcing the parts together throughout the entire length of the bore, and reduces the liability to abrasion and cutting between the surfaces. The parts are readily separated, since they become loose

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* Taken from *Machinery*, May, 1897.

when given a small relative lateral movement. The difficulty of securing tapers accurately the same on both parts is, of course, detrimental to their general application.

Table XXXII indicates the practice of the Chicago, Milwaukee, and St. Paul Railway in the Milwaukee shops, for both taper and cylindrical fits.

TABLE XXVIII.*

EXTRACT FROM LANE & BODLEY CO.'S RECORD BOOK OF FORCED FITS.

No.	Mean Diameter of Pin, inches.	Length of Fit, inches.	Mean Diameter of Hole, inches.	Total Allowance.	Allowance per Inch.	Area of Fitted Surface, square inches.	Volume under Fitted Surface, cubic inches.	Pressure to enter pin, tons.	Pressure at Mid-position, tons.	Maximum Pressure, tons.
1	1.8798	6.125	1.8767	.0031	.0017	36	16.7	2	10	20
2	1.8819	6.125	1.877	.0042	.0022	36	16.7	2	15	23
8	1.8774	4.875	1.8764	.001	.00052	24.4	13.7	.5	1	1
4	2.7455	4.5	2.7387	.0068	.00247	38.7	26.5	8	12	25
5	2.7465	4.5	2.7437	.0028	.001	38.7	26.5	5	12	23
6	3.261	5	3.2542	.0068	.0021	51	41.5	5	20	45
7	3.2625	5	3.2555	.007	.002	51	41.5	5	15	30
8	3.267	5	3.261	.006	.0018	51	41.5	5	15	20
9	4.2505	6.	4.2402	.0103	.0024	79.8	85.1	5	23	44
10	4.2388	6.625	4.2478	.0091	.0021	78.1	93.4	12	80	60
†11	4.2308	6.5	4.2224	.0079	.0019	95.8	91.	10	60	125
12	5.9348	4.0625	5.9216	.0127	.0022	75.7	112.2	6	16	25
13	5.9381	4.	5.9252	.0129	.0022	74.4	110.4	8	18	35
14	5.9294	4.125	5.9194	.01	.0017	76.7	113.8	5	15	25
15	6.8829	5.125	6.8697	.0132	.002	110.7	190.1	8	20	42
16	6.889	5.	6.8785	.0105	.0015	108.	18 .9	5	22	45
17	6.8692	4.875	6.855	.0142	.0021	104.8	180.4	5	85	65
18	7.8884	5.5	7.873	.0154	.002	135.9	267.8	5	32	64
19	7.8715	6.5	7.8575	.014	.0018	160.5	315.9	5	25	50
20	7.862	5.625	7.846	.016	.002	138.2	272.8	8	40	80
21	8.924	6.125	8.905	.019	.0021	170.8	378.9	20	45	68
22	8.9	6.75	8.8848	.0152	.0017	188.4	419.9	5	47	96
23	8.878	6.5	8.8669	.0112	.0013	180.7	401.	10	45	92

* Taken from *Machinery*, vol. III., No. 9, May, 1897.   † No. 11 was a cast-steel crank-disk.

TABLE XXIX.*

DATA FURNISHED BY C. O. HEGGEM, SUPT. RUSSELL & CO.,
MASSILLON, OHIO.

Allowance is for both press and shrinkage fits for steel cranks.

FOR PRESS FITS—CAST-IRON CRANKS.

Diameter of hole	4 to 5 inches.	Allow .0090 inch
" " "	5 " 7½ "	" .0060 "
" " "	7½ " 9 "	" .0055 "
" " "	10 " 12 "	" .0050 "
" " "	12 " 16 "	" .0040 "
" " "	16 " 18 "	" .0080 "

FOR SHRINK FITS—CAST-IRON CRANKS.

Diameter of hole	4 to 5 inches.	Allow .0045 inch
" " "	5 " 7½ "	" .0030 "
" " "	7½ " 9 "	" .0027 "
" " "	10 " 12 "	" .0025 "
" " "	12 " 16 "	" .0020 "
" " "	16 " 18 "	" .0015 "

TABLE XXX.*

FROM DATA FURNISHED BY H. BOLLINCKX, BRUSSELS, BELGIUM.

CRANK-PINS.

Diameter, inches.	Length, inches.	Total Allowance, inches.	Allowance per inch, inches.	Total Pressure, pounds.
2.2	2.76	.0039	.0018	43800
2.79	3.78	.0049	.0018	49500
3.145	4.32	.0058	.0018	55500

LEVER-PINS.

.865	1.57	.0023	.0027	6800
1.1	1.77	.0025	.0023	6800
1.18	1.965	.0027	.0023	6800
1.375	2.36	.0031	.0023	9060

CYLINDER-CASES.

11.4		.0098	.00086	37100
17.7		.0098	.00055	37100

PISTONS.

1.49	4.32	.0059	.004	37100
1.46	4.32	.0059	.004	49500
1.77	4.76	.0070	.004	55500
2.24	11.18	.0039	.0017	61800
2.68	11.18	.0039	.0015	74000
2.95	13.12	.0049	.0017	113200

* Taken from *Machinery*, vol. III., No. 9, May, 1897.

TABLE XXXI.*

BRIEF SUMMARY OF GENERAL PRACTICE.

	Diameter, inches.	Total Allowance, inches	Allowance per inch of Diam., in.	Pressure to enter-pin, tons per inch of diameter.	Remarks.
E. P. Allis Co., Milwaukee, Wis......	8	.008	.001	10 to 16	Allowance is more on smaller sizes, less [on larger.
Baldwin Locomotive Works, Phila., Pa.	5 to 9	.015 to .027	.008	10 to 15	Axles.
H. Bollinckx, Brussels, Belgium......	3 to 6	.0045 to .009	.0015	5 to 10	Wrist-pins.
	2.2	.0039	.0018	9	Crank-pins.
	1.18	.0027	.0023	2 to 8	Lever-pins.
	11.4	.0098	.0006	1 to 2	Cylinder-cases.
	1.49	.0059	.004	12	Piston-rods into piston.
Buckeye Engine Co., Salem, O......	8 to 10	.01 to .015	.008 to .0015	9 to 10	
Buffalo Forge Co., Buffalo, N. Y......	4 to 10	.01 to .01125	.0025 to .008		
	15 to 40	.047 to .0625	.008 to .0015		
Dickson Mfg. Co., Scranton, Pa......	4.5	.01	.0022	8	Diameter over 5 inches, 7 tons.
Juniata Shops, P. R. R., Altoona, Pa......	5.875	.017	.0081	10	With 80 pins, pressure varied from 35 to 80 tons; average, 56.
	6	.023	.0088	10	With 20 pins, pressure varied from 55 to 90 tons; average, 71.5.
	4	.013	.0089	10	With 60 pins, pressure varied from 30 to 60 tons; average, 45.
Lane & Bodley Co., Cincinnati, O......	2.74	.0028 o .0068	.001 to .0026	8.35 to 9.1	
	3.86	.006 to .0068	.0018 to .0021	6.15 to 13.8	
	4.25	.0103	.0024	10.3	See Table XXVIII for more details.
	5.98	.0127	.0022	4.21	
	6.88	.0132 to .0142	.002 to .0021	6.12 to 9.5	
	7.88	.0154 to .016	.002 to .0018	8.27 to 10.02	Least allowance takes greater pressure.
	8.90	.019 to .0112	.0021 to .0018	10.85 to 7.65	
Phila. Engineering Wks., Phila., Pa......	8 to 6	.002	.0007 to .00085	40 tons total.	
	6 to 9	.004	.0007 to .0005	75 to 80 total.	Same allowance and pressure for sizes on each line.
	9 to 12	.005	.0007 to .0005	100 total.	
	12 to 18	.009	.0007 to .00045	150 total.	
Rogers Locomotive Co., Paterson, N. J.	4 to 9	.015 to .022	.004 to .0024	12	See Table XXIX.
Russell & Co., Massillon, O	4 to 7.5	.009 to .005	.0023 to .0012	4	
	10 to 18	.005 to .008	.0005 to .00016	4	
Schenectady Loco. Wks., Schenec., N.Y.	5 to 8	.007 to .0075	.002 to .0019	10 to 11	Axles.
	3 to 6	.008 to .009	.0016 to .0015	5 to 8	Crank-pins.
Straight-Line Engine Co., Syracuse, N.Y.	3.25 to 8.9875	.007 to .0075	.002 to .0019		
	4.9875 to 5.9875	.008 to .009	.0016 to .0015		Both crank-pins.
	3. to 3.4375	.008 to .007	.002 to .0022		Both shafts.
Union Iron Works, San Francisco, Cal..	4.875 to 4.75	.0075 to .008	.0017 to .0017		Endeavor to obtain 200 to 500 lbs. pressure per square inch of contact surface.
	Generally	about	.001		

TABLE XXXII.

FORCED FITS.   PRACTICE OF CHICAGO, MILWAUKEE, AND
ST. PAUL RAILWAY.

	Diameter at large end, inches.	Approximate length, inches.	Taper = variation of diameter per foot of length.	Pressure to force parts together, tons.
Parallel-rod crank-pin *............	4¾	6¼	1/32	20
Main crank-pins*...	4	6	1/32	25
	4½	6	1/32	30
	5½	6	1/32	35
Engine driving-axles	7¾	7½	none	60 to 80;† 80 preferred
	7⅞	8⅛		
Engine truck-axle...	5¾	7	none	80 to 40; 25 will hold.
Car-axles.....	5⅝	7 to 9½	1/16	40 ‡

* End riveted over after forcing into place.
† If only 60 tons is used, more care is taken in fitting key.
‡ Only slight variations from this amount allowed.

In order to bring the parts into alignment for forcing them together, two diameters are sometimes used, one but very slightly smaller than the other. Each size extends half the length of the surfaces that bear together.

CHAPTER VIII.

SHAFTING, AND POSITIVE SHAFT-COUPLINGS.

93. Notation.—The following notation is used in the formulas for the strength and deflection of shafting:

$D =$ diameter of round shaft, or width of side of square shaft, inches;

$E_s =$ shearing modulus of elasticity of material of shaft, used for torsion, pounds per square inch;

$E_t =$ tensile modulus of elasticity of material of shaft, pounds per square inch;

$I =$ moment of inertia of cross-section of shaft about a gravity axis in the plane of the section, biquadratic inches;

$J =$ polar moment of inertia of cross-section of shaft, biquadratic inches;

$L =$ length of shaft, inches;

$M_b =$ bending moment, inch-pounds;

$M_t =$ twisting moment, inch-pounds;

$(M_b)_i =$ ideal bending moment which would induce the same fibre-stress as the combined bending and twisting moments ;

$(M_t)_i =$ ideal twisting moment which would induce the same fibre-stress as the combined bending and twisting moments;

$P =$ turning force, pounds;

$S_b =$ section modulus of shaft for bending $= I \div c$;

$S_t =$ section modulus of shaft for torsion $= J \div c$;

$W =$ bending force, pounds;

$c =$ distance from centre of shaft to most remote part of section, inches;

$d =$ diameter of bore in hollow shaft, inches;

$f =$ fibre-stress in material of shaft due to bending, pounds per square inch;

$l =$ length of lever-arm of turning force, inches;

$n =$ the ratio $d \div D$;

$s =$ shearing-stress in shaft due to turning force, pounds per square inch;

$w =$ weight of shaft per inch of length, pounds;

$\theta =$ angular deflection or twist of shaft in degrees $= 57.29578 \times$ (angular deflection in radians).

ROUND SHAFTING.

94. Torsional strength of round shafts.—When a shaft is used simply for transmitting power from one point to another, there being no intermediate devices, such as gears or pulleys and belts, for receiving or delivering power, the torsional strength and angular deflection of the shaft are all that ordinarily need consideration. This assumes that the bearings of a horizontal shaft are near enough together to prevent excessive sagging or bending between them, and that a vertical shaft is supported by thrust-bearings so as to prevent excessive local end thrust or buckling. The angular deflection or stiffness needs especial attention when steady running under a variable load is important.

The relation between the turning moment M_t and shearing-stress s in a shaft is expressed by the equation

$$M_t = Pl = s\frac{J}{c}. \quad \ldots \quad \ldots \quad (94)$$

For a solid round shaft $J = \dfrac{\pi D^4}{32}$ and $c = \dfrac{D}{2}.$

Substituting these values in the last equation, it becomes: For solid round shafts

$$M_t = Pl = .1963sD^3, \quad \ldots \quad \ldots \quad (95)$$

whence

$$D = \sqrt[3]{\frac{5.1M_t}{s}} = \sqrt[3]{\frac{5.1Pl}{s}} = 1.721\sqrt[3]{\frac{Pl}{s}}. \quad \ldots \quad (96)$$

For a hollow round shaft $J = \dfrac{\pi(D^4 - d^4)}{32}$, and $c = \dfrac{D}{2}$ as before. Equation (94) becomes, by substituting these values in it and putting $d = nD$, for hollow round shafts

$$M_t = Pl = .1963s\frac{D^4 - d^4}{D} = .1963sD^3(1 - n^4). \quad . \quad (97)$$

Transposing puts the equation into a convenient form to solve for D when the ratio n of the bore to the diameter is assumed. The equation thus becomes: For hollow round shafts

$$D = \sqrt[3]{\frac{5.1M_t}{(1 - n^4)s}} = \sqrt[3]{\frac{5.1Pl}{(1 - n^4)s}}. \quad . \quad . \quad (98)$$

The slight weakening effect of removing the centre of a shaft is worthy of note. A shaft having an axial hole of a diameter $d = .3D$ through its centre has $1 - (.3)^4 = .9919$ of the torsional strength of a solid shaft of the same material; if $d = .4D$, the strengths are as .9744 to 1; for $d = .5D$, the ratio is .9375 to 1; and for $d = .6D$ the ratio is .87 to 1.

The torsional strength of a hollow shaft having a bore half as large in diameter as the shaft is 1.443 times that of a solid shaft of an equal sectional area (i.e., the same weight per linear foot) and of the same material.

The above statements regarding the relative strength of solid and hollow shafting are based upon the assumption that the material in each is of the same strength. In shafts of the same sectional area, however, the material of the hollow-forged shaft can be made stronger than that of the solid one, because it can be worked more thoroughly under the hammer or forging-press. The ordinary method of making a hollow forging is to cast the ingot solid and bore out the centre. This takes out the poorest material, which is always at the centre. Practically the same result is obtained when the piece is forged solid and the centre bored out afterwards. For forgings of very large diameter, the method of boring before forging is undoubtedly preferable.

In modern practice a working shearing-strength as high as

12000 pounds per square inch is used for hollow-forged steel shafts for the screw-propellers of large vessels, etc.

Example.—The turning force F, applied at a distance $l = 40$ inches from the centre of a shaft 15 inches in diameter when working at a maximum shearing fibre-stress $s = 9000$ pounds per square inch, may be found by equation (95). By substituting the given values in this equation,

$$P \times 40 = .1963 \times 9000 \times (15)^3,$$

whence

$$P = 149000 \text{ pounds.}$$

Sectional area of shaft $= 176.71$ square inches.

Example.—The diameter of a hollow shaft having a hole half as large as the shaft, to resist a turning force $P = 149000$ pounds, acting on a lever-arm of a length $l = 40$ inches, and working at a maximum shearing fibre-stress of 9000 pounds per square inch, can be found by substituting in equation (98), thus obtaining

$$D = \sqrt[3]{\frac{5.1 \times 149000 \times 40}{[1 - (.5)^4] 9000}} = 15.33 \text{ inches.}$$

Sectional area of shaft $= 138.4$ square inches.

The area of a solid shaft for the same strength, as shown in the preceding example, is 27 per cent greater than this.

95. Twist of a shaft under torsional stress.—The relation between the angle of twist and the turning moment acting on a solid round shaft is expressed by the following equation, which gives θ in degrees:

$$\theta = 583.6 \frac{M_t L}{E_s D^4} = 583.6 \frac{P l L}{E_s D^4} \quad . \quad . \quad . \quad . \quad (99)$$

For hollow shafts the same relation is expressed by the following equation, giving θ in degrees:

$$\theta = 583.6 \frac{M_t L}{E_s (D^4 - d^4)} = 583.6 \frac{P l L}{E_s (D^4 - d^4)} \quad . \quad . \quad (100)$$

The following equation of the relation between the shearing-stress per unit area and the twist is applicable to both solid and hollow round shafts:

$$\theta = 114.6 \frac{sL}{E_s D} \text{ degrees.} \quad \ldots \ldots \quad (101)$$

Example.—The angular twist of a solid machine-steel shaft 15 inches in diameter and 100 feet = 1200 inches long, when subjected to a turning force $F = 149000$ pounds, applied at a distance $l = 40$ inches from the centre, assuming that the modulus of elasticity for shearing $E_s = 12000000$,* is by equation (99)

$$\theta = 583.6 \frac{149000 \times 40 \times 1200}{12000000 \times (15)^4} = 6.87°.$$

The data in this example, with the exception of the length of the shaft, are the same as used in the examples of the preceding section. The fibre-stress used in these examples is 9000 pounds per square inch. Substituting this value of s in equation (101), and solving, gives

$$\theta = 114.6 \frac{9000 \times 1200}{12000000 \times 15} = 6.87°,$$

which is, of course, the same as obtained by equation (99).

The shaft, if working at a shearing-stress of 12000 pounds per square inch, would have a twist of 9.17° per 100 feet of length ; the value of the turning force for this stress would be $P = 198700$ pounds.

96. Bending-strength of round shafting.—A round shaft that is subjected to bending only, as when a horizontal rotating shaft supports a load that does not exert a twisting moment upon it, can be calculated for strength as if it were a beam, the working fibre-stress f being selected with due regard to the fact that the stress is repeated at every revolution. Each fibre is alternately put into tension and compression during every revolution of the shaft. The safe working stress cannot be taken so high, on this account, as is

* $E_s = 10000000$ is more nearly correct for the materials generally used for line shafting in factories, etc.

permissible for a stationary beam supporting a static load. In the majority of cases the fibre-stress, as determined by the required rigidity of the shaft, is lower than the safe working strength for repeated stress.

The values of the maximum bending moment M_b acting on a round shaft when resisting a bending force W, applied according to the more common methods of loading, and for the weight of the shaft itself, are as follows:

Load applied to projecting end at a distance L from supporting bearing (cantilever),

$$M_b = WL. \quad \ldots \ldots \ldots \quad (102)$$

Load applied midway between two end supports,

$$M_b = \frac{WL}{4}. \quad \ldots \ldots \ldots \quad (103)$$

Load applied between two supports at distances a and b respectively from them $(a + b = L)$,

$$M_b = \frac{Wab}{L}. \quad \ldots \ldots \ldots \quad (104)$$

Two equal loads applied, each at the same distance a from the end nearest it,

$$M_b = Wa. \quad \ldots \ldots \ldots \quad (105)$$

Bending moment due to weight of horizontal projecting shaft (cantilever), length of projection $= L$,

$$M_b = \frac{wL^2}{2}. \quad \ldots \ldots \ldots \quad (106)$$

Bending moment due to weight of shaft, end supports,

$$M_b = \frac{wL^2}{8}. \quad \ldots \ldots \ldots \quad (107)$$

The relation between the bending moment M_b, tensile or compressive fibre-stress f, and the diameter of the shaft is given by the following four equations:

For solid round shafts,

$$M_b = .0982fD^3; \dots \dots \quad (108)$$

$$D = 2.168 \sqrt[3]{M_b \div f}. \dots \dots \quad (109)$$

For hollow round shafts,

$$M_b = .0982f(D^3 - d^3); \quad \dots \quad (110)$$

$$D = 2.168 \sqrt[3]{\frac{M_b}{f(1 - n^4)}}. \quad \dots \quad (111)$$

The actual point of support of a shaft is frequently difficult to decide upon. If the bearings are self-aligning, the point of support may be taken in the plane, perpendicular to the shaft, containing the point about which the bearing-sleeve swivels; this is generally at the middle of the sleeve. When a bearing is rigidly supported, however, the point of pressure against the shaft may be near one end of the box at the time the shaft is subjected to the greatest bending moment. This applies to a shaft working under a varying bending force, as the crank-shaft of a steam-engine.

A long, tight-fitting hub, as of a fly-wheel supported on a horizontal shaft, prevents the shaft from bending where it is encircled by the hub. If the shaft is of the same cross-section throughout, the maximum fibre-stress in it will be at the end of the hub. While this shortens the lever-arm of the bending force and reduces the maximum fibre-stress to a lower value than if the load were (theoretically) applied at the middle of the hub, it also localizes the strain at the ends of the hub, and thus increases the liability to rupture, on account of fatigue of the metal, by the repetition of stress due to the rotation of the parts. Probably the best way of reducing the liability to fracture at the end of the hub is to enlarge the shaft where the hub is placed upon it, joining the enlargement to the smaller part by a fillet of as large a radius as can be conveniently used.

It is assumed in the above discussion that the centre of the hub contains the centre of gravity of the fly-wheel.

In practice it has been found that shafts and axles which are

subjected to repeated stress, and especially to shocks and vibrations, are more durable when proportioned so that the stress and strain are distributed over them with at least an approximate degree of uniformity, and all re-entrant angles well filleted, than when made of uniform section throughout, or with sharp re-entrant angles.

It is common practice in car-axles to make them small in the middle in order that they may spring more readily, and thus reduce the strain upon the parts near the wheels and journals. This is an application of the same principle that makes a hammer-handle more durable when reduced in section between the head and the part that is held in the hand.

97. The lateral deflection of shafting on account of its own weight seldom needs consideration, except, possibly, for the smaller sizes. The remedy for the excessive deflections is to decrease the distance between bearings, or to increase the diameter of the shaft. The same is generally true of deflections caused by a load or belt pull.

The amount of the deflection can be calculated by the formulas ordinarily given for beams, but such a calculation is seldom needed.

It should be remembered, however, that a small shaft having supports so far apart as to allow it to bend considerably may, when rotated at a high speed, run out of true to a dangerous extent on account of the centrifugal action. This is due, not to the deflection of the shaft, which may be considered only as a measure of the liability to excessive bending, but to the fact that such a shaft when rotating will not keep its axis in exactly the same position. As soon as the shaft moves laterally to the least extent, the centrifugal action increases its deflection.

98. Shaft subjected to both torsion and bending, general case. —While this is the most common case that is found in practice with shafting, the experiments that have been made with a view to establishing the formulas that have been developed to meet it are very few and meagre. Of all the formulas that have been presented by different writers the following two, (112) and (113), seem to be the most convenient and correct.

The method adopted in these formulas is to find a bending or torsional moment equivalent to the actual moments acting simul-

taneously, and design the shaft accordingly. The shaft must, of course, be designed to resist the moment which has the greatest tendency to break it.

The value of the ideal or equivalent bending moment $(M_b)_i$ which will produce the same tensile or compressive fibre-stress in the material as the combined bending and twisting moments, M_b and M_t, is expressed by the equation

$$(M_b)_i = \tfrac{3}{8}M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_t^2}. \quad \cdots \quad (112)$$

And the value of the ideal torsional moment which will cause the same shearing-stress in the material as the combined bending and torsional moments is shown in the formula

$$(M_t)_i = \tfrac{3}{8}M_b + \sqrt{M_b^2 + M_t^2}. \quad \cdots \quad (113)$$

Solid Round Shafts.

99. Solid round shafts subjected to more than one force.—The value of the resisting moment required to withstand the ideal bending moment $(M_b)_i$ of equation (112) is expressed by the formula, for a solid round shaft,

$$(M_b)_i = \frac{\pi f D^3}{32} \quad \cdots \quad \cdots \quad (114)$$

By substituting this value of $(M_b)_i$ in equation (112) it reduces to equation (115).

For a solid round shaft:

$$fD^3 = 6.366[.6M_b + \sqrt{M_b^2 + M_t^2}], \quad \cdots \quad (115)$$

whence

$$D^3 = \frac{6.366}{f}[.6M_b + \sqrt{M_b^2 + M_t^2}] \quad \cdots \quad (116)$$

and

$$f = \frac{6.366}{D^3}[.6M_b + \sqrt{M_b^2 + M_t^2}]. \quad \cdots \quad (117)$$

The value of the resisting moment required to withstand the ideal twisting moment $(M_t)_t$ of equation (113) is, for a solid round shaft,

$$(M_t)_t = \frac{\pi s D^3}{16}. \quad \ldots \ldots \quad (118)$$

Substituting this value of $(M_t)_t$ in equation (113), it becomes, by a slight reduction,

For a solid round shaft:

$$sD^3 = 5.093[.6M_b + \sqrt{M_b^2 + M_t^2}], \quad \ldots \quad (119)$$

whence

$$D^3 = \frac{5.093}{s}[.6M_b + \sqrt{M_b^2 + M_t^2}] \quad \ldots \quad (120)$$

and

$$s = \frac{5.093}{D^3}[.6M_b + \sqrt{M_b^2 + M_t^2}]. \quad \ldots \quad (121)$$

In equations (115) to (121) the bracketed quantities are identical. In (119) the exponent 5.093 is 0.8 that of (115); therefore, if the shearing-strength of the material is taken as 0.8 of the tensile strength, which is very commonly done for iron and steel, the same value will be obtained for D^3 by both equation (116) and (120.) Hence either one of these equations may be used for determining D when $s = .8f$.

If the shearing-strength is taken greater than .8 of the tensile (i.e., $s > .8f$), then equation (116) should be used for finding D; but if the shearing-strength is taken as less than .8 of the tensile, (i.e., $s < .8f$), equation (120) should be used.

Example.—A belt whose turning force and total effective tensions are predetermined is to run on a pulley 5 feet in diameter, whose shaft is supported by bearings on both sides of the pulley, one at a distance of 5 feet and the other 3 feet from the middle of the pulley-hub; also that all the power is to be transmitted in one direction through the shafting leading from the pulley. What should be the diameter of the shaft for working strengths of 12000 pounds per square inch tensile, and 10000 pounds per square inch shearing for a belt wrapping 160° around the pulley?

The values of the turning force P and the belt-tensions T_n and T_o, as determined, may be taken as

$$P = 1467 \text{ pounds};$$
$$T_n = 2960 \text{ pounds};$$
$$T_o = 1493 \text{ pounds}.$$

The torsional moment M_t that the shaft must resist is

$$M_t = Pl = 1467 \times (2.5 \times 12) = 44010 \text{ inch-pounds};$$

and the bending force W that the shaft must resist is the resultant of T_n and T_o, which act at an angle of $(180° - 160°) = 20°$ with each other. This resultant is 4360 pounds. (The difference between this resultant and the sum of the two belt-tensions $(T_n + T_o) = 4453$ is so small comparatively as not to need consideration in general practice. The sum of the belt-tensions would ordinarily be used for so small an angle between the two stretches of belt. The actual resultant will be used, however, in the following calculations.)

The bending moment M_b which the shaft must resist is, according to equation (104),

$$M_b = \frac{4360(3 \times 12)(5 \times 12)}{8 \times 12} = 98100 \text{ inch-pounds}.$$

Since the ratio (10000 : 12000) is greater than .8, equation (116), involving the tensile strength of the material, must be used, whence

$$D' = \frac{6.366}{12000}[.6 \times 98100 + \sqrt{(98100)^2 + (44010)^2}];$$
$$D = 4.45 \text{ inches}.$$

Problem.—Fig. 118.1. A solid shaft 7 feet long between self-aligning bearing centres has a pulley 6 feet in diameter placed 3 feet from one bearing centre and 4 feet from the other. The pulley is to receive 200 horse-powers from a belt wrapping 150° around it. The shaft is to run at 250 revolutions per minute. Of the 200 horse-powers, 120 are to be delivered on the 3-foot shaft side of the pulley and 80 horse-powers through the 4-foot length of head shaft.

Find the diameter of the shaft for limiting fibre stresses of 10,000 pounds per square inch tensile and 8000 pounds per square inch shearing.
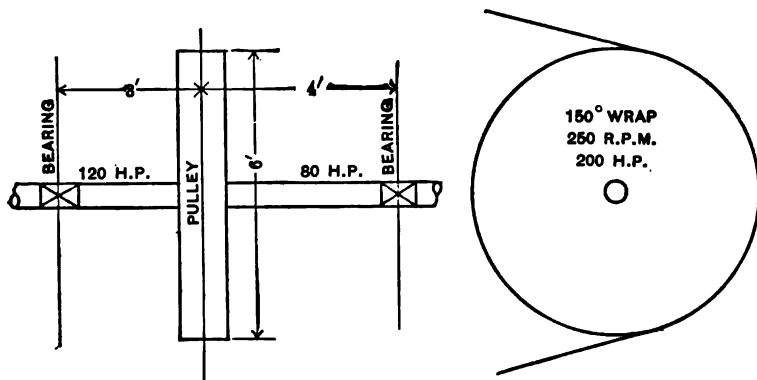
FIG. 118.1.

The linear velocity of the belt is

$$v = 250 \times \frac{6\pi}{60} = 78.54 \text{ feet per second.}$$

The turning force required is

$$P = \frac{550 \times \text{H.P.}}{v} = \frac{550 \times 200}{78.54} = 1400 \text{ pounds.}$$

Since the power is transmitted in both directions from the pulley, neither part of the shaft is subjected to the entire torsional effort necessary to transmit 200 horse-powers.

The torsions in the two parts of the shaft on each side of the pulley are proportional to the power transmitted through them. The greatest torsional moment will, of course, be on the 120 horse-power side. Its value is

$$M_t = \frac{120}{200} \times 1400 \times 3 \times 12 = 30,240 \text{ inch-pounds.}$$

Since the centrifugal tension in a belt has no effect in producing either torsion or bending in a shaft supporting the pulley upon which it runs, only the effective tensions in the belt need to be considered for designing the shaft. These effective tensions may

most readily be obtained by assuming that the belt has no weight and that therefore all the tension in the belt is effective. The turning force per square inch of belt having 150° wrap around a pulley with a coefficient of friction of .3 and working at a tension of 300 pounds per square inch is 163 pounds. The total effective tension on the tight side of the belt is

$$\text{Effective } T_n = \frac{1400}{163}\, 300 = 2570 \text{ pounds,}$$

and on the slack side

$$\text{Effective } T_0 = T_n - P = 2570 - 1400 = 1170 \text{ pounds.}$$

The resultant of 2570 and 1170 pounds, acting at an angle 30° with each other, is 3650 pounds.

The maximum bending moment in the shaft produced by the resultant of the two belt tensions is

$$M_b = 3650 \times \frac{(3 \times 12)\,(4 \times 12)}{7 \times 12} = 75{,}000 \text{ inch-pounds.}$$

The diameter of the shaft may now be found by either equation 116 or 120 for combined bending and torsion, since the limiting shearing stress is given as .8 of the tensile stress. Equation (160) will be used. By substitution in it

$$D^3 = \frac{6.366}{10000} \left[.6 \times 75000 + \sqrt{(75000)^2 + (30200)^2}\right]$$

$$= \frac{6.366}{10000} (45000 + 81000)$$

$$= \frac{6.366}{10000} \times 126000 = 80.2,$$

whence $D = 4.31$ inches.

100. Overhanging solid round crank-shafts and other overhanging shafts acted on by a single rotative force.—When a single rotative force P is applied to an overhanging shaft, the equations of turning and resisting moments take a simpler form than those in § 99. This is due to the fact that both the bending moment M_b and the twisting moment M_t are due to the same torsional force P instead of different forces. In this case $M_b = PL$ and $M_t = Pl$.

Substituting these values of M_b and M_t in equation (116), it reduces to—

For a solid round shaft:

$$D^3 = 6.366\frac{P}{f}[.6L + \sqrt{L^2 + l^2}] \; ; \quad \cdot \quad \cdot \quad \cdot \quad (122)$$

$$f = 6.366\frac{P}{D^3}[.6L + \sqrt{L^2 + l^2}] \; ; \quad \cdot \quad \cdot \quad \cdot \quad (123)$$

$$P = \frac{fD^3}{6.366[.6L + \sqrt{L^2 + l^2}]} \; ; \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (124)$$

and similarly for the shearing-stress equation (120) reduces to—

For a solid round shaft:

$$D^3 = 5.093\frac{P}{s}[.6L + \sqrt{L^2 + l^2}]; \quad \cdot \quad \cdot \quad \cdot \quad (125)$$

$$s = 5.093\frac{P}{D^3}[.6L + \sqrt{L^2 + l^2}]; \quad \cdot \quad \cdot \quad \cdot \quad (126)$$

$$P = \frac{sD^3}{5.093[.6L + \sqrt{L^2 + l^2}]} \cdot \quad \cdot \quad \cdot \quad \cdot \quad (127)$$

Example.—An overhanging crank-shaft is to be designed to drive a piston 35 inches in diameter against a constant pressure of 60 lbs. per sq. in. The following dimensions are also fixed: length of stroke, 40 inches; length of connecting-rod 6 feet; "overhang" of crank 15 inches, measured from middle of crank-pin to a point 3 inches back from the front of the crank-shaft bearing; tensile stress in material of shaft not to exceed 12000 lbs. per sq. in.; shearing-stress not to exceed 10000 lbs. per sq. in. The speed is to be so slow that the inertia of the moving parts need not be considered.

Since the limiting shearing-stress is greater than .8 of the tensile stress, the diameter of the shaft can be determined by equation

(122).   The values of the quantities entering into this equation for this particular case are:

$$P = \frac{\pi(35)^2}{4}60 = 57726.6 \text{ pounds;}$$

$$L = 15 \text{ inches;}$$

$$l = 20 \text{ inches;}$$

$$f = 12000 \text{ lbs. per sq. in.}$$

Therefore

$$D^3 = 6.366\tfrac{57727}{12000}[.6 \times 15 + \sqrt{(15)^2 + (20)^2}]$$
$$= 1041;$$

whence

$$D = 10.13 \text{ inches.}$$

Hollow Round Shafts.

. **101. The equations for hollow round shafts** are similar to those for solid ones and may be written directly from them.   The more important ones are given below for convenience of reference.

102. Hollow round shafts acted on by more than one force. For tensile fibre-stress:

$$D^3 = \frac{6.366}{f(1 - n^4)}[.6M_b + \sqrt{M_b^2 + M_t^2}]; \quad . \quad . \quad (128)$$

$$f = \frac{6.366D}{D^4 - d^4}[.6M_b + \sqrt{M_b^2 + M_t^2}]. \quad . \quad . \quad . \quad (129)$$

For shearing fibre-stress:

$$D^3 = \frac{5.093}{s(1 - n^4)}[.6M_b + \sqrt{M_b^2 + M_t^2}]; \quad . \quad . \quad (130)$$

$$s = \frac{5.093D}{D^4 - d^4} [.6M_b + \sqrt{M_b^2 + M_t^2}]. \quad \ldots \quad (131)$$

103. Overhanging hollow round shafts acted on by a single turning force. Crank-shafts, etc.

For tensile fibre-stress:

$$D^3 = \frac{6.366P}{f(1 - n^4)} [.6L + \sqrt{L^2 + l^2}]; \quad \ldots \quad (132)$$

$$f = \frac{6.366PD}{D^4 - d^4} [.6L + \sqrt{L^2 + l^2}]; \quad \ldots \quad (133)$$

$$P = \frac{f(D^4 - d^4)}{6.366D[.6L + \sqrt{L^2 + l^2}]}. \quad \ldots \quad (134)$$

For shearing fibre-stress:

$$D^3 = \frac{5.093P}{s(1 - n^4)} [.6L + \sqrt{L^2 + l^2}]; \quad \ldots \quad (135)$$

$$s = \frac{5.093PD}{D^4 - d^4} [.6L + \sqrt{L^2 + l^2}]; \quad \ldots \quad (136)$$

$$P = \frac{s(D^4 - d^4)}{5.093D[.6L + \sqrt{L^2 + l^2}]}. \quad \ldots \quad (137)$$

104. Experimentally determined values of the breaking tensile stress of round shafting subjected to combined bending and torsion.—Professor Gaetano Lanza made a series of experiments on shafting subjected to both bending and torsion while rotating.[*] Some of the values obtained by him are given in Table XXXIII. They represent the breaking stress by fatigue of the metal, but not the static strength.

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[*] Trans. Amer. Soc. Mech. Eng., .ol. VIII., 1887, p. 130.

TABLE XXXIII.*

BREAKING-STRENGTH OF SHAFTING SUBJECTED TO COMBINED
BENDING AND TWISTING.

Diameter of Shaft, inches.	Total Revolutions.	Horse-power Transmitted.	M_b Max. Bending Moment. Inch-lbs.	M_t Max. Twisting Moment. Inch-lbs.	f Fibre-stress Causing Fracture. Lbs. per sq. in.
1.25	7,040	11.717	11,514.1	3,926.4	62,168
1.25	38,839	8.181	10,507.8	2,656.8	55,876
1.25	81,641	5.291	9,891.0	1,714.6	52,062
1.25	108,002	4.881	9,241.7	1,399.2	48,589
1.25	80,694	6.276	9,241.7	2,027.6	48,911
1.25	19,383	6.342	8,917.1	2,038.2	47,245
1.25	82,741	6.283	8,917.1	2,029.7	47,246
1.25	108,739	6.192	8.592.5	2,081.6	45,582
1.25	88,208	6.338	8,267.8	2,026.8	43,914
1	185,283	6.283	8,781.5	2,029.7	41,768

* Taken from Transactions Am. Society Mechanical Engineers, 1887, vol. VIII., p. 136.

Samples of the shafting tested for combined stress were also
tested for static tensile strength. The results are given in Table
XXXIV.

TABLE XXXIV.

STATIC TENSILE STRENGTH OF SHAFTING.

Refers to shafting represented in Table XXXIII.

Breaking Tensile Strength,
lbs. per sq. in.

1.25″ diam. { No. 1............................ 46,800
            { No. 2............................ 49,865

Average..................... 48,333

1″ diam. { No. 1............................ 58,687
         { No. 2.... 61,812

Average..................... 60,250

105. Practically determined formulas for round shafting.—
There are many cases in practice where the relations between the
torsional and bending moments, or even the value of either, cannot
be estimated with much accuracy. To meet such conditions, for-
mulas have been devised to accord with transmission-machinery that
has given satisfactory service. These formulas are given in many

hand-books and treatises on power transmission. They may be found in Kent's Mechanical Engineers' Pocket-book, together with tables for different speeds, etc., calculated from the formulas.

SHAFTS OF SYMMETRICAL SECTIONS OTHER THAN ROUND.

106. Shafts of any section other than round are seldom used in machine construction. Since they are sometimes used, however, the method of determining their strength for combined stress will be pointed out.

In order to obtain the equation of strength of any form of section, it is only necessary to substitute the resisting moment of the shaft against bending for the ideal bending moment $(M_b)_i$ in equation (112), or its resisting moment against torsion for the ideal torsional moment $(M_t)_i$ in equation (113), and solve as for round shafts after making the substitution.

Calling f the working strength of the material for tensile or compression stress, and S_b the section modulus of the shaft for resisting bending, the following equation may be written:

$$(M_b)_i = fS_b ; \quad \ldots \ldots \ldots \quad (138)$$

and, similarly, calling s the working shearing-strength of the material, and S_t the section modulus of the shaft for resisting torsion,

$$(M_t)_i = sS_t. \quad \ldots \ldots \ldots \quad (139)$$

The values given in these two equations are the ones to be substituted in equation (112) or (113) in order to obtain equations of the same general nature as (115), (116), and (117).

POSITIVE COUPLINGS FOR SHAFTS.

107. Rigid shaft-couplings.—When no allowance is to be made for lack of alignment of the connected shafts, a rigid coupling is commonly used.

The most common form of rigid coupling for large shafts is a

pair of flanged hubs or collars. The hub of each is solid and bored to a diameter to fit the shaft, to which it is generally keyed. The flanges are bolted together after putting in place. The shafts are generally brought into line either by allowing one shaft to enter into the bore of the half coupling on the other, by a shoulder on one flange fitting into a corresponding recess in the other, or by making an accurate fit of the bolts in the flanges. Other less common methods are also used.

For the medium and smaller sizes of shafting used for power transmission, couplings bored to a slightly smaller diameter than that of the shaft, and split longitudinally, either upon one side or both, are very commonly used.

A very simple form of split-sleeve coupling is shown in Fig. 119. It consists of a sleeve bored to the same diameter from end to end,
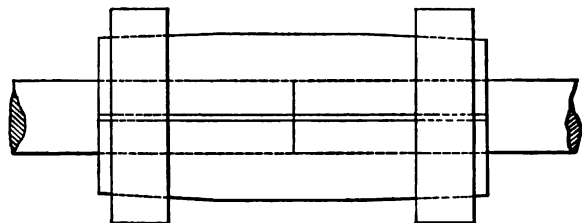
FIG. 119.

and given a slight conical taper toward each end on the outside. It is either split longitudinally down one side, or cut completely in two longitudinally, so as to form two halves. When used, it is placed upon the two sections of shafting, which are brought together, and then a ring, bored to the same taper as the outside of the sleeve, is either driven or shrunk upon each end of the coupling, thus causing it to grip the shafts tightly. When the torsional force to be transmitted is small, the coupling may be used without a key, but for heavy work a key is generally used after the manner common to solid couplings.

It can readily be seen that a split-sleeve coupling of much the same nature as the above one can be made by using bolts, instead of the rings, to clamp the two halves together.

A largely used form of coupling, commonly known as Seller's

coupling, consists of an outer sleeve, bored at each end with a conical hole to receive a bushing with a corresponding taper on the outside. The bushing has a bore slightly smaller than the shaft. Bolts, extending from end to end through the sleeve, serve to draw the two conical bushings towards each other, and thus cause them to grip the shaft. These bolts also serve as keys to prevent the rotation of the bushings in the outer sleeve. Keys may be used or not between the shaft and bushing, according to the nature of the work.

108. Flexible shaft-couplings.—When there is a probability that there will be a slight relative movement of the shafts coupled together, as in the case of the shafts of a dynamo and engine resting on separate foundations and direct-connected together, it is necessary to have a coupling which will adjust itself to the slight throwing out of alignment of the shafts that occurs under such conditions.

The coupling Fig. 120 has been successfully applied for mod-
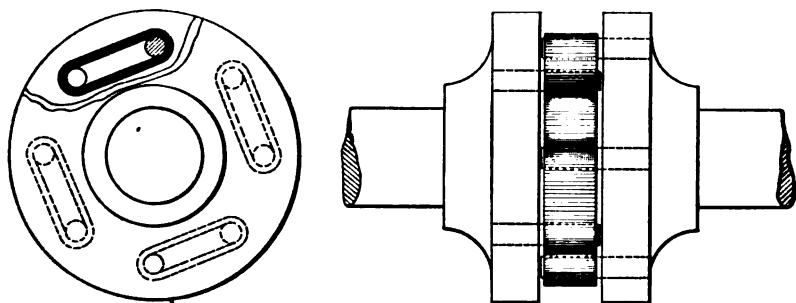
FIG. 120.

erate amounts of power transmitted. It consists of a pair of disks very similar to those of the rigid coupling, but instead of being held firmly together by bolts they are separated as shown, and each has a number of strong pins projecting from it so as to almost touch the other. The other disk has the same number of pins. The pins are spaced uniformly around both flanges near their peripheries. Each pin of one flange is connected to one of the other by a short link of some elastic material, as rubber-or leather. As one flange rotates it draws the other after it by means of the connecting-links, whose elasticity allows them to adjust themselves to the lack of

alignment of the shafts.   The links can be made longer or shorter than shown in the figure if desired.

Another flexible coupling, which allows the shafts to have bending motion relatively to each other, but does not allow lateral motion, is shown in Fig. 121.* It consists of a large flange upon one of the shaft-ends to be coupled, and a small one upon the other. The two flanges are connected together by a washer-shaped ring, which may
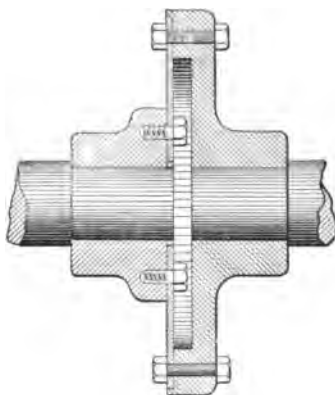
Fig. 121.

be made by cutting it from sheet metal, the outer diameter being made much larger than the inner.   This flat ring, being of an elastic material, allows a slight bending motion of the shaft as stated without injury to the parts.†

109. Positive clutch-couplings are often used for connecting parts of machinery which are to be disengaged at times. The most common form consists of two parts resembling, in a manner, the halves of the coupling shown in Fig. 120, but having projecting teeth or jaws instead of pins. These jaws are made so that they will engage with each other for transmitting rotative motion. The coupling is generally disengaged by slipping one of its halves along the shaft so as to separate the jaws.

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* Trans. Amer. Soc. Mech. Eng., vol. VII., p. 526.

† For Oldham's and Hooke's universal couplings see Part I. of Machine Design.

CHAPTER IX.

110. In the operation of machinery it is often desirable to start and stop some part which receives its motion from a constantly running driver, or to bring two parts into engagement without changing their relative positions. For this purpose "friction-couplings" are used almost universally. Such a coupling transmits power by means of two smooth surfaces, held together with sufficient pressure to make their frictional resistance to slipping great enough to produce the required transmission of power.

The **friction-brake**, for retarding the motion of a part, is practically the reverse of the friction-coupling, so far as its general principle is concerned, but differs from it in that there is generally much more rubbing between the surfaces, and consequently the materials must be selected with more attention to their qualities of not abrading and cutting. This applies to brakes such as are used on hoisting-machinery, but hardly to those for railway-wheels, etc., where the conditions are such that it is impossible to prevent abrasion on account of foreign matter getting between the rubbing surfaces.

FRICTION-COUPLINGS

111. Cone friction-couplings, consisting of two parts having conical surfaces, one external and the other internal, fitting together as in Fig. 122, are often used in machine construction, and especially in machine tools, for connecting the ends of two shafts, or for transmitting power from a shaft by means of a gear or other device attached to the part of the coupling which is free to rotate upon the shaft when the clutch is open, but is driven by the shaft when the two parts of the clutch are forced together.

The notation in the equations for a cone friction-clutch is:

M = torsional moment transmitted through clutch;

N = total normal pressure between the conical surfaces of the clutch;

P = turning force acting at a distance R from the centre of the shaft;
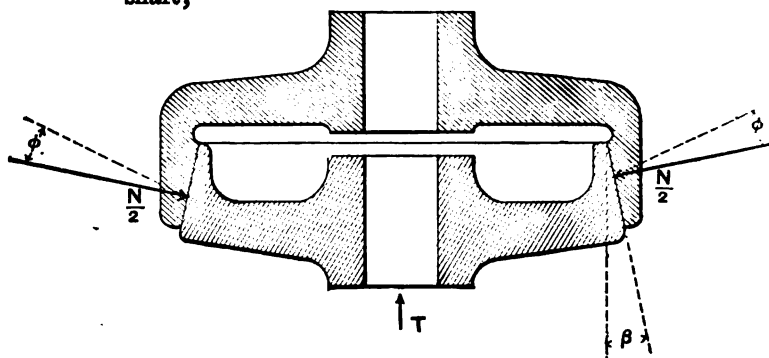
FIG. 122.

R = mean radius of conical surfaces;

T = axial force acting to close the clutch;

β = angle between the conical surfaces and the axis of shaft; the apex angle of the conical surfaces is therefore 2β;

μ = tan ϕ = coefficient of friction between the conical surfaces;

ϕ = angle of friction between the conical engaging surfaces of the clutch.

If the two parts of the coupling are forced together when they have no relative rotation, the pressure between the conical surfaces would make an angle ϕ with a normal to the surfaces, as indicated by the dotted lines in Fig. 122. This frictional resistance would have to be taken into consideration if it were desired to set the clutch so that there would be no slip whatever between the parts at the time of closing or afterwards. But if the clutch is closed while one part is rotating relatively to the other, or if a slight slip at the time of throwing on the load is not objectionable, it is not necessary to consider the angle of friction when determining the axial force T required for closing it. A slight slip at the time of closing is almost invariably, probably always, allowable. Of the following equations, the ones not including ϕ are therefore applicable to cone friction-clutches in nearly all cases.

For cone friction-clutches closed while having a rotative slip between the engaging surfaces:

$$N = T \text{ cosec } \beta; \qquad \ldots \ldots \ldots \ldots \ (140)$$

$$P = \mu N = \mu T \text{ cosec } \beta; \qquad \ldots \ldots \ldots \ (141)$$

$$M = PR = \mu TR \text{ cosec } \beta. \qquad \ldots \ldots \ldots \ (142)$$

For a cone friction-clutch closed without rotative slipping between the engaging surfaces:

$$N = T \text{ cosec } (\beta + \phi); \qquad \ldots \ldots \ldots \ (143)$$

$$P = \mu N = \mu T \text{ cosec } (\beta + \phi); \qquad \ldots \ldots \ (144)$$

$$M = PR = \mu TR \text{ cosec } (\beta + \phi). \qquad \ldots \ldots \ (145)$$

When $\beta = \phi$ the parts will separate of their own accord with no load on the clutch and when $T = 0$.

Problem.—Design a cone friction-clutch of the type shown in Fig. 122 for transmitting 25 horse-powers at 250 revolutions per minute.

The mean radius of the cone surface may be assumed as 6 inches. The turning force necessary on this radius is, by equation (17),

$$P = \frac{33000 \times 12 \times \text{H.P.}}{2\pi RV} = \frac{33000 \times 12 \times 25}{2\pi \times 6 \times 250} = 1054 \text{ pounds.}$$

The coefficient of friction μ may be taken as .15. Then

$$N \quad M = P \div \mu = 1054 \div .15 = 7027 \text{ pounds.}$$

The clutch will just open if tan $\beta = \mu$. Since μ may at times become greater than the assumed value of .15, its maximum value may be taken as .2; then

$$\tan \beta = \tan \phi = \mu = .2,$$
$$\beta = 11.33°.$$

The axial closing force must be, from equation (140),

$$T = N \div \csc\beta = N \sin \beta = 7027 \times .196 = 1380 \text{ pounds.}$$

For a pressure of 50 pounds per square inch between the cone surfaces the area of each surface will be

$$7027 \div 50 = 140.5 \text{ square inches.}$$

The mean diameter of the cone surface is 12 inches. The width will therefore be

$$140.5 \div 12\pi = 3.75 \text{ inches.}$$

Note. The resultant of the turning forces acting tangent to the conical surface is actually at a slightly greater distance from

the axis of the clutch than the mean radius of the cone surface. The error introduced by using the mean radius is small and on the safe side.

112. Multiple-ring friction-coupling.—When it is desirable to keep the size of the coupling small, especially in diameter, the form shown in Fig. 123 is applicable.* As illustrated, the outer
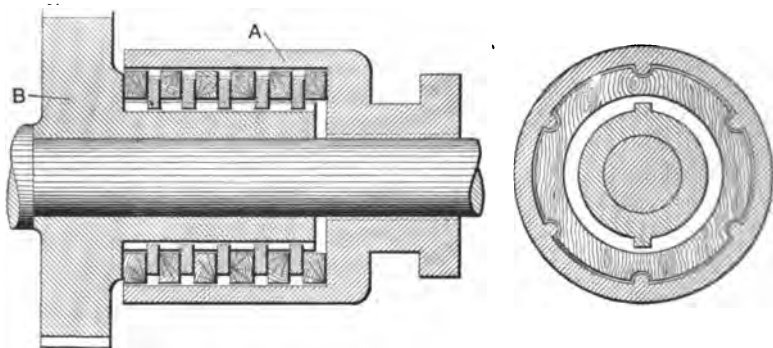
FIG. 123.

cylindrical sleeve A is fitted on the shaft with a feather key, so that it may slip along it. The hub B has a running fit on the shaft, but is prevented from moving along it by a shoulder. A number of rings are placed between the sleeve and hub, each alternate one engaging with the sleeve by means of several projections on the latter corresponding to feather keys, so as to be free to slip along the sleeve and hub. The remaining rings engage with the hub in a similar manner. By applying an axial force to press the sleeve toward the hub, a pressure, equal to the axial force, is induced upon each side of every ring.

Putting, for multiple-ring friction-couplings: $M = $ torsional moment transmitted through coupling; $P = $ turning force exerted; $R = $ mean radius of rubbing surfaces of rings; $T = $ axial force exerted to press the sleeve toward the ring; $n = $ number of pairs of rubbing surfaces in contact; $\mu = $ coefficient of friction of the rubbing surfaces; then

$$P = \mu n T; \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (146)$$
$$M = PR = \mu n \, TR. \quad . \quad . \quad . \quad . \quad . \quad (147)$$

As represented in the figure, there are as many pairs of rubbing

* Weston friction-coupling used by Yale & Towne Mfg. Co. on hoisting-machinery.

surfaces as there are rings.   This is not always true of such coup-. lings, however.

113. Materials and coefficient of friction for friction-couplings. —In machine tools and the counter-shafts for driving them, unless they are exceptionally large, the material of a cone friction-coupling is generally cast iron for both rubbing surfaces.   When accurately ground together with a fine abrasive, as flour-emery, the cast iron gives excellent satisfaction for this use.   The coefficient of friction μ must be taken with regard to whether the surfaces are oily or dry.   Since there is nearly always a certainty of oil getting on them at some time, the value $\mu = 0.15$ is probably as high as it is safe to assume for cast iron on cast iron.

When a friction-coupling is to perform heavy service with considerable slipping at the time of setting the machinery into motion, cast iron on wood or leather gives good service.   The wood may be either set on end or with the grain parallel to the direction of rubbing.   Any of the metals that run well together for journal-bearings may be used if the coupling is kept clean and lubricated. Otherwise wood or leather rubbing against metal is safer.   For dry wood on cast iron μ may be taken from 0.15 to 0.18; for oily wood μ falls as low as 0.10.   The value of μ for leather on cast iron does not seem to be well determined for such service as is required for friction-clutches, but it can probably be taken as high as 0.20 for oily leather; it is much higher for dry leather.   Leather should not be used where there is sufficient slipping to burn it.

The best service for heavy work and hard usage seems to be obtained with wood rubbing on metal.   In practice the latter is generally cast iron.

FRICTION-BRAKES.

114. Strap brake.—A form of strap brake much used for hoisting-machinery is shown in Fig. 124.   The brake-drum, whose centre is at O, is partly encircled by a strap whose ends are attached to a lever for tightening it upon the drum.   The strap may be made of any material, according to the requirements to be met.   For heavy service on mine-hoists, a strap of Swedish wrought iron, lined with blocks of basswood placed with the grain parallel to the length of the strap, is very commonly used.   Experience has

shown that a steel strap is very liable to fracture under such service, apparently on account of fatigue of the metal, caused by the repeated application of stress due to tightening and loosening the
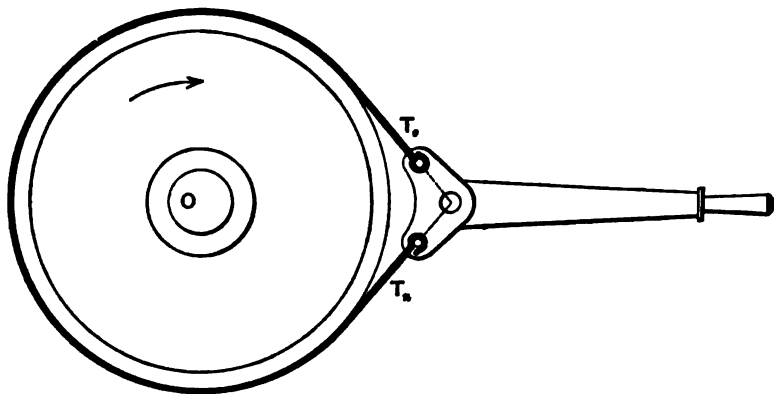
Fig. 124.

drum, and to the almost incessant vibrations common to such service. Basswood is selected because in it are combined both durability and uniform gripping power. For lighter service, a wrought-iron strap is often used directly against the cast-iron brake-drum. This can be done successfully only where the rubbing surfaces can be kept lubricated.

The tensions T_a and T_o in the two free portions of the brake-strap between the drum and brake-lever, for a given force P resisting the rotation of the drum, and applied at the surface of the latter, may be found by the equations for belting. The value of V in these equations is zero for a brake-strap, and no centrifugal force due to its own weight acts upon it.

Equations (39) to (45) are applicable to the solution. In these equations the coefficient of friction μ may be taken as 0.1 for wet or oily surfaces of wood on cast iron, and from 0.15 to 0.18 for dry rubbing surfaces of the same materials.

The pressure that can be safely put on wood-lined strap or crab brakes for the heavy duty of mine hoists is 25 lbs. per sq. in. or less.

115. The Prony brake, Fig. 125, is a special form of strap brake. Its almost universal application is as an absorbent dynamometer for tests of the power developed by motors. As shown in

the figure, it consists of an iron strap A encircling a brake-drum whose centre is at O. The strap is lined with wooden blocks which bear against the surface of the drum. The ends of the straps are held together by a bolt which affords a means of adjustment of
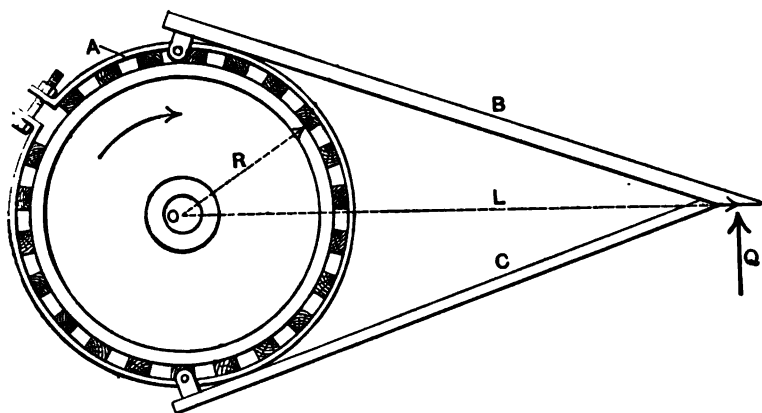
FIG. 125.

tension in the strap. The two members B and C each have one end attached to the brake-strap, and are joined together so as to form a rigid arm. As the brake-drum rotates in the direction of the arrow, the tendency of this arm to rotate with it is resisted by a force Q acting against the end of the arm.

If the two members of the brake-arm are attached to the strap at points diametrically opposite, so that each portion of the strap covers 180° of the drum, and if Q acts vertically upward and is of a value equal to the portion of the weight of the brake that is supported by the drum, then the following equations are applicable for determining the relation between the maximum tension T_n in the brake-strap and the force Q. It can be seen that these equations are of the same nature as those for belting. The brake-strap corresponds to two portions of a belt having an arc of contact of 180° upon the pulley.

The notation for the Prony brake is:

$L =$ distance from centre of drum to line of action of Q (not necessarily the distance from the centre of the drum to the point of application of Q);

P = total frictional resistance at surface of brake-drum;
Q = force applied at end of brake-arm to resist its rotation;
R = radius of brake-drum;
T = greatest tension in brake-band;
ϵ = 2.71828;
π = 3.1416.

By taking moments about O

$$PR = QL \quad \text{or} \quad P = \frac{L}{R}Q. \quad . \quad . \quad . \quad . \quad (148)$$

In accordance with the equations for belting, taking into account the fact that there is no centrifugal force acting on the brake-strap,

$$T = \frac{P}{2} \frac{\epsilon^{\mu\pi}}{\epsilon^{\mu\pi} - 1} = \frac{P}{2} \frac{10^{.00758\mu180}}{10^{.00758\mu180} - 1}. \quad . \quad . \quad (149)$$

Whence, by substituting the value of P given in equation (148),

$$T = \frac{QL}{2R} \frac{\epsilon^{\mu\pi}}{\epsilon^{\mu\pi} - 1} = \frac{QL}{2R} \frac{10^{.00758\mu180}}{10^{.00758\mu180} - 1}. \quad . \quad . \quad (150)$$

If the points of attachment of the two members of the brake-arm to the brake-strap are not diametrically opposite, it is hardly possible to get equations for the maximum tension in the strap that are of any practical value. If the material of the strap were totally inelastic, such equations might be deduced, but on account of its elasticity they cannot be. The equations just given may be applied with a fair degree of accuracy, however, when the arc between points of attachment does not differ greatly from 180°, and if Q does not differ greatly from the portion of the weight of the brake supported by the drum.

The coefficient of friction μ, and the materials for the Prony brake, may be taken the same as for the strap brake of the preceding section. Since the service required of a Prony brake is seldom so trying as that of the strap brake for hoisting-machinery, it is generally not necessary to exercise so much care in selecting the materials.

CHAPTER X.

FLY-WHEELS AND PULLEYS.

116. A fly-wheel, as commonly used in machine construction, is a rotating wheel which serves as a storehouse for energy. At certain periods during the operation of the machine the rotative speed of the fly-wheel is increased, thus increasing its store of kinetic energy, and at other periods the speed is decreased, the wheel thus returning or supplying energy to the machine.

Examples of the application of fly-wheels are very common. Three cases may be cited, however, to point out different classes of service.

In the ordinary double-acting steam-engine with a single cylinder the fly-wheel has its speed slightly increased twice during each revolution, and also slightly decreased twice during the same revolution, provided the average speed of rotation does not change during a series of successive revolutions. The function of the fly-wheel in this case is to prevent great fluctuation of speed during a single revolution, as well as to maintain an approximately uniform speed when the load against which the engine works varies suddenly.

In a punching and shearing machine the fly-wheel receives a store of energy while being brought up to its normal speed. As the punch or shears are driven through the metal operated upon, the speed of the fly-wheel is decreased and a portion of its energy given up to perform the work upon the metal and to overcome frictional resistances. The time interval of working on the material punched or sheared, which corresponds to the period of reduction of speed, is small in comparison with the time the machine is running idly. After punching the material the fly-wheel again receives energy while being brought back to its normal speed.

In the Howell torpedo a fly-wheel is brought up to an exceedingly high speed. When the torpedo is launched into the

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water, the kinetic energy stored in the wheel is utilized to drive a screw propeller for forcing the torpedo through the water. The fly-wheel gradually stops as it gives up its energy.

117. Determination of moment of inertia and kinetic energy of a webbed wheel.—The total kinetic energy, K.E., stored in a rotating fly-wheel is expressed by the equation

K.E. = $\frac{1}{2}$(mass) × (angular velocity)^2 × (radius of gyration)^2,   or

K.E. = $\frac{1}{2}$(moment of inertia) × (angular velocity)^2.

The form of fly-wheel that is most generally adopted has a rim of rectangular cross-section with slightly rounded corners, a hub that is approximately cylindrical, and arms tapering uniformly from the hub to the rim, growing smaller toward the rim; for small sizes of wheels a solid web is often used, the web generally being thinner at the rim than at the hub.

The moment of inertia of a wheel with uniformly tapering arms can be found by reducing the arms to an equivalent web. This reduction can be made with an accuracy far within the limits required for ordinary practice, and the results obtained by this method of substituting a web for the arms are ordinarily more accurate than those obtained by dealing with the arms directly. (See § 120.) The method of finding the moment of inertia of a webbed wheel of the form shown in Fig. 126 will therefore be given
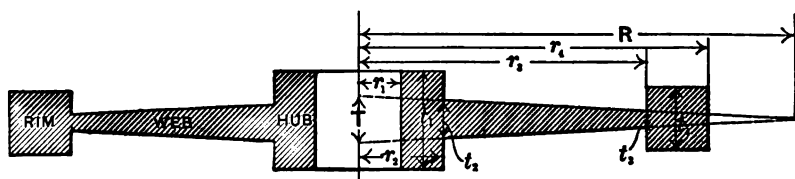
FIG. 126.

as being more generally applicable than that for any other form of wheel.

*The notation for the equations applying to a webbed wheel is:

E = energy given out by wheel for a given change of speed of rotation, foot-pounds;

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* See § 120 for notation of fly-wheel with arms.

I = moment of inertia of entire wheel about its axis, foot-pound-seconds;

I_{hub}, I_{web}, and I_{rim} = moment of inertia of the hub, web, and rim, respectively, foot-pound-seconds;

K.E. = total kinetic energy stored in the rotating wheel, foot-pounds;

$N = \omega \div 2\pi$ = revolutions per second;

N_1 = initial speed of rotation, revolutions per second;

N_2 = speed of rotation after slowing down, revolutions per second;

$R = \dfrac{r_2 t_3 - r_3 t_2}{t_3 - t_2}$ = radial distance at which sides of web would

intersect if extended, used for convenience only, feet;

$T = \dfrac{t_2 R}{R - r_2} = \dfrac{t_3 R}{R - r_3}$ = length of axis intercepted between

sides of web if extended to axis of wheel, used for convenience only, feet;

g = acceleration due to force of gravity = 32.2 feet per second per second;

p = tensile stress in rim due to rotation, pounds per square inch;

r = mean radius of rim of wheel, feet;

r_1 = radius of bore of hub, feet;

r_2 = outer radius of hub = inner radius of web, feet;

r_3 = outer radius of web = inner radius of rim, feet;

r_4 = outer radius of rim, feet;

t_1 = thickness (or length) of hub, feet;

t_2 = thickness of web at inner edge, feet;

t_3 = thickness of web at outer edge, feet;

t_4 = thickness of rim measured parallel to axis of wheel, feet;

v = velocity, feet per second;

w = weight of material, pounds per cubic foot;

λ = increase in radius of ring due to centrifugal action, feet;

π = 3.1416;

$\omega = 2\pi N$ = velocity of rotation, radians per second;

ω_1 = initial speed of rotation, radians per second;

ω_2 = speed of rotation after slowing down, radians per second.

The moments of inertia of the three parts, the hub, web, and rim, are:

$$I_{hub} = \frac{w\pi t_1}{2g}(r_2{}^4 - r_1{}^4); \quad \cdots \quad \cdots \quad (151)$$

$$I_{web} = \frac{2\pi w T}{g}\left[\frac{r_3{}^4 - r_2{}^4}{4} - \frac{r_2{}^5 - r_3{}^5}{5R}\right]; \quad \cdots \quad (152)$$

$$I_{rim} = \frac{w\pi t_4}{2g}(r_4{}^4 - r_3{}^4). \quad \cdots \quad \cdots \quad (153)$$

The moment of inertia of the entire wheel is the sum of the I's for all the parts; whence

$$I = I_{hub} + I_{web} + I_{rim}. \quad \cdots \quad \cdots \quad (154)$$

The total kinetic energy K.E. stored in the wheel when rotating at an angular velocity of ω radians, or N revolutions, per second, is

$$\text{K.E.} = \tfrac{1}{2}\omega^2 I = 2\pi^2 N^2 I. \quad \cdots \quad \cdots \quad (155)$$

The energy E transformed from kinetic energy into mechanical energy or heat, or both, when the speed of rotation of the fly-wheel drops from ω_1 to ω_2 radians, or from N_1 to N_2 revolutions, per second, is shown by the expression

$$E = \tfrac{1}{2}(\omega_1{}^2 - \omega_2{}^2)I = 2\pi^2(N_1{}^2 - N_2{}^2)I. \quad \cdots \quad (156)$$

The same amount of mechanical energy E must, of course, be applied to the wheel to bring it back from N_2 to N_1 revolutions per second, friction neglected.

Example.—It is required to find the kinetic energy K.E. at a speed of 300 revolutions per minute of a cast-iron fly-wheel of the form shown in Fig. 126 and having the following dimensions:

$$t_1 = 6 \text{ in.} = .5 \text{ ft.}; \quad r_1 = 1\tfrac{1}{4} \text{ in.} = \tfrac{5}{48} \text{ ft.};$$
$$t_2 = 1\tfrac{1}{2} \text{ in.} = .125 \text{ ft.}; \quad r_2 = 3 \text{ in.} = .25 \text{ ft.};$$
$$t_3 = 1 \text{ in.} = \tfrac{1}{12} \text{ ft.}; \quad r_3 = 18 \text{ in.} = 1.5 \text{ ft.};$$
$$t_4 = 5 \text{ in.} = \tfrac{5}{12} \text{ ft.}; \quad r_4 = 24 \text{ in.} = 2 \text{ ft.}$$

The weight of cast iron may be taken as 450 pounds per cubic foot.

If the acceleration due to force of gravity is taken as 32.2 feet per second per second, then the units of measurement must be the foot, pound, and second.

The moment of inertia of the three parts may first be determined.

The moment of inertia of the hub is, by equation (151),

$$I_{hub} = \frac{450\pi \times .5}{2 \times 32.2}\left[(.25)^4 - \left(\frac{5}{48}\right)^4\right]$$
$$= 10.97(.0039062 - .0001174)$$
$$= .0416.$$

For the web equation (152) can be applied. In this equation the quantities T and R enter. T may be measured on the drawing with considerable accuracy, but the intersection of the lines determining R is generally difficult to determine. The values of both will be calculated for this problem according to the equations given in the notation; whence

$$R = \frac{18 \times 1.5 - 3 \times 1}{1.5 - 1} = 48 \text{ inches} = 4 \text{ feet,}$$

and

$$T = \frac{1.5 \times 48}{48 - 3} = 1.6 \text{ inches} = \tfrac{2}{15} \text{ foot.}$$

By substituting in equation (152)

$$I_{web} = \frac{2\pi 450 \times 2}{32.2 \times 15}\left[\frac{(1.5)^4 - (.25)^4}{4} - \frac{(1.5)^5 - (.25)^5}{5 \times 4}\right]$$
$$= 11.7 \times .885$$
$$= 10.4.$$

And for the rim, by equation (153),

$$I_{rim} = \frac{450\pi 5}{2 \times 32.2 \times 12}[(2)^4 - (1.5)^4]$$
$$= 100.3.$$

For the entire wheel the moment of inertia is therefore

$$I = .0416 + 10.4 + 100.3$$
$$= 110.74.$$

It can be seen that the effect of the hub on the total moment of inertia is practically inappreciable, and may therefore be neglected in a wheel whose rim diameter and weight of rim are as great in proportion to the similar quantities for the hub as for the wheel just considered.

The kinetic energy of the entire wheel when rotating at 300 revolutions per minute is, according to equation (155),

$$\text{K.E.} = 2\pi^2 N^2 I$$
$$= 2\pi^2 (\tfrac{300}{60})^2 \, 110.74$$
$$= 54648 \text{ foot-pounds}$$
$$= 27.32 \text{ foot-tons.}$$

118. Problem.   To design a fly-wheel for a given moment of inertia and according to a given form.—Let the required moment of inertia $I = 300$, the wheel to be similar to Fig. 126.

Since the required wheel is to be similar to the one considered in the preceding problem, it is only necessary to change the dimensions of the wheel, all in the same proportion, to such an extent as will give the required I—in other words, to apply such a scale to the drawing as will give the required I.

The proportionate change of the dimensions can readily be determined by making use of the fact that the I of similar wheels is proportional to the fifth power of their linear dimensions.   Therefore the linear dimensions of the wheel in the preceding example, which has an $I = 110.74$, must each be multiplied by

$$\sqrt[5]{\frac{300}{110.74}} = 1.22$$

in order to obtain the required $I = 300$.

This scale gives the outer radius of the rim

$$r_a = 1.22 \times 24 = 29.28 \text{ inches,}$$

and the other dimensions must be increased in the same proportion.

The above solution shows that when it is desired to design a wheel for a required I it can be done by making a drawing according to the form desired, considering it full size, and finding its I accordingly. The scale which must be applied to the drawing to obtain the required I can then be determined by dividing the required I by the I of the drawing, considered full size, and extracting the fifth root of the quotient.

119. Problem. To design a fly-wheel which will furnish a given amount of energy for a given variation of speed.—Let it be required that the wheel shall furnish 40000 foot-pounds of energy for a 5% speed reduction, and that the full speed shall give the outer circumference of the rim a velocity of 4800 feet per minute.

This problem can most readily be solved by making use of the fact that in similar wheels having the same circumferential linear velocity the kinetic energy is proportional to the cubes of similar linear dimensions.

For convenience it may be assumed that the form of wheel selected is that of Fig. 126, and that the proportions selected for the first trial are those of § 117.

For the wheel considered in § 117, $I = 110.74$. When this wheel runs at a circumferential velocity of 4800 feet per minute, its angular velocity is

$$\omega_1 = \frac{4800}{60r_4} = \frac{4800}{60 \times 2} = 40 \text{ radians per second.}$$

When the speed has dropped 5%, the angular velocity is

$$\omega_2 = 40 - .05 \times 40 = 38 \text{ radians per second.}$$

The energy given out by the wheel while slowing down 5% is, by equation (156),

$$E = \tfrac{1}{2}[(40)^2 - (38)^2]110.74$$
$$= \tfrac{1}{2} \times 156 \times 110.74$$
$$= 8638 \text{ foot-pounds.}$$

In order to secure a fly-wheel, similar to the one just considered, which, when running at 4800 feet per minute circumferential speed, will furnish 40000 foot-pounds of energy for a 5% reduction of speed, the proportions given in § 117 must all be multiplied by the same factor, whose value is

$$\sqrt[3]{\frac{40000}{8638}} = 1.6668 = 1\tfrac{2}{3} \text{ about.}$$

The outer radius of the rim must therefore be $1.6668 \times 2 = 3.3336$ feet, and the other dimensions must be increased in the same proportion.

In the fly-wheel considered above, part of the energy given out by it during its reduction of speed is converted into heat on account of journal friction, etc. If the frictional resisting forces remain constant, the mechanical energy converted into heat while the speed is being checked is proportional to the number of revolutions made during the change of speed. If the speed of the wheel is checked by frictional resistance only, all of the energy given up by the change of speed is converted into heat.

120. Moment of inertia of a fly-wheel with arms.—The I of the hub and rim can be found by equations (151) and (153), the numerical solution being similar to that in § 117. Only the method of dealing with the arms will therefore be considered. It is assumed that the hub and rim have the form shown in Fig. 126.

Suppose that the arms are sheared off from the hub and rim so as to leave the surfaces of the latter two parts smooth, and that the arms are flattened out to form a web which will just fit in between the hub and rim. The moment of inertia of the web will be practically the same as that of the arms from which it was made, and the area of its outer edge will be the same as the total area of all the sheared outer ends of the arms; the area of the inner edge will be equal to the total of all the sheared surfaces at the inner ends of the arms.

If, in a pulley with arms:

A_1 = total sheared area of the inner ends of the arms;
A_2 = total sheared area of the outer ends of the arms;
t_1 = thickness of inner edge of equivalent web having the same area as the sheared area of all the inner ends of the arms;
t_2 = thickness of outer edge of equivalent web;

and the remainder of the notation is the same as for the webbed wheel, Fig. 126; then

$$t_2 = \frac{A_2}{2\pi r_2} \quad \text{and} \quad t_3 = \frac{A_3}{2\pi r_3}. \quad \ldots \quad \ldots \quad (157)$$

As an example, let it be required to find the moment of inertia of six pulley-arms according to the following data:

$$A_2 = 80 \text{ square inches;}$$
$$A_3 = 48 \text{ square inches;}$$
$$r_2 = 11 \text{ inches (see Fig. 126);}$$
$$r_3 = 40 \text{ inches (see Fig. 126).}$$

By equation (157)

$$t_2 = \frac{80}{2\pi 11} = 1.157 \text{ inches;}$$

$$t_3 = \frac{48}{2\pi 40} = .1909 \text{ inch.}$$

These are the values of t_2 and t_3, according to Fig. 126, for the web whose moment of inertia is the same as that of all the arms. The corresponding values of R and T are:

$$R = \frac{40 \times 1.157 - 11 \times .1909}{1.157 - .1909} = 45.73 \text{ inches;}$$

$$T = \frac{1.157 \times 45.73}{45.73 - 11} = 1.524 \text{ inches.}$$

The moment of inertia of all the arms is, therefore, by equation (152), using the values for the equivalent web,

$$
\begin{aligned}
I_{arms} = I_{web} &= \frac{2\pi 450 \times 1.524}{32.2 \times 12.}\left[\frac{(40)^4 - (11)^4}{4(12)^4} - \frac{(40)^5 - (11)^5}{5 \times 45.73(12)^4}\right] \\
&= 11.151\left[\frac{2545359}{82944} - \frac{102238949}{4741300}\right] \\
&= 11.151(30.688 - 21.566) \\
&= 11.151 \times 9.122 \\
&= 101.7.
\end{aligned}
$$

It is not generally convenient to obtain the areas of the sheared ends of the arms, since these ends are curved surfaces which are portions of cylindrical surfaces of radii equal to the distances from the centre of the pulley to the outside of the' hub and inside of the rim. In any ordinary design the error is inappreciable when the area taken is that of a plane section of the arm at right angles to its length.

Even if the arms taper uniformly from end to end, and the areas of the curved sheared surfaces are taken, there is a slight error in this method ; for if the arms were flattened out so that each particle remains at its original distance from the centre of the wheel, the web thus formed will not have straight lines for its sides, as shown in Fig. 126, but the sides will be slightly concave.

The greater the width of the arm in the direction of the wheel's rotation, the greater the curvature of the side of the web.   In ordinary wheel designs, however, this curvature is so slight as to be negligible.

The two errors, one due to taking plane cross-sections of the arm, and the other to assuming that the equivalent web has straight lines bounding its axial section, are compensative, instead of accumulative.

121. Stresses in fly-wheels with arms.—The stresses which occur in a fly-wheel of the ordinary design when in operation are so complicated that it is not believed they can be computed with even a practical degree of accuracy.   The general nature of the stresses may be shown in such a manner, however, as to be a guide to the designer when considering the methods of reducing them with a view to decreasing the liability of the wheel to rupture on account of excessive speed or sudden variation of speed.

Fig. 127 is a portion of a built-up pulley.   The hub is complete, but the ends of sections of the rim are not fastened.   The pulley is represented as being under stress applied by a weight W hanging from the end of an arm attached to the hub or shaft which supports the wheel.   The wheel is prevented from rotating by cords or tension-bars attached to pins at A, B, and C, extending out from both sides of the rim and having a cord attached to each end, the cords all having the same tension and pulling at right angles to the arms.

Each arm acts as a cantilever to resist the bending action of the cords pulling on the pin near its end, and is accordingly bent or deflected as a cantilever.   This bending makes the centre line of
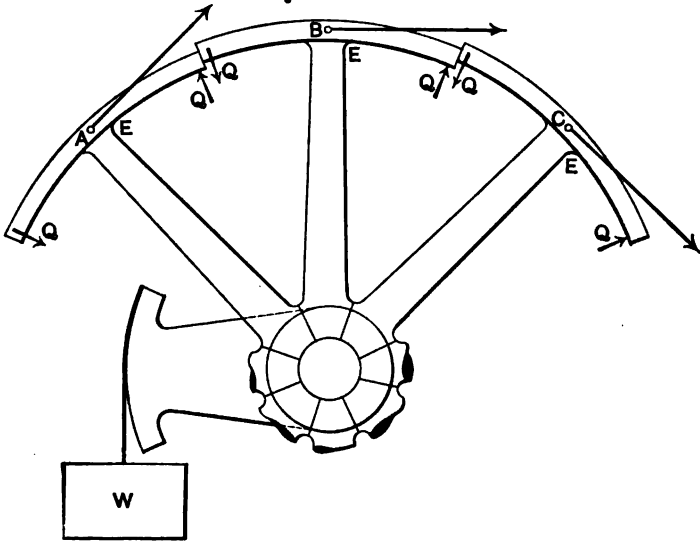
Fig. 127

each arm convex on the side opposite that toward which the cords are pulling, assuming that the centre lines were straight before stress was applied, and the sections of the rims are consequently tipped forward so that their ends are slipped over each other as shown.

Now suppose that, while the arms are still under stress due to the hanging weight W, the ends of the rim-sections are brought into line by radial forces Q applied near the ends of each section as shown.   This will cause stress in the rim and arms.   In the neighborhood of the angles E where the arms join the rim, the material will be in tension, whose maximum value will occur near E.   The maximum value of this stress may occur in the arm or rim, according to their relative strengths.

The material of each arm is in tension near the hub on the side opposite E on account of the pull of the cords.

The bending of the arm shortens the radial distance from the

centre of the pulley to the outer end of the arm ; hence, when all the sections of the entire pulley have their ends brought into line, the rim will be drawn in somewhat at the end of each arm, and, instead of being circular, will be slightly wavy around its circumference. The effort of the rim to assume its circular form, assuming that this is the form when not under stress, will cause tension in the arm; there will also be bending tensile stress in the rim near E on account of its being bent inward.

If, instead of having cords attached to the pins in the rim of the stationary fly-wheel, it were started or stopped from rotating by power applied through its shaft, the inertia of the parts would induce stresses similar to those caused by the pull of the cords. A belt running on the rim and transmitting power to or from the shaft has much the same effect.

In addition to the stresses corresponding to those caused by the pull of the cords on a stationary wheel, there are in a rotating wheel others due to the centrifugal tendency of the rotating parts. These stresses may be considered by dealing with the rim and arms separately.

When a ring, thin radially, of a mean radius r, rotates about its geometrical axis with a linear velocity of V feet per second, circumferential tension is induced in the material. The value of this stress p in pounds per square inch is expressed by the following equation, in which $(w \div 144)$ equals the weight of a piece of the material 1 inch square and 1 foot long:

$$p = \frac{w}{144g}v^2 = \frac{w}{144g}(\omega r)^2 = \frac{w}{144g}(2\pi Nr)^2. \quad . \quad . \quad (158)$$

Equation (158) shows that the tensile stress in a thin rotating ring is proportional to the square of the linear velocity v, and that, if V remains constant, the tensile stress is constant whatever the radius of the ring.

The radius of the ring is increased on account of this stress by an amount λ in feet given by equation (159), in which $E_t =$ tensile modulus of elasticity of the material:

$$\lambda = \frac{pr}{E_t}. \quad . \quad . \quad \quad . \quad . \quad . \quad (159)$$

If the ring is thick radially, say 1 foot thick for 20 feet of diameter, the mean radius $(r_s + r_i) \div 2$ can be used without an error great enough to require practical consideration.

For the arms equations of the same nature as (158) and (159) can be written. They are probably of no practical value, however, so they will be omitted. The general nature of the effect of rotation upon the arms may be expressed, however, by the statement that, if a thin, prismatic bar is rotated about an axis at one end and normal to the length of the bar, the maximum tensile stress in the bar is one half as great as in a ring having a radius equal to the length of the bar and rotating at the same angular velocity; the increase of the length of the bar is one third that of the increase of the radius of the ring.

Suppose that a ring of metal, corresponding to the rim of a fly-wheel, is placed on a horizontal table, and a "spider," corresponding to the hub and radial arms, is placed in the ring, the arms being of such a length as to just fit in the ring without pressure against it. If the table is rotated about a vertical axis coincident with that of the ring and spider, carrying these parts with it, the ring, on account of the centrifugal action, will increase in diameter more rapidly than the arms will increase in length, and will therefore separate from them, leaving a space between the end of each arm and the ring. If, on the contrary, the arms are attached to the ring so that no separation can occur, the arms will be somewhat elongated on account of the pulling-out action of the ring, and the ring will be bent inward where the arms are attached. Tensile stress will therefore be induced in the arms and in the inner side of the ring at and near the arms. The relative intensity of the stress in the arms and ring will depend on their relative strength.

The pulling-in action of the arms, when there are several, reduces the tension in the rim caused by centrifugal action.

As a summary of the above, it may be stated that, if a fly-wheel which has no stress in its parts when at rest is rotated uniformly about its axis, tensile stress will be induced in the arms and rim, that in the rim having the greatest intensity midway between the arms and at or near the arms; also, if the speed of rotation is varied by a driving or resisting force applied through the shaft or hub, there will be additional tensile stress induced about one of the angles

between the end of each arm and the rim. A pulley carrying a belt that is transmitting power has similar stresses.

Only tensile stresses have been considered in the discussion of the fly-wheel, because they are greater than those of compression, and also because most fly-wheels, and a considerable proportion of large pulleys, are made of cast iron, which has a much lower tensile than compressive strength.

When a fly-wheel is supported on a horizontal shaft, there is an additional tension in the lower arms. The tension in an arm extending vertically downward from the hub is much greater than that due to the portion of the rim belonging to the arm, i.e., greater than that due to a weight equal to that of the whole rim divided by the number of arms.

122. Numerical example of stress in, and enlargement of, a rotating ring due to centrifugal action.—The tensile stress in a thin cast-iron ring of any diameter, rotating about its geometrical axis at a circumferential velocity of 6000 feet per minute, is, by equation (158), taking $w = 450$,

$$p = \frac{w}{144g} V^2 = \frac{450}{144 \times 32.2} \times \left(\frac{6000}{60}\right)^2 = 970 \text{ lbs. per sq. in.}$$

The increase of the radius caused by the rotation is for a ring 20 ft. in diameter, by equation (159), taking $E_t = 14000000$ lbs. per sq. in.,

$$\lambda = \frac{970 \times 10}{14000000} = .000693 \text{ ft.} = .0083 \text{ in.}$$

The corresponding increase in the circumference is

$$2\pi \times .0083 = .052 \text{ in.}$$

The arms in a fly-wheel of ordinary design are considerably smaller near the rim than at the hub. The elongation is therefore not uniformly distributed along an arm, but is greatest at or near its smallest cross-section. For a rotating wheel having a heavy, stiff rim, it seems fair to assume that the greatest elongation per unit length in the arm is at least as great as would be obtained per unit length in a prismatic bar of a total length equal to the diameter of the rim, by subjecting it to a tensile stress sufficient to elongate it

to the same extent as the diameter of the free rim would be increased by the rotation.

Upon this assumption the tension in each arm supporting the ring just considered, taking the arm as 83 inches long and allowing for no distortion of the hub, would be

$$\text{Tension in arm} = \frac{.0083}{83} \times 14000000 = 1400 \text{ lbs. per sq. in.}$$

The rim-tension of 970 lbs. per sq. in., and the tension of 1400 lbs. per sq. in. in the arms are of comparatively small value when considered separate from stresses due to other causes. But, since the rim- and arm-tension increase as the square of the velocity, they may attain considerable values for very high speeds, such as sometimes occur when an engine "races."

123. Sectional-rim fly-wheels and pulleys.—When the rim of a fly-wheel or pulley is divided into sections by being cut through midway between each pair of arms, as in Fig. 127, the ends of the sections must be held together, of course, by some kind of fastening. As has already been stated, centrifugal action tends to bend or bow the rim out between the arms. If the wheel is rotating uniformly and has no belt upon it, each part of the rim between two arms acts much like a beam supported at the ends and uniformly loaded from end to end. The centrifugal action in the wheel corresponds to a uniformly distributed load acting radially outward. This bending outward of the rim has a tendency to open up the joint more at the outer than the inner side of the rim. For this reason the fastenings, which are tension-members, should be placed as near as possible to the face or outer circumference of the rim, and should have a considerable thickness of metal, measured radially, between them and the inner circumference of the rim.

If the rim is thin, as for band-wheels, lugs or flanges must generally be added for the fastenings. These lugs should be of sufficient radial height to make the distance from the fastenings to the inner side of the lug large in comparison with the distance from the centre of the fastenings to the face of the pulley, and they should be braced well with circumferential ribs running back towards the arms. In order to give the rim rigidity to resist bending, these ribs should also have a considerable radial depth.

In recent years a number of large pulleys have been designed with sectional rims divided at the end of each arm, there being no joint in the rim between the arms.

124. Bursting tests of small cast-iron fly-wheels by centrifugal action.—A number of tests were made upon small cast-iron fly-wheels by Prof. C. H. Benjamin in order to determine the speed at which they would burst when rotating with practical uniformity.* The wheels tested were all of cast iron, and most of them were miniatures of large fly-wheels designed by leading manufacturers of such machinery. Some of the miniatures were 15 inches, and the others 24 inches, in diameter. Part had solid rims, others had rims made in two sections and fastened together with bolts, while still others had rims in two sections fastened together with links. The wheels were direct-connected to a steam-turbine, which drove them at a gradually increasing speed until they flew to pieces. No brake or pulley was applied to cause tangential resistance to rotation, and no belt was run on the wheel.

The tests showed that, as would naturally be expected, the wheels having sectional rims fastened together midway between two arms were much weaker than those having solid rims ; and that, although the tension-members of the fastenings at the ends of the sections were of less tensile strength than the rim itself, they gave way in but one instance, in which case the bolts broke; the rupture occurred in the jointed rims of all the other wheels.

It was also shown that a wheel cast from the same pattern as another, but having its rim turned down so as to be much thinner radially, broke at a much lower speed than the one having the rim left thick, as it had come from the mould.

Tests upon wheels similar in every respect except the number of arms, some having three and some six, showed that the three-arm pulleys were weaker than those having six. The pattern for the three-arm wheels was made by removing the alternate arms from the patterns for the six-arm wheels.

Tables XXXV to XLI show the dimensions of the fly-wheels tested and the results of the tests. When bolts were used to fasten the joints, flanges, extending across the inner surface of the rim,

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* The Bursting of Small Cast-iron Fly-wheels. Trans. Amer. Soc. Mech. Eng., vol. xx., 1899.

furnished holes for the bolts. When two links were used, they were imbedded in the side of the rim, after the method of Fig. 128. The third link, when used, gripped a pair of lugs extending out from the inner surface of the rim.

TABLE XXXV.

FIFTEEN-INCH WHEELS.

No.	Rim.					Arms.		Weight of Wheel, pounds.
	Style.	Diameter, inches.	Breadth, inches.	Depth, inches.	Area, sq. in.	No.	Area, sq. in.	
1	Solid	15¼	2	.70	1.4	6	.46	20.87
2	"	15⅛	2	.65	1.3	6	.46	20.44
3	"	15	2	.615	1.23	6	.46	19.12
4	"	14¹¹⅛	2	.52	1.04	6	.46	16.62
5	Sectional	15₁	....	....	....	6	.46	20.87
6	Solid	15₁	2	.69	1.38	8	.46	19.25
7	"	15	2	.615	1.23	3	.46	16.56
8	"	14¼	2	.475	.95	3	.46	13.68
9	"	14¼	1⅞	.400	.75	6	.46	12.68
10	"	14¼	1⅛	.347	.65	6	.46	18.00

TABLE XXXVI.

FIFTEEN-INCH WHEELS.

No.	Bursting Speed.		Centrifugal Tension $= \dfrac{V^2}{10}$.	Remarks.
	Revolutions per Minute.	Feet per Second $= V$.		
1	6,525	430	18,500	
2	6,525	430	18,500	
3	6,035	395	15,600	Thin rim.
4	5,872	380	14,400	" "
5	2,925	192	3,700	Joint.
6	5,600*	368	13,600	Three arms.
7	6,198	406	16,500	" "
8	5,709	368	13,600	" "
9	5,709	365	13,300	Thin rim.
10	5,709	361	13,000	" "

* Doubtful.

TABLE XXXVII.

TWENTY-FOUR-INCH WHEELS.

No.	Shape and Size of Rim.					Weight of Wheel, pounds.
	Diameter, inches.	Breadth, inches.	Depth, inches.	Area, sq. in.	Style of Joint.	
11	24	2⅛	1.5	3.18	Solid rim.	75.25
12	24	4₁₁₆	.75	3.85	Internal flanges, bolted.	93
13	24	4	.75	3.85	" " "	91.75
14	24	4	.75	3.85	" " "	95
15	24	4₁₁₆	.75	3.85	" " "	94.75
16	24	1.2	2.1	2.45	Three lugs and links.	65.1
17	24	1.2	2.1	2.45	Two " " "	65

TABLE XXXVIII.

FLANGES AND BOLTS.

No.	Flanges.			Bolts.		
	Thickness, inches.	Effective Breadth, inches.	Effective Area, inches.	No. to Each Joint.	Diameter, inches.	Total Tensile Strength, pounds.
12	11/16	2.8	1.92	4	5/16	16,000
13	11/16	2.75	1.89	4	5/16	16,000
14	15/16	2.75	2.58	4	5/16	16,000
15	15/16	2.5	2.34	4	3/8	20,000

BY TESTING-MACHINE.

Tensile strength of cast iron   = 19,600 pounds per square inch.
Transverse strength of cast iron = 46,600 pounds per square inch.
Tensile strength of 5/16 bolts   = 4,000 pounds.
Tensile strength of 3/8 bolts   = 5,000 pounds.

TABLE XXXIX.

FAILURE OF FLANGED JOINTS.

No.	Area of Rim, sq. in.	Effective Area Flanges, sq. in.	Total Strength Bolts, pounds.	Bursting Speed.		Centrifugal Tension		Remarks.
				Rev. per Minute.	Ft. per Second = V.	Pounds per Sq. In. $= \dfrac{V^2}{10}$.	Total Pounds.	
11	3.18	....	......	8,672	885	14,800	47,000	Solid rim.
12	3.85	1.92	16,000	.....	....	......	......	Flange broke.
13	3.85	1.89	16,000	1,760	184	3,400	13,100	" "
14	3.85	2.58	16,000	1,875	196	3,850	14,800	Bolts broke.
15	3.85	2.34	20,000	1,810	190	3,610	13,900	Flange broke.

TABLE XL.

LINKED JOINTS.

No.	Lugs.			Links.				Rim.	
	Breadth, in.	Length, in.	Area, sq. in.	Number Used.	Effective Breadth, in.	Thick-ness, in.	Effective Area, sq. in.	Max. Area, sq. in.	Net Area, sq. in.
16	.45	1.0	.45	3	.57	.327	.186	2.45	1.98
17	.44	.98	.43	2	.54	.380	.205	2.45	1.98

<div align="center">

BY TESTING-MACHINE.

Tensile strength of cast iron $= 19,600$.
Transverse strength of cast iron $= 40,400$.
Av. tensile strength of each link $= 10,180$.

</div>

TABLE XLI.

FAILURE OF LINKED JOINTS.

No.	Strength of Links, pounds.	Strength of Rim, pounds.	Bursting Speed.		Centrifugal Tension.		Remarks.
			Rev. per Minute.	Ft. per Second = V.	Pounds per sq. In. $= \dfrac{V^2}{10}$.	Total, Pounds	
16	30,540	38,800	8,060	320	10,240	25,100	Rim broke.
17	20,360	38,800	2,750	290	8,410	20,600	Lugs and rim broke.

124.1. Effect of ribs on pulley rims.—In the use of circumferential ribs on the rims of pulleys care should be taken to see that they do not weaken the rim instead of strengthening it. They will generally stiffen the rim whether it is made stronger or weaker. In Table XLIA three cross-sections of equal area are shown. The section modulus or resistance factor of each is given for both steel with its tensile and compressive strengths equal, and for cast iron four times as strong in compression as in tension.

With the steel the section modulus of a given form of section is the same with the ribs in tension as in compression. This is not true of the cast iron for a section ribbed on one side only.

Deep ribs should be used in steel and cast-iron sections whose ribs are in tension. In cast-iron sections whose ribs are in compression the ribbed section is always stronger than the rectangular.

124.2. Problem.—Find the speed of rotation of a steel armature ring with 24 lugs of 100 pounds each, attached as shown in the Fig. 127.1, which will cause a tensile stress of 14,500 pounds per square inch in the ring, and 12,000 pounds per square inch in the lug-bolts.

The following notation will be used in addition to that in § 117 :

C = centrifugal force of one lug, pounds;

$P = P + P'$ = total tension in rotating armature ring due to the speed and weight of both itself and the attached lugs, pounds;

P = total tension in the rotating armature ring due to its own speed and weight, pounds;

P' = total tension, due to the speed and weight of the lugs, in the armature ring, pounds;

a = effective area of bolts to resist tension, square inches;

d = diameter of body of bolts for attaching the lugs to the armature ring, inches;

p = tension, due to its own speed and weight, in a rotating ring of one square inch section, pounds.

If v is taken as the linear velocity of the ring at its mean radius of 52.5 inches, then the linear velocity of the middle of the lugs, which lie in a circle of 57 inches radius, will be

$$\frac{57}{52.5} \times v = 1.086v.$$

TABLE XLIA.

STRENGTH FACTORS OF DIFFERENT SECTIONS OF EQUAL AREA.

FORM OF SECTION. DIMENSIONS IN INCHES. — NEUTRAL AXIS.	AREA OF SECTION SQ. INCHES.	MOMENT OF INERTIA			SECTION MODULUS OR RESISTANCE FACTOR OF SECTION.		
					STEEL COMP. STRENGTH=TENSILE STRENGTH. RIBS IN TENS. OR COMP.	CAST IRON COMP. STRENGTH = 4 × TENS. STRENGTH	
			Y	Z		RIBS IN TENSION	RIBS IN COMPRESSION
(section ⅛, 2½, 30)	65	38.8	1¼	1¼	27.1	27.1	27.1
(section 1⅛, 2¼, 30, 2)	65	58.2	1.387	2.918	18.8	18.8	39.8
(section 1⅜, 2, 30, 6, 4)	65	128.8	1.6	4.4	29.2	29.2	80.2

The force necessary to prevent a lug weighing 100 pounds from flying off the armature ring, when the ring rotates at a linear velocity of v feet per second, is expressed by the equation

$$C = \frac{100(1.086v)^2}{g \times 57 \div 12} = \frac{100 \times 1.1794v^2 \times 12}{32.2 \times 57} = .771v^2 \text{ pounds.}$$

Fig. 127.1.
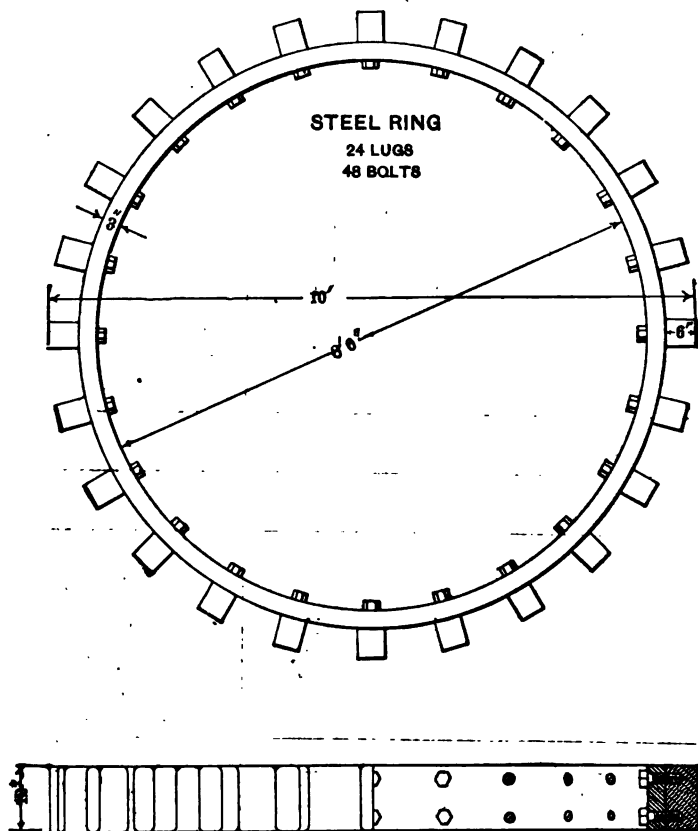
The effective sectional area of the bolt that holds the lugs in place is that at the bottom of the threads. Since the diameter of the bolt is not yet determined, the exact ratio of the diameters of

the body and of the root of the thread is not known. It is on the safe side to assume that the body is 1.18 times as large in diameter as the root of the thread. This is a somewhat larger ratio than really exists for bolts from 1.75″ to 3″ diameter of body.

Under this assumption a bolt in tension, and of a body diameter d, has an effective area of bolt

$$a = \frac{\pi d^2}{4(1.18)^2}.$$

Two bolts are to be used for each lug. Their combined tensile strength must equal the centrifugal force of one lug, whose value has just been determined as $.771v^2$.

The equation of the bolt area and the centrifugal force of the lug is therefore, for two bolts in each lug, at a tensile stress of 12,000 pounds per square inch,

$$\frac{12000 \times 2\pi d^2}{4 \times (1.18)^2} = 771v^2;$$

whence

$$d = 1.18v \times \sqrt{\frac{2 \times .771}{12000\pi}} = .007547v \text{ inches.}$$

The tension per square inch in the ring due to its rotation, taking the weight of steel as 490 pounds per cubic foot, is, for its own mass,

$$p = \frac{wv^2}{144g} = \frac{490v^2}{144 \times 32.2} = .1057v^2 \text{ pounds per square inch.}$$

The given cross-section of the ring is

$$3'' \times 12'' = 36 \text{ square inches.}$$

The total tension in the ring due to its own weight and speed is therefore

$$P = 36 \times .1057v^2 = 3.804v^2 \text{ pounds.}$$

The tension in the ring due to the weight and speed of the attached lugs may be found by a method analogous to that commonly used for determining the circumferential tension in cylindrical boiler shells, tubing, pipes, etc., due to the internal pressure of an enclosed fluid or gas. The relation between the circumferential tension and the internal pressure is

$$\text{Total circumferential tension in tube} = \frac{\text{press. per unit area} \times \text{diam.} \times \text{length}}{2}$$

$$= \frac{\text{total radial pressure}}{2\pi}.$$

Adapting this to the armature, the centrifugal force of the 24 lugs corresponds to the total internal pressure of the tube. The centrifugal force for one lug of 100 pounds has been found to be $.771v^2$ pounds. The tension in the ring due to the speed and weight of the 24 lugs is therefore

$$P' = \frac{24 \times .771v^2}{2\pi} = 2.95v^2.$$

The total tension in the armature ring is the sum of the tensions P and P'.

The tension per unit area in the ring is found by dividing this amount by the effective area. The least effective area of the armature ring is on a section through the centre lines of a pair of bolt holes. The solid area of the ring is here reduced by an amount equal to the thickness of the ring times twice the diameter of the bolt. Therefore, for a fibre stress of 14,500 pounds per square inch, the following equation can be written:

$$14,500 = \frac{P + P'}{\text{effective area of armature ring}}$$

$$= \frac{3.804v^2 + 2.95v^2}{36 - (3 \times 2 \times .007547v)}.$$

$$v = 233.9 \text{ feet per second}$$

and

$$V = 14030 \text{ feet per minute,}$$

which corresponds to a

$$\text{Speed of rotation} = \frac{14030}{9\pi} = 496.2 \text{ R.P.M.}$$

The outer diameter of the armature ring is here used in order to roughly counterbalance the error introduced by using the mean diameter of the ring and the diameter of a circle through the middles of the lugs, instead of slightly larger diameters, in the calculations for tension due to rotation.

The diameter of the bolts may now be obtained by introducing the value of v just determined into the equation for d developed earlier in the problem; thus

$$d = .007547v = .007547 \times 233.9 = 1.766 \text{ inches.}$$

A diameter d of 1.75 inches would naturally be used in practice.

The values of the tensions in the ring and bolts may now be determined as a check, using 1.75-inch bolts.

$$\left. \begin{array}{l} \text{Net sectional} \\ \text{area of ring} \end{array} \right\} = 3 \times 12 - 3 \times 2 \times 1.75 = 36 - 10.5 = 25.5 \text{ square ins.}$$

$$\left. \begin{array}{l} \text{Tension in} \\ \text{ring} \end{array} \right\} = \frac{P + P'}{\text{net section}} = \frac{6.750v^2}{25.5}$$

$$= \frac{6.750 \times (233.9)^2}{25.5} = 14480 \text{ pounds per square inch.}$$

A bolt of 1.75″ body has, according to tables of standard bolts, an

.   Area at root of thread $= 1.746$ square inches.

Therefore

$$\left. \begin{array}{l} \text{Tension} \\ \text{in bolts} \end{array} \right\} = \frac{.771v^2}{2 \times 1.746} = \frac{.771 \times (233.9)^2}{2 \times 1.746} = 12080 \text{ pounds per sq. in.}$$

There are, of course, bending stresses in the ring, due to the localization of centrifugal force at the points where the lugs are attached. The arms of the spider supporting the ring are also a cause of bending stresses.

At any other speed of rotation the tensions in the ring and bolts are proportional to the square of the velocity.

Special Forms of Fly-wheels and Pulleys.

125. Hollow cast-iron arms with wrought-iron or steel tension-rods for fly-wheels and pulleys.—On account of the comparatively low tensile strength of cast iron and the predominance of tensile stress in the arms of pulleys and fly-wheels, it is obvious that the liability of the arms to fracture will be reduced if, by some means, the cast iron can be relieved partly or wholly of tensile stress, although by doing so it may be subjected to high compressive stress. A simple and comparatively inexpensive way of doing this is to cast the arms hollow and put a steel or wrought-iron tension-rod through the centre of each from rim to hub. The rod is put in tension by means of its end fastenings when the pulley is constructed. The surrounding cast-iron walls act as a column to resist this tension and are thereby subjected to compression. A pulley so constructed is unquestionably much safer under heavy service than one having only cast-iron arms. This device has been adopted by at least one concern which has placed many large pulleys in commercial service.

126. "Tangent" arms for pulleys and fly-wheels.—Pulleys of this description have arms which, instead of being radial, are tangent to a circle of fixed diameter, as in the construction common to bicycle-wheels. Tangent-arms have been adopted for wheels of considerable size performing service where the demands upon the fly-wheel for energy are sudden and severe. Such fly-wheels have been applied to mine hoisting-machinery. The arms, instead of being round, are generally of rectangular cross-section. Being purely tension-members, they are of course made of some such material as wrought iron or steel, cast iron being totally unfit for this purpose. The tangential direction of the arms prevents any necessity of their resisting the torsional moment of the shaft or rim by a bending moment in the arm. It is not probable that even a very approximate calculation of the stresses in the arms can be arrived at.

127. "Built-up" plate fly-wheel.—A fly-wheel made of rolled structural-steel plates is in use in the power-station of the Union Railroad Co. at Providence, R. I. The web of the wheel is made of a number of segmental plates placed so as to break joints, and held together by rivets passing through from side to side. The rim is built up of several pieces of plate metal cut so as to form a ring

when placed together. These sections are placed together so as to break joints and are riveted through from side to side. Steel plate 1 inch thick is used for the web, and 1¾ and 1½ inches thick for the rim. The hub is of cast iron 72 inches in diameter, made of two disks. It can be seen that the pulley has what might be called a built-up web. The pulley is 18 feet in diameter and has a rim 15¼×16 inches. The web is made of two thicknesses of one-inch plate. The factor of safety is about forty.

128. Wire-wound fly-wheel.—A fly-wheel having a rim composed of wire wound circumferentially under tension around a web was constructed for use in a rolling-mill at Ladore, Wales, using the Mannesmann process of rolling tubing from the solid bar. In this class of rolling a very great amount of power must be supplied to the machinery in a short time. The strain upon the fly-wheel is accordingly very great on account of the sudden reduction of speed which it must undergo in order to deliver its energy, as well as on account of the high speed at which it runs in order to have sufficient energy stored in it.

The construction adopted is the strongest and safest that can be devised. Two steel disks, 20 feet in diameter, are bolted to a cast-iron hub. The outer edges of the disks form a groove into which wire is wound to form the heavy rim. The groove is filled with 70 tons of No. 5 steel wire wound on under a tension of 50 pounds. The wheel is run at 240 revolutions per minute, which corresponds to a circumferential velocity of 15080 feet=2.85 miles per minute.

129. Other special forms of pulleys.—Many large pulleys are now constructed with built-up rims and cast-iron arms. The rims are frequently made heavy enough to serve as fly-wheels.

Smaller pulleys are often made completely of wood built up in sections and having but a few arms, generally two or four.

Medium-size pulleys are often made with wrought-iron or sheet-steel rims and cast-iron arms. Small-size pulleys are also frequently constructed in this way.

An "all-steel" pulley has recently been placed on the market. It is made up completely of sheet metal, the arms and hub being pressed into form.

Two sets of arms are very commonly used for wide pulleys, and in some cases as many as three or more sets are used.

TABLE XLII.*

DIMENSIONS OF STANDARD ENGINE-PULLEY ARMS OF ELLIPTICAL CROSS-SECTION. (SIX ARMS IN EACH PULLEY.)

For very wide faces two sets of arms should be used. The diameter of hub is taken as twice that of the shaft on which it fits.

Diameter of Pulley, inches.	Light Pattern, Size of Arms.				Medium Pattern, Size of Arms.				Heavy Pattern of Fly-wheels, Pulleys with Thick Rims, and Pulleys with Faces up to 26 inches Wide—Size of Arms.			
	At Rim.		At Hub.		At Rim.		At Hub.		At Rim.		At Hub.	
	Dimens., inches.	Area, sq. in.	Dimens., inches.	Area, sq. in.	Dimens., inches.	Area, sq. in.	Dimens., inches.	Area, sq. in.	Dimens., inches.	Area, sq. in.	Dimens., inches.	Area, sq. in.
10	2 × 1	1.57	2¼ × 1¼	2.2	2 × 1	1.57	2¼ × 1¼	2.2				
12	2 × 1	1.57	2¼ × 1¼	2.2	2 × 1	1.57	2¼ × 1¼	2.2				
14	2 × 1	1.57	2¼ × 1¼	2.2	2 × 1	1.57	2¼ × 1¼	2.2				
16	2 × 1	1.57	2¼ × 1¼	2.2	2 × 1	1.57	2¼ × 1¼	2.2				
18	2 × 1	1.57	2¼ × 1¼	2.2	2 × 1	1.57	2¼ × 1¼	2.2				
20	1¼ × ⅞	.858	2 × 1	1.57	2⅛ × 1¼	2.08	2⅜ × 1⅜	2.56				
22	1¼ × ⅞	.858	2 × 1	1.57	2⅛ × 1¼	2.08	2⅜ × 1⅜	2.56				
24	1¼ × ⅞	.858	2 × 1	1.57	2⅛ × 1¼	2.08	2⅜ × 1⅜	2.56				
26	1¼ × ⅞	.858	2 × 1	1.57	2¼ × 1¼	2.20	2½ × 1½	2.94				
28	1¼ × ⅞	.858	2 × 1	1.57	2⅜ × 1¼	2.32	2⅞ × 1½	3.38				
30	2 × 1⅛	1.767	2½ × 1⅜	2.69	2⅜ × 1¼	2.32	2⅞ × 1½	3.38				
32	2 × 1⅛	1.767	2½ × 1⅜	2.69	2½ × 1½	2.94	3 × 1½	3.53				
34	2 × 1⅛	1.767	2½ × 1⅜	2.69	2⅞ × 1⅜	3.10	3¼ × 1½	3.82	3¼ × 1½	3.82	3¾ × 2	5.89
36	2¼ × 1⅛	1.98	2¾ × 1⅜	2.96	3 × 1½	3.53	3¼ × 1⅝	4.14	3¼ × 1⅝	4.14	3¾ × 2	5.89
38	2¼ × 1⅛	1.98	2¾ × 1⅜	2.96	3 × 1½	3.53	3¼ × 1⅝	4.14	3¼ × 1⅝	4.14	3¾ × 2	5.89
40	2¼ × 1⅛	1.98	2¾ × 1⅜	2.96	3 × 1½	3.53	3¼ × 1⅝	4.14	3¼ × 1⅝	4.14	3¾ × 2	5.89
42	2⅜ × 1¼	2.32	2⅞ × 1½	3.38	3¼ × 1½	3.82	3¾ × 1¾	5.15	3½ × 1¾	4.81	4 × 2⅛	6.67
44	2⅜ × 1¼	2.32	2⅞ × 1½	3.38	3¼ × 1½	3.82	3¾ × 1¾	5.15	3½ × 1¾	4.81	4 × 2⅛	6.67
46	2⅜ × 1¼	2.32	2⅞ × 1½	3.38	3¼ × 1½	3.82	3¾ × 1¾	5.15	3½ × 1¾	4.81	4 × 2⅛	6.67
48	2¾ × 1⅜	2.96	3¼ × 1⅝	4.14	3⅝ × 1½	4.27	4¼ × 1¾	5.84	3¾ × 2	5.89	4½ × 2¼	7.94
50	2¾ × 1⅜	2.96	3¼ × 1⅝	4.14	3⅝ × 1½	4.27	4¼ × 1¾	5.84	3¾ × 2	5.89	4½ × 2¼	7.94
54	3⅝ × 1¾	4.97	4¼ × 2	6.67	4 × 1⅝	5.10	4¾ × 1⅞	6.99	4¼ × 2⅛	7.09	5 × 2½	9.79
58	3⅝ × 1¾	4.97	4¼ × 2	6.67	4 × 1⅝	5.10	4¾ × 1⅞	6.99	4¼ × 2⅛	7.09	5 × 2½	9.79
60	3¾ × 2	5.89	4¾ × 2⅜	8.85	4¼ × 2	6.67	5 × 2⅝	10.3	4⅜ × 2¼	7.73	5⅜ × 2¾	11.60
66	3⅞ × 2⅛	6.46	4⅞ × 2½	9.56	4¼ × 2	6.67	5 × 2⅝	10.3	4⅜ × 2¼	7.73	5⅜ × 2¾	11.60
68	3⅞ × 2⅛	6.46	4⅞ × 2½	9.56	4¾ × 2⅜	8.85	5½ × 2⅞	13.4	4⅞ × 2½	9.56	5⅝ × 2⅞	12.66
72	4 × 2¼	7.06	5 × 2½	9.79	4¾ × 2⅜	8.85	5½ × 2⅞	13.4	5 × 2¾	10.8	5¾ × 3	13.54
78	4 × 2¼	7.06	5 × 2½	9.79	4¾ × 2⅜	8.85	5½ × 2⅞	13.4	5 × 2¾	10.8	5¾ × 3	13.54
84	4¼ × 2⅜	7.92	5¼ × 2⅝	10.82	4⅞ × 2½	9.56	5⅞ × 3	13.83	5 × 2¾	10.79	6 × 3¼	15.31
96	5 × 2¾	10.8	6 × 3	14.13	5½ × 2⅞	13.4	6½ × 3¼	16.58	7 × 3½	19.24	8½ × 4½	29.71

* Pulley and Fly-wheel Construction, by Theo. F. Scheffler, Jr., *Power*, April, 1898, p. 8.

Designs and Proportions of Fly-wheels and Pulleys Taken from Practice. *

130. Since it is evidently impossible to make calculations for fly-wheels and pulleys with regard to their strength, it seems advisable to present some of the larger sizes, as made by leading builders, which have shown themselves to be satisfactory in use; also to give some of the proportions of the smaller sizes of pulleys. Table XLII gives the sizes of arms for pulleys from 10 to 96 inches in diameter and having rims with faces up to 26 inches wide. These are the proportions adopted by Struthers, Wells & Co. for the pulleys used upon their engines. Table XLIII gives the thicknesses of rims adopted by the same company.

TABLE XLIII.
AVERAGE THICKNESS OF ENGINE-PULLEY RIMS.

Wide Pulleys.			Narrow Pulleys.		
Diameter, inches.	Face Width, inches.	Thickness of Rim, inches.	Diameter, inches.	Face Width, inches.	Thickness of Rim, inches.
12	8¼	5/16	6	3¼	1/4
18	4¼	3/8	8	4¼	5/16
20	6¼	7/16	10	5¼	3/8
28	8¼	1/2	14	7¼	7/16
36	9¼	9/16	16	10¼	7/16
44	10¼	1/2	20	10¼	7/16
44	12¼	9/16	32	10¼	1/2
48	12¼	9/16	32	10¼	5/8
54	12¼	11/16	36	10¼	5/8
60	14¼	3/4	36	12¼	7/8
60	14¼	15/16	42	12¼	7/8
72	16¼	7/8	48	12¼	1
72	16¼	15/16	48	14¼	1¼
72	16¼	1¼	54	14¼	1¼
78	16¼	1¼	54	16¼	1¼
78	18¼	1⅜	54	16¼	1½
78	20¼	1¼	60	16¼	1½
78	22¼	1⅜	72	18¼	1⅜
84	24¼	1¼	72	20¼	1¾
84	26¼	1½	78	22¼	2
96	31	1⅞	78	24¼	2
96	31	2			
108	34	2¼			
108	37	2¼			
120	42	2¾			
120	48	3			

* The following illustrations of fly-wheels and pulleys are taken from drawings and blue-prints kindly supplied by the establishments whose names are mentioned in connection with them.

Fig. 128 is a 20-foot band-wheel with a 62-inch face and having a rim divided into eight sections as made by the Edward P. Allis Co. The rim-sections meet at the ends of the arm. The weight is 65000 pounds.
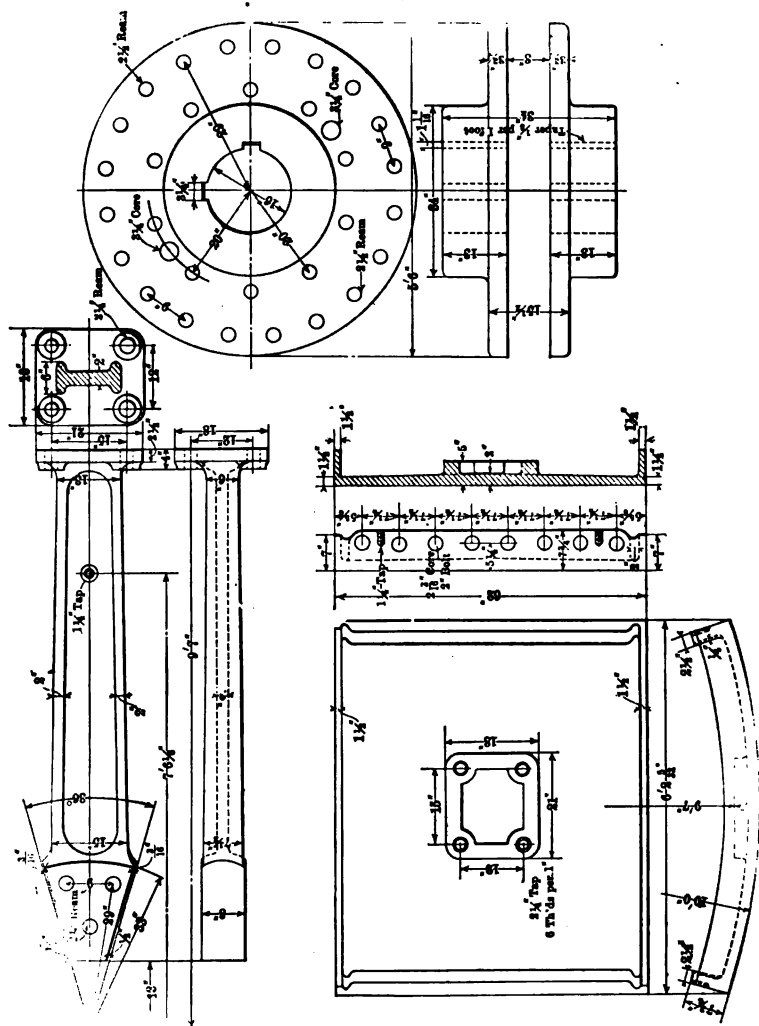
Fig. 129 is the working drawing of the hub, one arm, and one

Fig. 128.

20-foot band-wheel, 62-inch face. Cast iron. 8 rim sections. Weight

Various dimensions labeled on the engineering drawing:

22"
1⅝"
1⅝"
1⅝"
Chord 7'7"
2⅛ Bolt
C 18" D
11"
8'-10"
3'0" Rad.
A 18" B
15"
27"
36
Q 2½"
Q 2½"
1¼"
1¼"
Face

To face page 368.

,000 pounds. Edward P. Allis Co., Milwaukee, Wis.
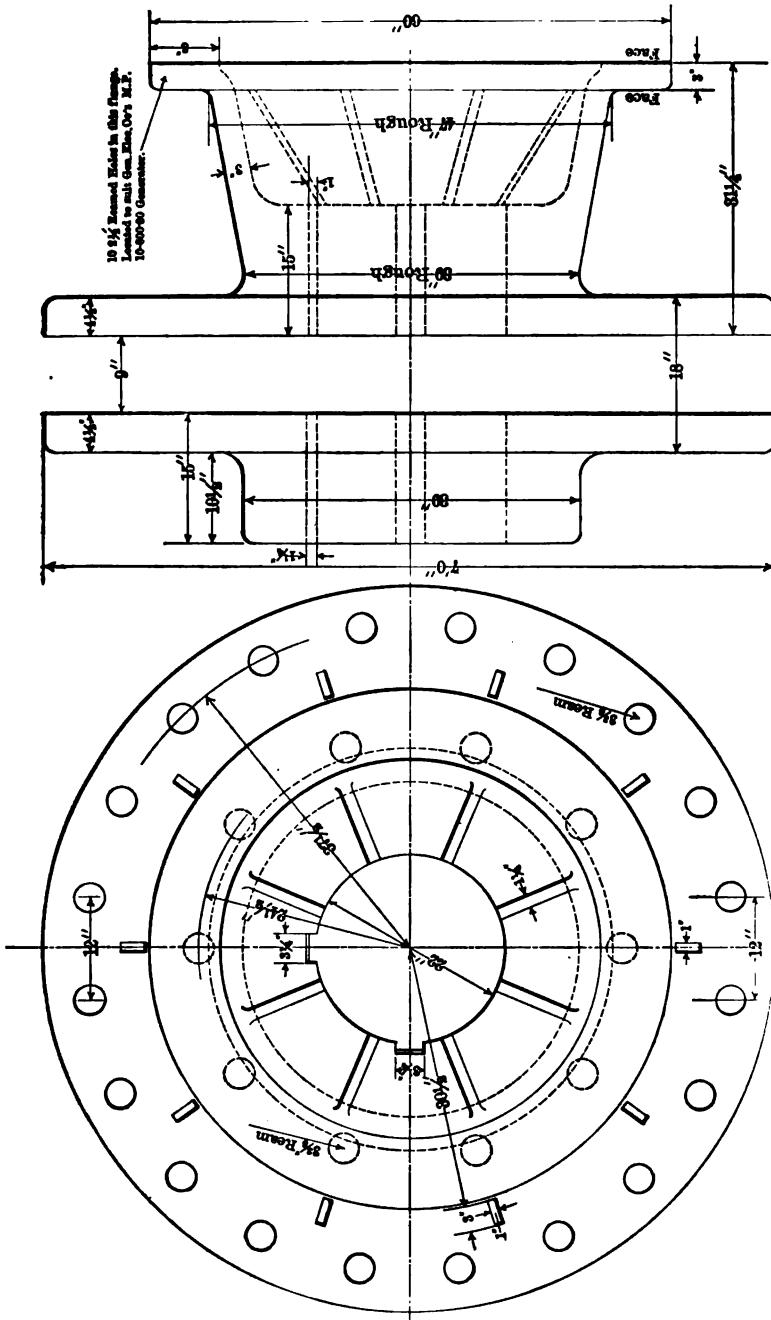
FIG. 180.

Hub for 28' 6" Fly-wheel with barring-holes. Cast iron. 10 sections. Weight 128,421 pounds. Hoover, Owens & Rentschler Co. Hamilton, Ohio, 1896

section of the rim of a 10-foot band-wheel having a 62-inch face
and weighing 60000 pounds as made by the Hooven, Owens &

FIG. 181.
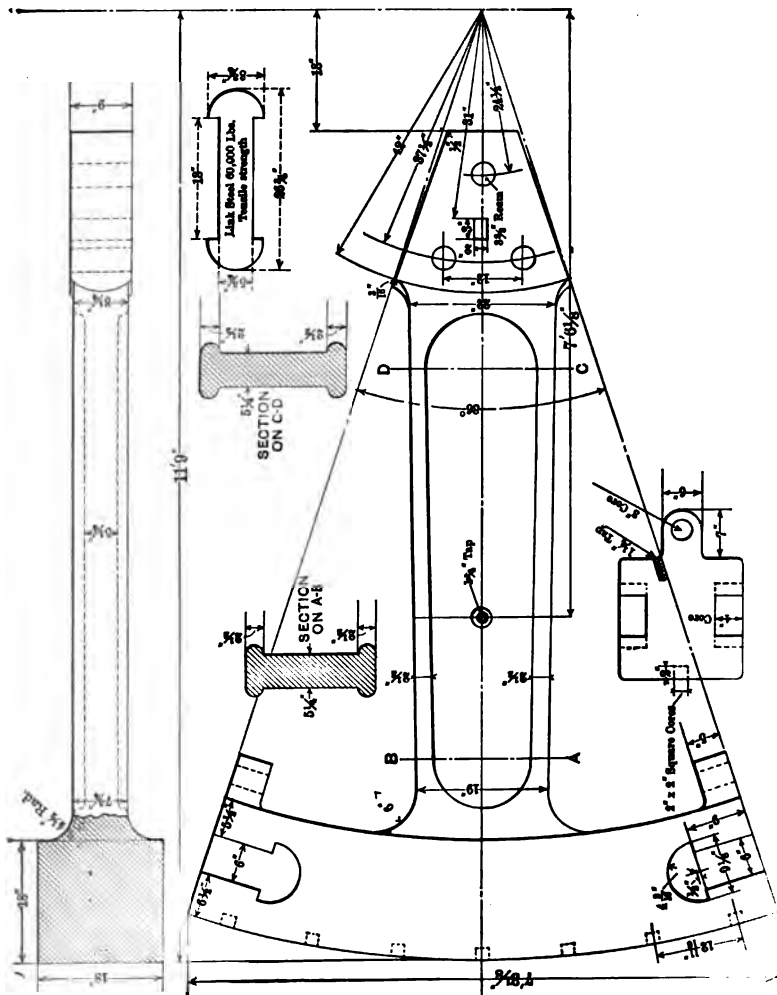
Arm and 1 section of rim for 28′ 6″ fly-wheel with barring-holes. Cast iron. 10 sections. Weight 138,491 pounds. Hooven, Owens & Rentschler Co. Hamilton, Ohio.

Rentschler Co.   The rim is divided into ten sections, each section
having one arm.   The arm in this design is different from those

LINK, SWEDISH IRON.

8'6"
2'8"
10"
7½"

24"

18"

25'-0" Dia.

3½ BOLT FITTED IN REAMED HOLE.
TAPER 1/16 IN DIA. PER FOOT.
WROUGHT-IRON BOLT.

19
10"
21

48"

7'-6"
4'-2"
27 15/16"
5'-1"
0'-9"
4'-2"

2½ R.
SEC

2¾ R.
SE
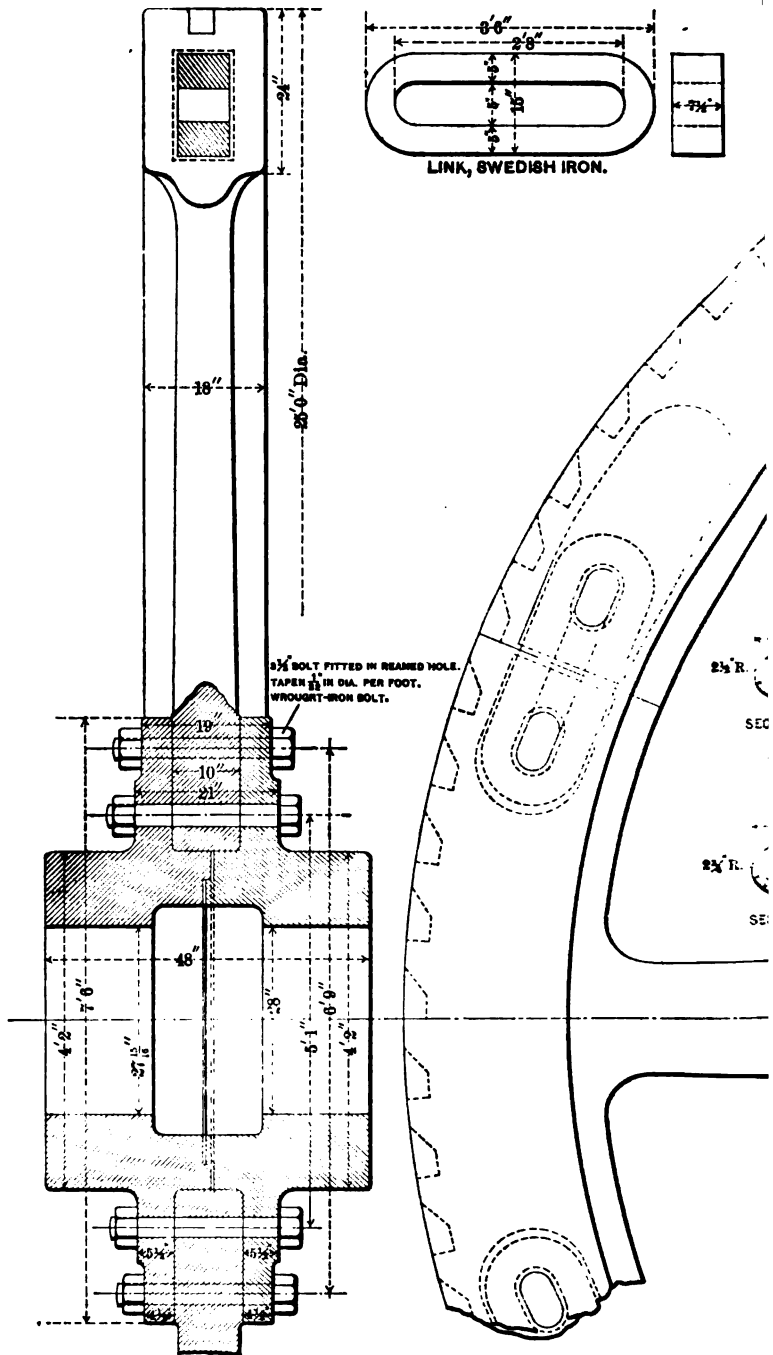
FIG. 1

25-foot fly-wheel.   Cast iron.   8 sections.   Weight 160,
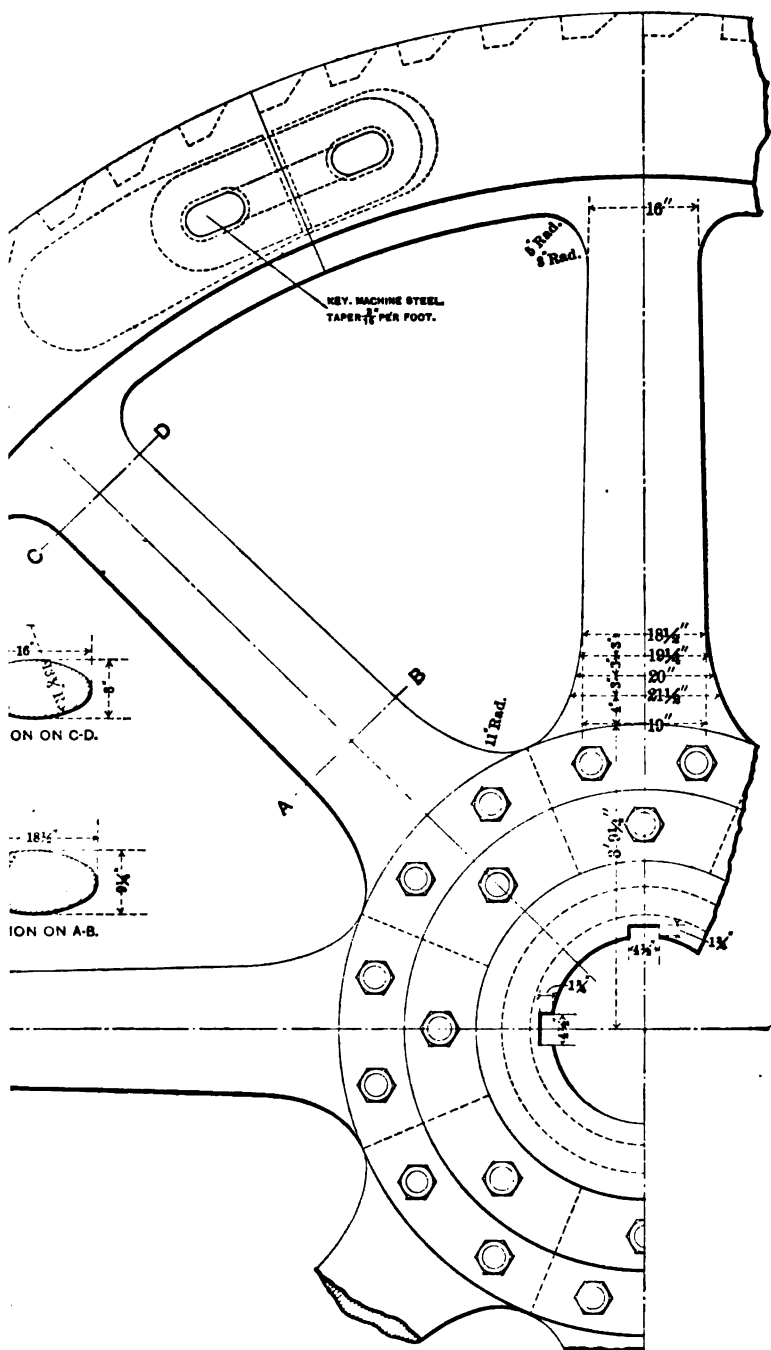
KEY. MACHINE STEEL.
TAPER ⅛ PER FOOT.

5" Rad.
5" Rad.

16"

D

C

16"

ON ON C-D.

B

1' Rad.

18¼"
19¼"
20"
21½"
10"

A

18½"

ION ON A-B.

3' 6¼"

4½" 1¼"

1¼"

4½"
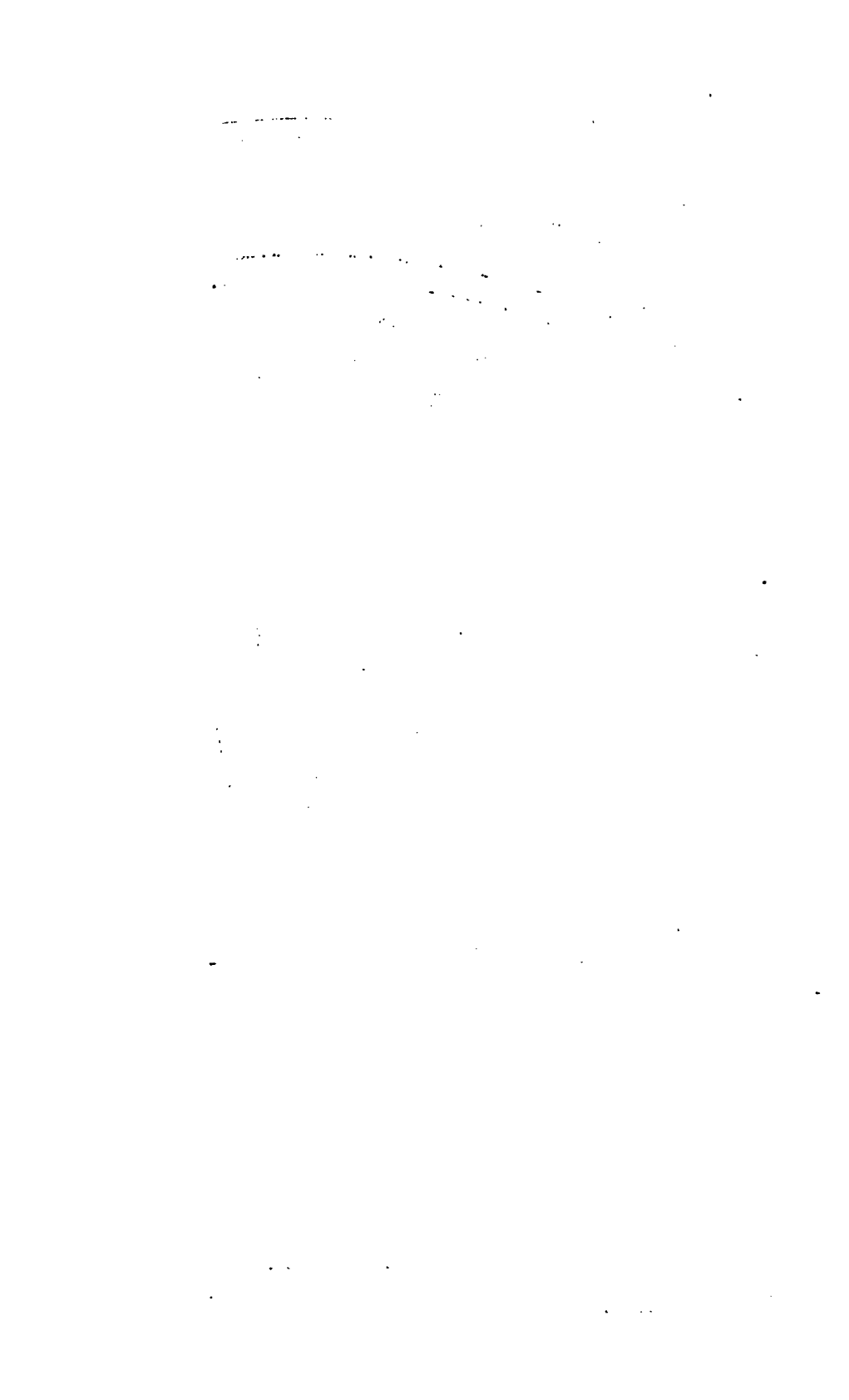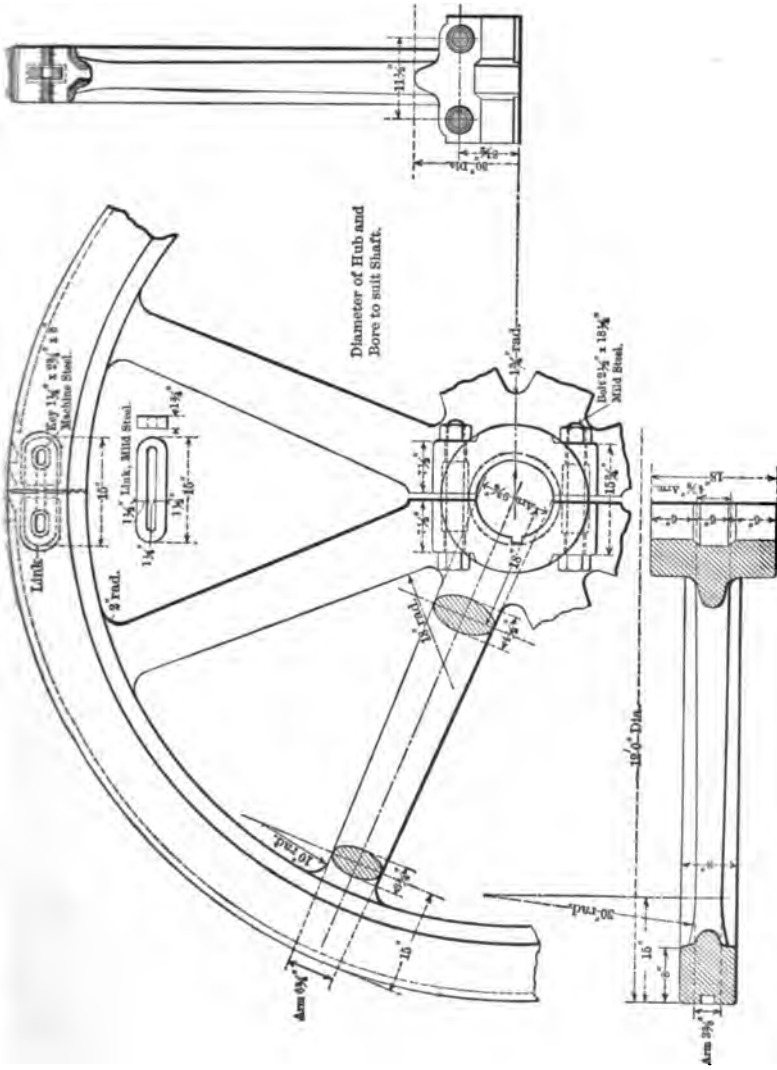
FIG. 188.

12-foot fly-wheel with barring teeth. Cast iron. Split in halves. Estimated finished weight about 12,000 pounds. Fraser & Chalmers, Chicago, Ill.

usually adopted.   It is made in the form of a cast-iron I beam.
This form presents advantages in the way of getting a sound cast-
ing and in making an arm which is strong enough to resist a turn-
ing force acting upon the rim or hub.   A wheel of this design is
operating successfully at 75 revolutions per minute.
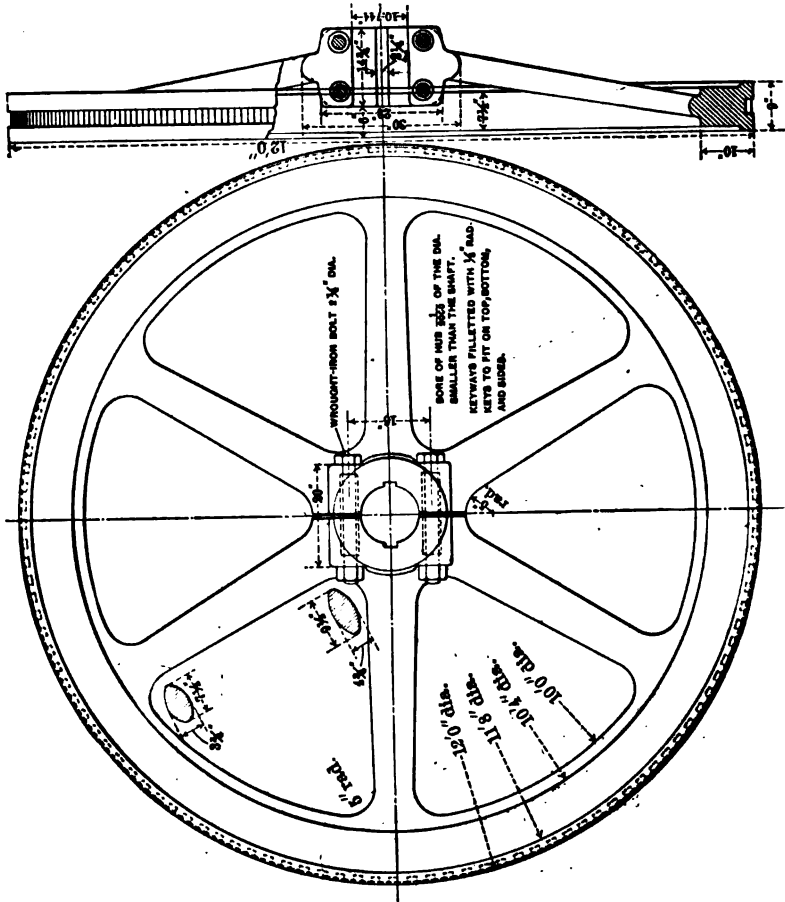
Fig. 184. 12-foot overhung fly-wheel for Riedler pumping-engine.   Cast iron.   Weight 12,500 pounds.   Fraser & Chalmers, Chicago, Ill.

Figs. 130 and 131 are from the working prints of a fly-wheel 23
feet 6 inches in diameter as made by the same company.   This
wheel is also divided into ten sections.   The net weight is 128421
pounds.   One of the wheels made after this design is direct-con-

nected to an 800-kilowatt generator speeded at 80 revolutions per minute.

Fig. 132 is a 25-foot fly-wheel weighing 160000 pounds and divided into eight sections. Four of these wheels have been made
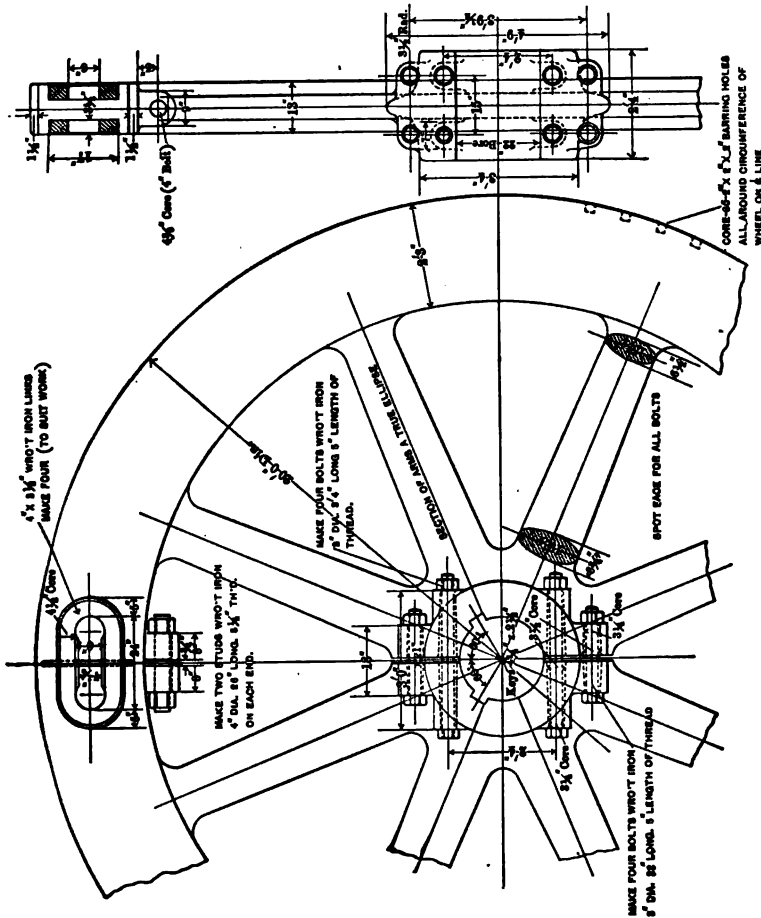
FIG. 185.

20-foot fly-wheel with barring-holes. Cast iron. Two parts. Weight 80,000 pounds. Southwark Foundry and Machine Co., Philadelphia, Pa.

by Fraser & Chalmers, and put into operation in the power-station of the West Chicago Street Railway Co. They are on 34 and 54 × 60 inches compound condensing Corliss engines which are direct-connected to electric generators.

Fig. 133 represents another fly-wheel as built by the same company.   It is 12 feet in diameter and made in two parts.   The weight is about 12000 pounds.

Fig. 134 is another 12-foot fly-wheel made by Fraser & Chalmers.   The design is peculiar in that the rim is overhung in order to bring its weight over the bearing which supports it.   The weight of this pulley is 12500 pounds.   Two of these wheels are in use on each of the pumping-engines for delivering water at high pressure to the hydraulic accumulators in the works of the Pope Tube Co.   The water pumped is utilized for tube-drawing benches, etc.

Fig. 135 represents a 20-foot fly-wheel weighing 80000 pounds as constructed by the Southwark Foundry & Machine Co.   The wheel is made in two parts, held together by links at the rim and bolts at the hub.

Fig. 136 is the wheel used on the direct-connected electric generating units of the Metropolitan Railway, New York City.

Fig. 136.1 is a band-saw wheel with staggered arms which give rigidity and allow at the same time for shrinkage of the casting.

Fɪɢ. 136 (

STEEL RIVET

8 7.83 CHORD

5 REAMED HOLES

160 PLATES
1⅛ THICK

140 R.

11'7"

98'0 D. Turn

13'6½ R

12'0½ R

STEEL CASTING

160 STEEL PLATES IN RIM
1⅛ THICK
ALL DRILLED ALIKE

TURNED BOLTS
EACH END
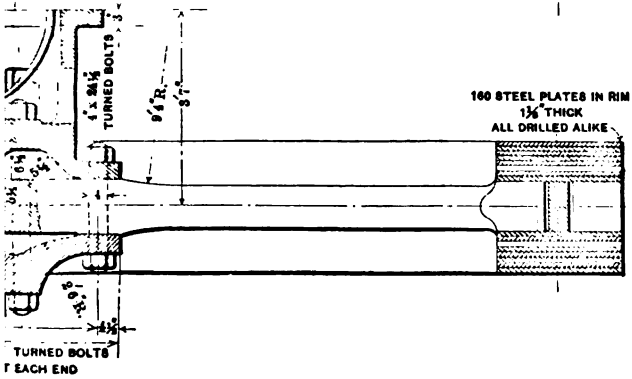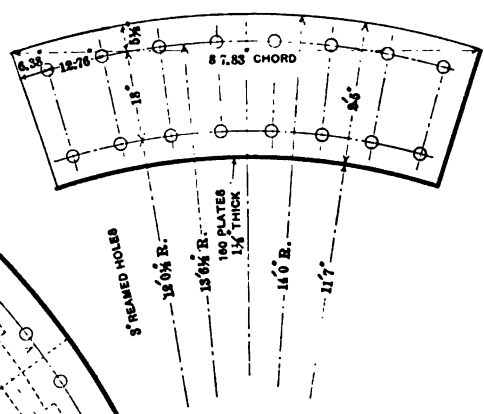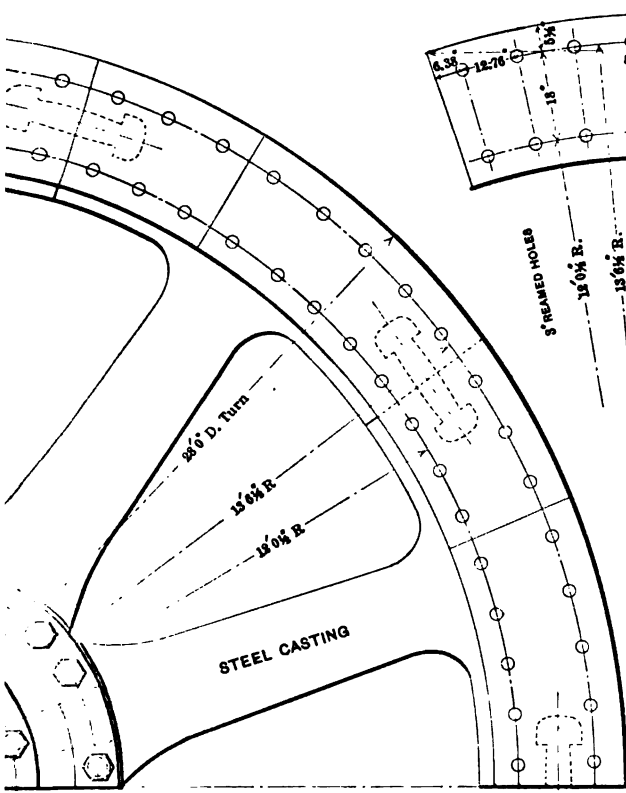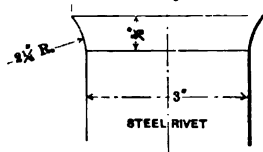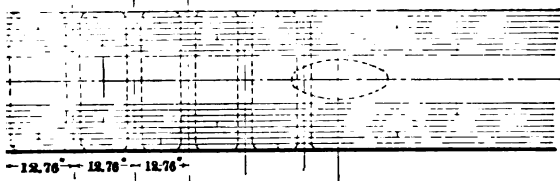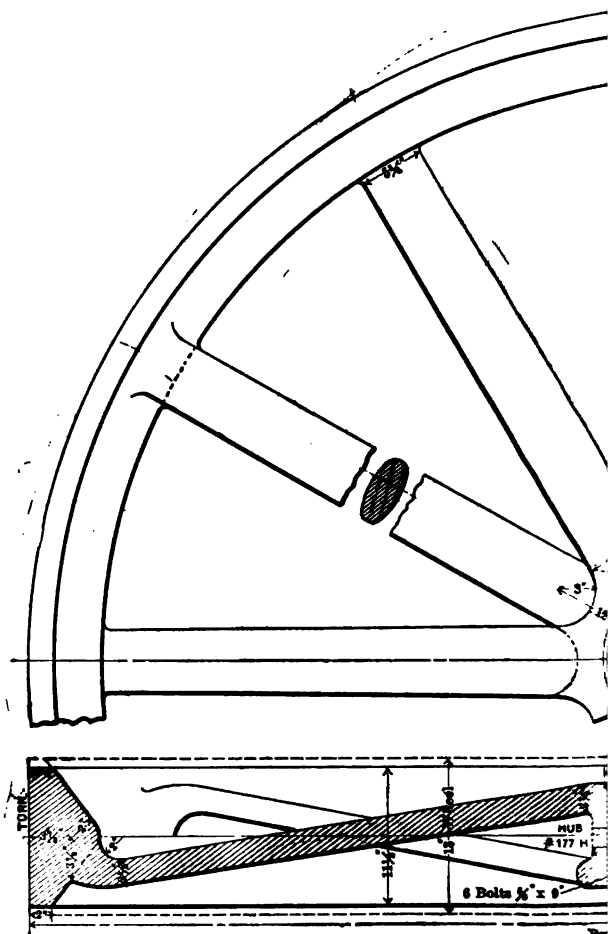
FIG.

Lower band wheel 9 feet diam. by 11½-inch face for band
lines.   Allis Chalmers C

Bore

36.1 (at page 346).

aw-mill.   A 18-inch rim for the same hub and arms is shown in broken
, E. P. Allis Works, Milwaukee, Wis.

CHAPTER XI.

CYLINDERS, TUBING, PIPES, AND PIPE-COUPLINGS.

131. In the following discussion of cylinders, tubing, and pipes, they are separated into two classes, according to whether the walls are thin or thick, on account of the difference in the nature of the stress occurring in thin and thick walls. It is assumed that all the cross-sections discussed are circular.

The notation for cylinders, tubing, and pipes is as follows:

D = external diameter, inches;
F' = cross-sectional area of wall of cylinder, square inches;
T = total circumferential tension per inch of length in the wall of the cylinder, pounds;
T' = total longitudinal tension in the wall of the cylinder, pounds;
a = thickness of wall, inches;
d = internal diameter, inches;
p = internal pressure, pounds per square inch;
t = circumferential tension, pounds per square inch;
t' = longitudinal tension, pounds per square inch;
π = 3.1416.

132. Tension in a thin circular cylinder due to internal pressure.—When a pipe or tube is closed at the end and contains a liquid or gas under pressure, there is a tendency to burst the pipe which produces tensile stress in its walls.

If the pipe is long and the walls thin in comparison with its diameter, the relation between the internal bursting-pressure and the total circumferential tension in an inch of length, taken at some distance from the end so that there will be no strengthening effect of the cap or head, is expressed by the equation

$$T = \tfrac{1}{2}pd. \quad \ldots \ldots \quad (160)$$

This expression is obtained in the same manner as that for the tension in a shrunk-on ring (see § 90 and Fig. 118).

The circumferential tension per square inch in the walls when they are thin in comparison with the diameter is

$$t = \frac{T}{a} = \frac{1}{2}\frac{pd}{a} \text{ (approximately).} \quad . \quad (161)$$

The total longitudinal force tending to separate the ends of the pipe is

$$T' = \frac{p\pi d^2}{4}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (162)$$

The wall area resisting this force is that of the cross-section cut by a plane normal to the length of the pipe, and is

$$F' = \pi\frac{D^2 - d^2}{4}. \quad . \quad . \quad . \quad . \quad . \quad (163)$$

The walls being very thin in comparison with the diameter of the cylinder, their cross-sectional area is

$$F' = \pi da \text{ (approximately)} \quad . \quad . \quad (164)$$

The longitudinal tensile stress per square inch, as obtained by combining equations (162) and (163), is

$$t' = \frac{pd^2}{D^2 - d^2}. \quad . \quad . \quad . \quad . \quad . \quad (165)$$

And the approximate longitudinal stress per square inch, as obtained by combining equations (162) and (164), is

$$t' = \frac{1}{4}\frac{pd}{a} \text{ (approximately)} \quad . \quad . \quad (166)$$

Equations (161) and (166) show that for pipes with thin walls the circumferential tensile stress per unit area is twice as great as

the longitudinal. In other words, the tendency to burst the pipe longitudinally is twice as great as circumferentially.

The cylinder-head, flange, or cap upon the end of the cylinder or pipe aids the material near the end of the pipe in resisting the circumferential tensile stress due to internal pressure. If, therefore, the cylinder is short in comparison with its diameter, its capacity to resist pressure will be greater than that of a long one.

Example.—The circumferential tensile stress in a pipe 7 inches inside diameter and $7\frac{5}{8}$ inches outside, when withstanding an internal pressure of 2500 lbs. per square inch, is, for thin walls, by equation (161),

$$t = \tfrac{1}{2}(2500 \times 7) \div \tfrac{5}{16} = 2735 \text{ lbs. per square inch.}$$

The total end pressure on the pipe may be obtained by equation (162), whence

$$T' = \frac{2500\pi(7)^2}{4} = 96212 \text{ lbs.}$$

And the longitudinal tension per unit area is, by equation (166),

$$t' = \tfrac{1}{4}(2500 \times 7) \div \tfrac{5}{16} = 1367 \text{ lbs. per square inch.}$$

133. Cylinder with thick walls. Stress in material due to internal pressure.—When the walls of the cylinder are thick in comparison with its diameter, the circumferential tensile stress in them due to the internal pressure is not uniformly distributed throughout the material, but, on account of the elasticity of the walls, is greatest at the inner layer of metal and least at the outer, gradually diminishing from the inside toward the outside.

Equations (167) and (168) agree with those given in several books on the mechanics of materials, etc. They are based upon the assumption that the volume of the material forming the cylinder does not change when put under stress (i.e., the material is assumed to be incompressible). Under this assumption the circumferential stress in any fibre of the material is inversely proportional to the square of its distance from the axis of the cylinder.

The equations for cylinders with thick walls are:

$$t = \frac{p(d + 2a)}{2a}; \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (167)$$

$$a = \frac{1}{2}\frac{pd}{t - p}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (168)$$

The influence of flanges and closed ends is doubtless greater in thick cylinders than in thin ones. For such short cylinders as are commonly used for hydraulic presses, etc., the strengthening effect of the closed end is very great.

134. Bursting tests of cylinders and pipes. Lap - welded wrought-iron pipe.*—The results of a series of tests made by R. W. Hildreth & Co. of New York upon a large number of pieces of lap-welded wrought-iron pipe illustrate so well the manner in which piping and couplings may give way under pressure that a very brief abstract of the log of the tests will be given.

The pipes tested were all 7 inches inside diameter and 7⅝ inches outside. The sections were held together by cast-iron flanges of the form shown in Fig. 137. Three 1-inch bolts were used in each pair of flanges. The pipes and flanges were threaded with 8 threads per inch, having a taper of $\frac{1}{16}$ of an inch in 2½ inches. When screwed together, the pipe protruded beyond the flange $\frac{1}{16}$ of an inch.

The gaskets were of gutta-percha 8.7 inches external diameter, so as to fit into the recess of the flange.

In the first series of experiments six lengths of pipe, each about 20 feet long, selected at random, were coupled together and tested under high pressure. The longitudinal tension due to the water-pressure against the heads of the pipe was resisted by the pipe and flanges, no auxiliary devices being used to resist this end thrust. One flange broke at a pressure of 1700 pounds per square inch, and after this was repaired another broke at an internal pressure of 2000 pounds per square inch.

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* *Engineering News*, March 21, 1895.

In the second series of tests the ends of the pipes were tied together from end to end with two 3-inch rods having sleeve-nuts for tightening; the pipe was also secured against buckling. At the
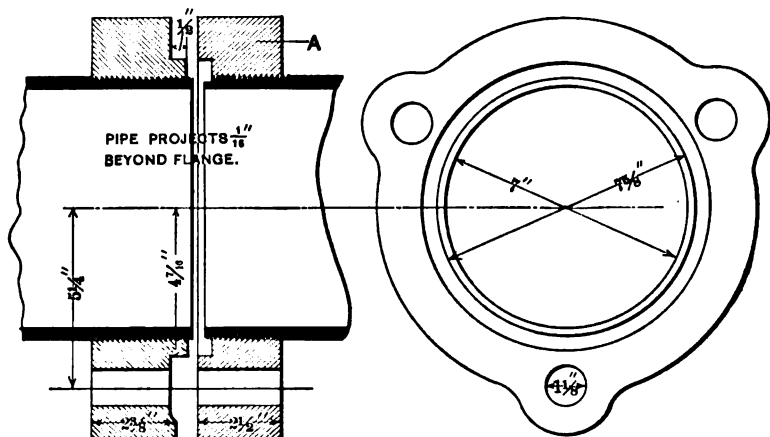
Fig. 187.

first trial one pipe burst six feet from the end at an internal pressure of 2400 pounds per square inch. The fracture showed a blister about 12 inches long. At the second trial a flange cracked at 800 pounds pressure. It was supposed that the bolts had not been tightened properly. At the third trial another flange broke with violence at 2300 pounds pressure. And at the fourth trial still another flange broke at 2500 pounds pressure per square inch.

All the flanges that broke were of the form A, Fig. 137. Each broke about midway between the lugs through which the bolts passed.

As a result of the tests it was recommended that a round flange with five or six ¾-inch bolts be specified instead of that with three lugs and bolts. It was stated that "with such flanges perfect pipe of the same quality and dimensions as those ordered should withstand a pressure of from 2500 to 3500 pounds per square inch."

Tests of drawn tubing. — A number of so-called "seamless"

steel tubes were tested at Sibley College.*   The tubes were made
by welding or brazing the edges of steel skelp after the manner of
making seamed tubing, and then drawing down in pipe-drawing
dies, as is common for seamless-pipe manufacture.   During the
tests on the finished tubes no indication of a seam could be found,
and in no case was there rupture along a seam.

The only tube which is recorded as having burst was 10 inches
long, 1.254 inches external diameter and 1.202 inches internal, the
thickness corresponding to a gauge of between 22 and 23.   The
ends were closed by clamping the tube endwise between two metallic
plates, one of which was perforated and connected to the pressure-
pump by tubing.   The tube burst at an internal pressure of 4700
pounds per square inch, corresponding to a circumferential tensile
stress of 108642 pounds per square inch.   It gave way along one
side about 3 inches from one end.   The external diameter was in-
creased about $\frac{1}{14}$ of an inch by the internal pressure.

Other similar tests were made upon pieces of tubing 30 inches
long, but in each case the tube deflected so that the pressure could
not be carried to the bursting-point.   These tests showed that a
tearing-stress of 80000 pounds per square inch could be applied with-
out rupturing the material.   It was also shown that a tearing-stress
of 70000 pounds per square inch did not sensibly increase the di-
ameter of the tube.

Cast-iron cylinders.—A series of experiments upon short cast-
iron cylinders to determine their bursting-strength was conducted
by Professor C. H. Benjamin.†

The form of the cylinders tested is shown in Fig. 138, and
the proportions are given in Table XLIV.   The lengths are ap-
proximately twice the diameters.   The flanges and heads were made
extra heavy in order that rupture might occur in the cylindrical
shell.   Each cylinder was examined for flaws, and if any small
blowholes were found they were filled with lead or tin, hammered
in, and the surface was then covered with a coating of paraffin.

The gaskets gave much difficulty on account of leaking and

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* Trans. Amer. Soc. Mech. Eng., vol. XIX., 1898.   Endurance, bending,
crushing, and other tests were also made.

† Trans. Amer. Soc. Mech. Eng , vol.. XIX., 1898.

TABLE XLIV.

DIMENSIONS OF CAST-IRON CYLINDERS TESTED FOR BURSTING-STRENGTH.

(Refers to Fig. 138.)

No.	A	B	C	D	E	Depth Counter-bore.	G	H	I	K	No. of Bolts in Each Head.
a	12.16	26.05	.70	16.25	......	....	1.07	1.12	1.0	....	24
b	9.16	17.95	.60	13.06	......	...	1.09	.70	1.0	....	16
c	6.09	12.19	.50	10.05	......	....	1.12	.70	1.0	....	8
d	12.45	26.5	.56	16.21	13.25	.12	1.75	1.35	1.5	....	24
e	9.12	19.0	.61	12.96	10.08	.11	1.5	1.25	1.25	....	16
f	6.12	13.0	.65	10.02	7.08	.11	1.25	1.00	1.25	....	8
1	9.58	18⅝	.402	13.33	10.88	⅛	1⅛	1⅛	1⅝	11¾	16
2	9.375	18⅝	.573	13.13	10.63	⅛	1⅛	1⅛	1⅝	11¾	16
3	9.13	18⅝	.596	12.88	10.88	⅛	1⅛	1⅛	1⅝	11¾	16
4	12.53	25⅝	.571	16.4	13.34	⅛	1.34	1½	1⅛	14¾	24
5	12.56	25⅝	.531	16.56	13 56	⅛	1.34	1⅞	1⅛	14½	24
6	12.16	25⅝	.93	16.22	13.41	½	1.18	1½	1⅛	14¾	24

NOTE.—The rough dimensions in this table are averages from a number of measurements. Cylinders bored to make walls of approximately uniform thickness, and the ends faced.

Cylinders a, b, and c made of ordinary foundry-iron showing a tensile strength of about 18000 pounds per square inch in the test-piece.

Cylinders d to 6 made of a special foundry mixture which showed a tensile strength of about 24000 pounds per square inch in the test-piece.

blowing out when used upon the flat-faced end of the cylinder. By counterboring, as shown in the figure, and packing with a
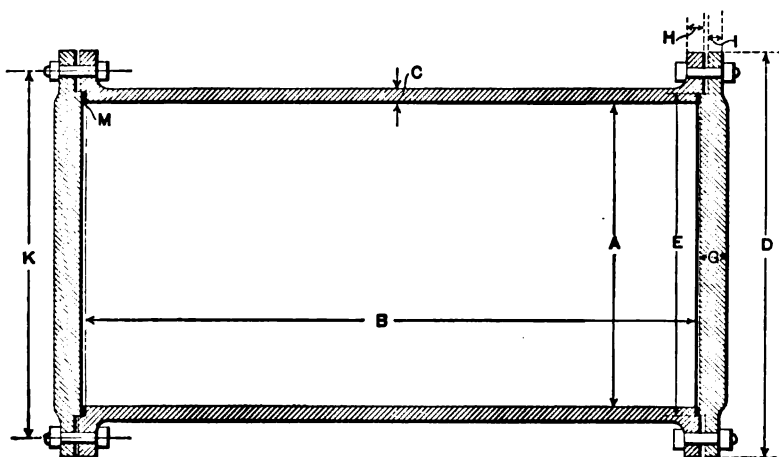
FIG. 138.

gasket of strawboard saturated with oil, this difficulty was largely obviated. This packing held except when the projection on the head was smaller in diameter than the counterbore in the cylinder.

One of the serious difficulties met during the test was leakage through the cylinder walls on account of minute blowholes. Some of them were almost invisible to the naked eye, but at high pressure the water would spurt out several feet in a slender stream. Water oozed through the walls at all points.

The log of the tests is as follows: *

" *Cylinder a.*—Wire-gauze packing; leaked at 400 pounds. Substituted copper wire No. 22, A. W. G.; this leaked at 600 pounds. Substituted soft-rubber gasket; pressure carried to 800 pounds several times. Leak at blowhole stopped by peening. On raising pressure to 775 pounds cylinder failed on a circumference just below the upper flange, the crack starting at blowhole and running each way about 90 degrees.

" *Cylinder b.*—Gasket of lead fuse wire $\frac{3}{32}$-inch diameter with ends fused together. Leakage at pressure of 450 pounds, and the flange cracked. Substituted rubber and graphite packing; leak at crack with pressure of 600 pounds; no further rupture.

" *Cylinder c.*—Rubber and graphite packing inserted, heated to 250 degrees Fahr. by live steam; bolts screwed down and packing left one day to harden. Leaked badly at 600 pounds; renewed packing, but it leaked again at 550 pounds. Flanges showed signs of failure and experiment was abandoned.

" *Cylinder d.* — Counterbored joint, with gasket of straw-board soaked in linseed-oil. Leakage at blowholes with 700 pounds pressure. Blowholes peened and coated with paraffin, when pressure was raised to 800 pounds several times. One blow-hole calked on outside; on applying pressure of 700 pounds rupture occurred on longitudinal line through blowhole. Several small blowholes found in line of fracture.

" *Cylinder e.* — (On this and all subsequent cylinders the counterbore and straw-board gasket were used.) Pressure raised gradually to 1325 pounds, when rupture occurred on circumference under flange. The crack began at several small blowholes.

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* Illustrations and references to them are omitted.

" *Cylinder f.*—Pressure raised gradually to about 2500 pounds (above graduation of gauge), when cylinder failed in same manner as preceding one; cylinder leaking badly at time of rupture.

" *Cylinder No. 1.*—Broke at 600 pounds on a longitudinal line along a row of blowholes.

" *Cylinder No. 2.*—Broke at 1050 pounds around a circumference just under flange. Fracture very clean.

" *Cylinder No. 3.*—Broke at 975 pounds in the same manner as No. 2, the crack beginning where there was a slight flaw. Fracture clean.

" *Cylinder No. 4.*—A number of small blowholes near the centre of shell caused considerable trouble by leakage, and had to be calked inside and out. Rupture finally occurred at 700 pounds pressure along a longitudinal line.

" *Cylinder No. 5.*—Rupture occurred at 875 pounds, a crack starting under the flange running part way around and then up through flange and head.

" *Cylinder No. 6.*—At 475 pounds pressure the bottom head broke. On renewing this and raising pressure to 900 pounds the top head failed in the same manner. These heads had been used for several cylinders, and were probably weakened. The test was abandoned at this point for lack of time.

Great pains were taken in casting these cylinders, and they may be considered good examples of cast-iron cylinders as made for engine- or pump-work. The blowholes mentioned were most of them very minute, and under ordinary circumstances would have remained unnoticed."

The percolation of liquids througn the walls of cast-iron cylinders at high pressure is of common occurrence. Such cylinders may be lined with copper, brass, etc., to prevent this leakage.

135. Special forms of pipes.—The high steam-pressures common to modern practice, as well as the high pressures used for hydraulic and other purposes, have necessitated the use of piping made of a stronger material than cast iron. For such purposes pipe made of steel plates riveted together, as is ordinarily done in the construction of shell boilers, has been adopted to a considerable extent. The bursting-strength of such a pipe is measured by the strength of the riveted joints.

"Spiral-riveted" pipe is used for nearly all the sizes of pipes common to practice, and especially for those which are thin in comparison with their diameters. It is made by wrapping a long strip of flat material, as a thin strip of paper would be wound around a lead-pencil, so that the edges overlap. In the pipe the overlap is made sufficient for a row of rivets used to fasten the successive convolutions together. The edges of the plate must be prepared before wrapping, by thinning or offsetting them, so that the diameter of the pipe will not increase with each turn of the strip from which it is made. On account of the necessity of such offsetting it is difficult to make this kind of pipe of any great thickness in comparison with its diameter. The helical joint is sometimes welded instead of riveted.

136. Pipe-couplings and -flanges.*—The most essential features of a pipe-coupling are that it shall be strong enough to prevent fracture when under pressure, and that it shall not leak either between the faces of the two flanges or between the pipe and flange.

The stresses which a flange must withstand are those due to internal pressure in the pipe, together with those caused by bolting the flanges together, expansion and contraction of the pipe, and its tendency to bend under its weight or to buckle on account of the elongation of the pipe by expansion when heated by steam, etc. Steel flanges are used for the high steam-pressures of modern practice, as well as for pipes for high hydraulic pressures.

The packing must be so held in position that it will not blow out under pressure. If it is held between two perfectly plane flange-faces, the friction between it and the flanges, together with its own strength, is all that holds it in position. The end thrust due to internal pressure has a tendency to separate the flanges, thus reducing their clamping force on the packing. This is apt to cause leakage and possibly to tear the packing to pieces and blow it out. This was plainly shown in the experiments on cast-iron cylinders cited in § 134. By recessing or counterboring one flange and forming its mate to fit into the counterbore, the packing, if of the

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* The dimensions of the standard flanges adopted by the Amer. Soc. Mech. Engrs. are given in Kent's "Mechanical Engineers' Pocket-book," together with numerous other designs of flanges.

same external diameter as the counterbore and fitted into it, will be held in place much more effectively.

Fig. 139 represents a method of attaching pipe-flanges that is suitable for the high steam pressures of modern engineering.   The

FLANGE PIPE COUPLING.
STEEL FLANGE SHRUNK ON.
END OF PIPE PEENED OVER
AND FACED FLUSH WITH
FACE OF FLANGE.
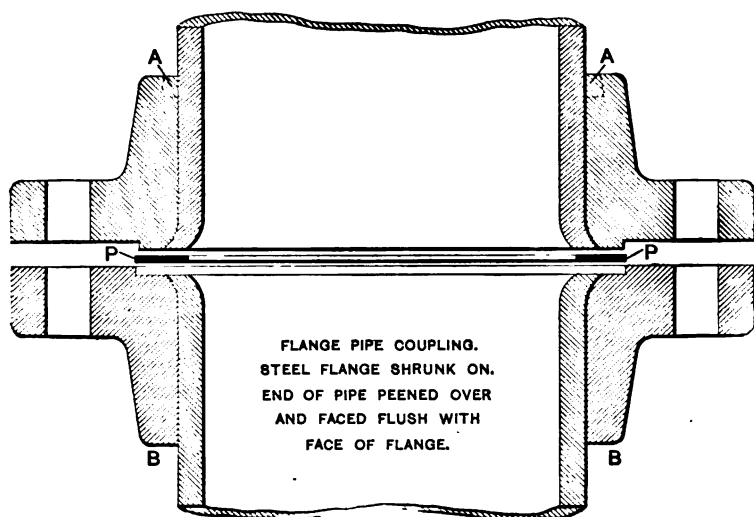
FIG. 139.

proportions shown in the figure are those for rolled-steel flanges. In order to attach the flange it is expanded by heat and then shrunk on the pipe so that the end of the latter protrudes beyond the face of the flange.   The pipe is then peened or spun out so as to expand it over the rounded corner of the flange, and is then turned off so that it is just flush with the face of the flange.   The packing P fits in the counterbore and covers the joint between the flange and pipe, thus serving the double purpose of preventing leakage between the flange-faces and also between the pipe and bore of the flange.   As an extra precaution a dovetail counterbore is sometimes made as shown by the dotted lines at A, into which a piece of some malleable material, such as copper, can be calked to prevent leakage between the pipe and flange.

For exceedingly high pressures and correspondingly thick pipe, the pipe and flange are both threaded and screwed together, the rest of the details remaining the same.

Fig. 1 0 and Table XLV show the form and proportions of weldless rolled-steel flanges for pipes from 8 to 72 inches outside diameter and for pressures up to 150 pounds per square inch.
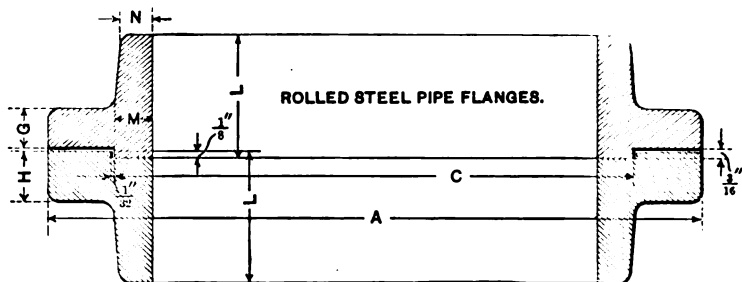
FIG. 140.

Fig. 140.1 shows two pipe couplings of nearly similar design. The proportions are given in Table XLVA. The bolt holes are 1/48 of the diameter of the bolt larger than the bolt. Pattern A
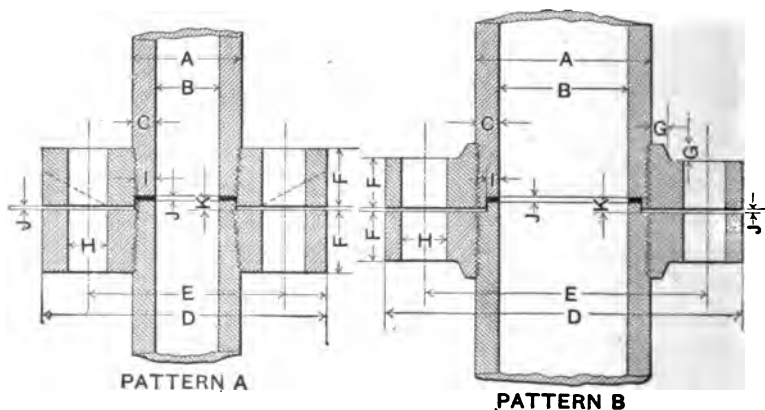
PATTERN A

PATTERN B

FIG. 140.1.

is for pipes from ½ inch to 7 inches diameter, inclusive. Pattern B is for sizes larger than 7 inches up to about 17 inches. The flanges are of rolled steel. When used on the thicker pipes these joints have been satisfactory under an internal pressure of 7000 pounds per square inch. It is suggested that the flanges could be reduced in weight without injuring their efficiency by beveling the corner on the inner surface as shown by the dotted lines of Pattern A.*

A pipe is sometimes riveted to the flange by a circumferential row of rivets extending radially through a lip running back over the pipe on the side of the flange opposite its face.   This lip corresponds to the ones extending out toward A and B in Fig. 139.   In such a joint the pipe extends only partly through the flange and is calked at the end to prevent leakage.

Flanges forged or cast as an integral part of the pipe are used to some extent, chiefly for thick pipes and high pressures.

Fig. 140.2 shows a form of pipe coupling designed by Mr. A. H. Emery and used on small sizes of pipe for high hydraulic pressure.   The illustration is full size.   The small pipe shown is steel and the other parts bronze.   The joint is made tight by giving slightly different angles of bevel to the ends of the pipes so that
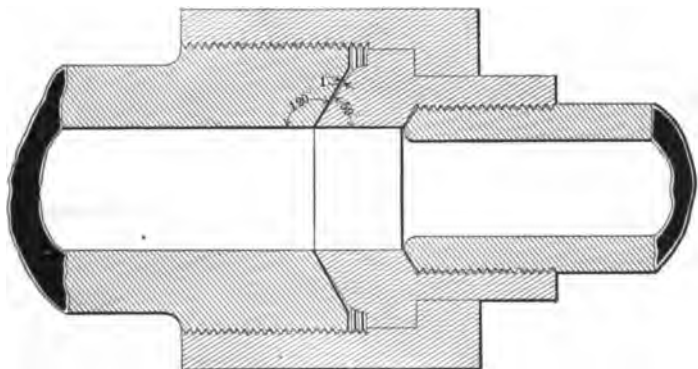
Fig. 140.2.

the parts coming together are in contact only in a narrow ring of the smallest diameter posssible.   A fine thread is generally used, 12 or 20 per inch, preferably the latter, except for cast iron.

The particular joint shown was subjected to an internal pressure of 6000 pounds per square inch and showed no leak.   Joints similarly constructed have held without leak under 10,000 pounds per square inch internal pressure.*

137. Expansion-couplings for pipes.—In order to allow for the change of length which always occurs in pipes with change of temperature, some form of expansion-coupling must be introduced at intervals in order to prevent excessive stresses in the material. Numerous devices have been adopted for this purpose.   Those most

* Communication from Mr. A. H. Emery, Jr.

TABLE XLV.

DIMENSIONS OF WELDLESS ROLLED-STEEL FLANGES FOR PIPE
CARRYING PRESSURE UP TO 150 POUNDS PER SQUARE INCH.

(Dimensions in Inches.) (Refers to Fig. 140.)

Diameter, inside.	8	9	10	11	12	13	14	15	16	17	18
A	14½	15½	16¾	17¼	18½	20¼	21¼	22¼	23½	24½	25¾
C	9½	10½	11½	12½	14	15	16	17	18	19	20
G	1⅜	1¼	1⅜	1⅝	1⅜	1½	1⅝	1⅝	1¹¹⁄₁₆	1¾	1¹¹⁄₁₆
H	1⅝	1⅞	1½	1⅞	1⅝	1¹¹⁄₁₆	1¾	1¹¹⁄₁₆	1⅞	1¹¹⁄₁₆	2
L	3½	3⅞	3¾	3⅞	4	4⅛	4⅜	4½	4¾	4⅞	5
M	1	1	1⅛	1⅛	1⅜	1⅜	1¼	1¼	1⅜	1⅜	1½
N	⅝	⅝	¾	¾	¾	¹³⁄₁₆	¹³⁄₁₆	⅞	⅞	¹³⁄₁₆	1
Approx. { Rough Wt.	67	77	91	97	113	136	154	172	192	209	235
{ Fin. Wt. Male	54	62	76	81	95	115	131	149	166	180	204
Finished Wt. Female	57	65	79	85	100	120	137	156	174	188	213

Diameter, inside.	19	20	24	30	36	42	48	54	60	66	72
A	27¼	28¼	32¾	39	45½	51½	57¾	64¼	70¾	76¾	83½
C	21	22½	26½	32½	39	45	51	57½	63½	69½	76
G	1⅞	1¹¹⁄₁₆	2¾	2¼	2¼	2¼	2¼	2¼	2¼	2¾	2¼
H	2⅛	2⅝	2⅞	2⁷⁄₁₆	2⁷⁄₁₆	2⁷⁄₁₆	2⁷⁄₁₆	2⁷⁄₁₆	2⁷⁄₁₆	2⁷⁄₁₆	2⁷⁄₁₆
L	5½	6	6	6	6½	6½	6½	6½	6½	7½	7½
M	1⅝	1⅛	1⅝	1⅝	1⅝	1¾	1¾	1¾	1¹³⁄₁₆	1¹³⁄₁₆	1¾
N	1	1⅛	1¼	1¼	1⁷⁄₁₆	1⁷⁄₁₆	1⁷⁄₁₆	1⅝	1¹³⁄₁₆	1⁷⁄₁₆	1¾
Approx. { Rough Wt.	265	295	416	525	665	766	913	1053	1245	1444	1661
{ Fin. Wt. Male	232	257	371	466	595	686	820	949	1120	1301	1501
Finished Wt. Female	242	268	387	482	612	705	842	972	1148	1332	1534

used may be divided into two general classes, however: 1st. Those
which operate by telescopic action, generally having an accurately
turned tube which slides back and forth through a stuffing-box
packed in a manner suitable to allow such motion after the method
common for piston-rod glands of steam-engines. 2d. Those in
which the elasticity of one of the parts, called the expansion-piece,
is depended upon to allow a slight end motion of the pipes. One
form of this coupling consists of a short length of pipe made of
rather thin copper, corrugated circumferentially. The corrugations
allow the piece to change its length endwise under moderate end
pressure. Another form consists of a pair of flanges, one much
larger in diameter than the other, fastened together in a manner
somewhat similar to that shown for the shaft-coupling in Fig. 121.
Instead of having a plane disk, however, as shown in this figure, a
corrugated copper disk is generally used, the corrugations running
circumferentially around the disk. The corrugations allow a com-
paratively free end motion of the parts relatively to each other.

TABLE XLVA.
PIPE FLANGES FOR HEAVY PRESSURES, FIG. 140.1.
Inch- and Pound-units.

Pattern "B."	Pattern "A."	Nominal Diam.	X Pipe for 1500 Pounds per Square Inch Pressure					XX Pipe for 3000 Pounds per Square Inch Pressure				Thread Threads per Inch	Flange							Packing	
			Actual Out-side Diam. A	Actual Inside Diam. B	Thick-ness C	Inter-nal Area	W'ght per Foot	Actual Inside Diam. B	Thick-ness C	Inter-nal Area	W'ght per Foot		Diam. Over All D	Diam. of Bolt Circle E	Thick-ness F	Thick-ness G	No. of Bolts	Diameter of Bolts*	Width of Groove I	Thickness of Copper Gasket J	Length of Tongue K
		⅜	0.84	0.542	0.149	0.231	1.09	0.244	0.298	0.017	1.7	14	3⅜	2⅝	¾	…	4	½	⅜	3/32	3/16
		½	1.05	0.736	0.157	0.452	1.39	0.422	0.314	0.139	2.44	14	4	3¼	⅞	…	4	⅝	⅜	3/32	3/16
		¾	1.315	0.951	0.182	0.71	2.17	0.587	0.364	0.271	3.65	11½	4½	3⅝	1	…	4	⅝	½	⅛	¼
		1	1.66	1.272	0.194	1.27	3.	0.885	0.388	0.615	5.2	11½	4½	3⅞	1	…	4	⅝	½	⅛	¼
		1¼	1.9	1.494	0.203	1.75	3.63	1.088	0.406	0.93	6.4	11½	5¼	4½	1⅛	…	4	¾	½	⅛	¼
		1½	2.375	1.933	0.221	2.93	5.02	1.491	0.442	1.744	9.02	11½	5½	4½	1¼	⅞	4	¾	⅝	⅛	5/16
		2	2.875	2.315	0.28	4.21	7.67	1.755	0.560	2.419	13.6	8	6½	5¼	1⅜	1	4	¾	⅝	⅛	5/16
		2½	3.5	2.892	0.304	6.57	10.25	2.284	0.608	4.037	18.56	8	7¼	6¼	1½	1	4	¾	¾	3/16	⅜
		3	4.	3.358	0.321	8.86	12.47	2.716	0.642	5.794	22.75	8	8½	7¼	1½	1	6	¾	¾	3/16	⅜
		3½	4.5	3.818	0.341	11.45	14.97	3.135	0.682	7.724	27.4+	8	9	7¾	1¾	1⅛	6	¾	¾	3/16	⅜
		4	4.5	4.813	0.375	18.19	20.54	4.033	0.75	12.865	38.12	8	9½	8¼	1¾	1⅛	6	¾	¾	3/16	⅜
	4	5	5.563	5.75	0.437	25.97	28.58	0.875	18.665	53.11	8	11¼	9¾	1⅞	1¼	6	⅞	⅞	¼	7/16
	5	6	6.63	6.75	0.5	34.52	34.52		2.00	2.125			13¾	11¾	2¼	1⅜	6	⅞	⅞	¼	7/16
	6	7	7.63	6.63	0.5				0.575	2.125			15	13	2⅛	1⅜	6	1	1	¼	7/16
	7	8	8.00	7.00	0.500				1.00			6	16½	14¼	2¼	…	8	1	1	5/16	½
8		…	8.63	7.50	0.565				1.125	1.375		6	17¼	15½	2⅛	…	8	1	1⅛	5/16	½
9		…	9.69	8.00	0.625				1.25	1.50		6	19	16⅛	2¼	⅞	8	1	1⅛	5/16	½
10		…	10.50	8.43	0.63				1.625			8	20¼	17½	2½	⅞	10	1⅛	1⅛	5/16	½
		…	10.50	9.00	0.75				1.375	1.75		8	20½	18¼	2⅝	1	10	1⅛	1¼	⅜	9/16
		…	11.75	9.35	0.70				1.50	1.875		8	22½	19½	2⅞	1	10	1¼	1¼	⅜	9/16
		…	13.00	10.00	0.875				1.625	2.00		8	24	21½	3	1⅛	10	1¼	1¼	⅜	9/16
		…	14.25	11.25	0.875				1.75	2.125		6	28½	24¾	3¼	1¼	12	1¾	1⅜	7/16	⅝
		…	15.50	12.50	1.00				1.875			4	27¾	23¾	3¾	1¼	12	2	1⅜	7/16	⅝
		…	16.75	13.50	1.125				1.875			4	30½	22½	3¾	1¼	12	2¼	1½	7/16	⅝
		8	18.00	14.50	1.125				2.00			4	31½	24½	4	1⅜	12	2¼	1½	7/16	⅝
		9 10	19.25	16.75	1.25				2.00 2.125			4	32½	27¼	4⅛	1⅜	14	2½	1½	7/16	7/16

* Diameter of bolt hole = 1/16 × diameter of bolt.

CHAPTER XII.

138. It is common practice in engineering to fasten together the edges of plates, angles, beams, etc., with rows of rivets. If the plates riveted together are to be used for the retaining-walls of a vessel to contain a fluid under pressure, the joint must be designed for tightness as well as strength. This means that the rivets must be put close enough together to prevent the plates from springing apart between the rivets to any considerable extent. In addition to placing the rivets near together it is generally necessary to calk the joints with a calking-tool, whose typical form is that shown in Fig. 141. This tool resembles a cold-chisel whose chipping edge has been ground off blunt. It is held in the posi-
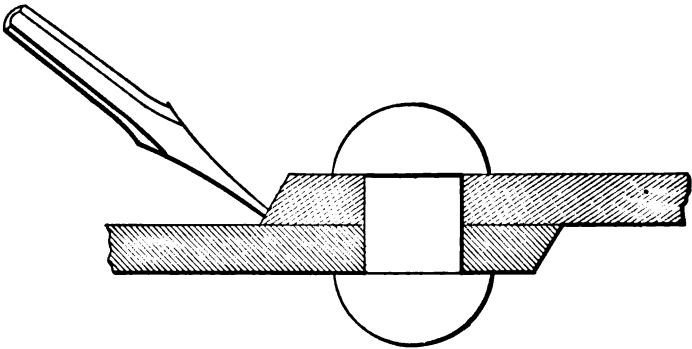
FIG. 141.

tion indicated in the figure and struck with a hammer to calk the edges of one plate down against the other.

If the joint is for vessels which are to hold solid material, or for

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the members of bridges and buildings, it is necessary to design it for strength only, there being no necessity for tightness.

In hand-riveting the rivet is heated to a full red heat and then passed through holes previously punched or drilled in the plates and brought into proper position to receive the rivet. The rivet is then held in place with a " dolly-bar " while the opposite end is first upset with hammers and then finished with a " set " or " snap." The " set " resembles a sledge-hammer with a depression cut in its face to conform with the kind of head that is desired, as button-head, cone-head, pan-head, etc. It is held against the rivet and struck with a sledge. Very small rivets are often put in place cold.

Machine-riveting is used much more in modern practice than hand-riveting. A machine-riveter has two dies, cupped to the form the rivet-head is to have, which press on the opposite ends of the rivet-blank after it has been put in place. The body of the rivet is swelled out in the hole and the head formed by the heavy pressure that is exerted. The dies of riveting-machines are commonly operated by either steam, air, or hydraulic pressure. In hydraulic riveting one method is to press the rivet down so as to form the heads while it is still very hot, then relieve it from the pressure of the die for a short time until it has cooled somewhat, and then put it under pressure again until it has become cold enough to prevent its stretching materially under the tendency which the plates may have to spring apart. In steam and pneumatic riveting the rivet-head is first formed by a steady pressure of the die, then allowed to cool somewhat, after which it is struck a few sharp blows with the die driven out by the action of the steam or air against it in order to bring the plates together so that they will be gripped tightly when the rivet has cooled completely. For very heavy riveting the riveter sometimes has a pair of closing- or gripping-dies, called a " plate-closer," which are used for pressing the plates together and holding them until the rivet has cooled.

If the work is well done, the rivets should fill the holes completely. Since the rivet blank is always smaller in diameter than the hole it is to fill, this necessitates its being upset from end to end so that it may swell to the size of the hole. It has been found that this can be accomplished better by having the blank hottest at the head end, so that it will swell first under the head, gradually fill-

ing the hole from the head toward the point, and finally forming the second head.

When the holes are made by punching, they are larger at one end than the other, being roughly conical on account of the punch being smaller than the die on which the plate rests during the operation of punching. The plates should be placed so that the small ends of the holes come together. When so placed, the end contraction of the rivet tends to draw it more tightly against the conical walls, thus eliminating, in a measure at least, the loosening effect of the decrease in diameter due to cooling, and giving the rivet a better fit in the hole; the rivet also grips the sides of the holes and draws the plates together, thus relieving the heads of a portion of the strain. If, on the contrary, the large ends of the holes are placed together, the swelling of the rivet when under the dies tends to force the plates apart, and its contraction to loosen it in the hole; also the end contraction is resisted by the heads only.

Numerous styles of riveted joints are in common use. Some of the simpler ones will be shown in order to explain the nature of the stresses that act upon the plate and rivets. The seams most com-

LAP JOINT, SINGLE RIVETED
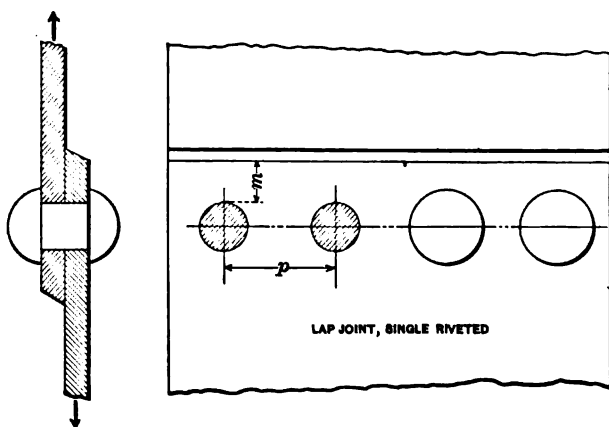
FIG. 142.

monly used are of two general classes, namely, lap-joint and butt-joint. The names are derived from the manner in which the edges of the plates are placed relatively to each other. In the lap-joint

the plates overlap each other, examples of this form of seam being shown in Figs. 142, 143, and 144; in the butt-joint the edges of

LAP JOINT, DOUBLE RIVETED, STAGGERED RIVETS.
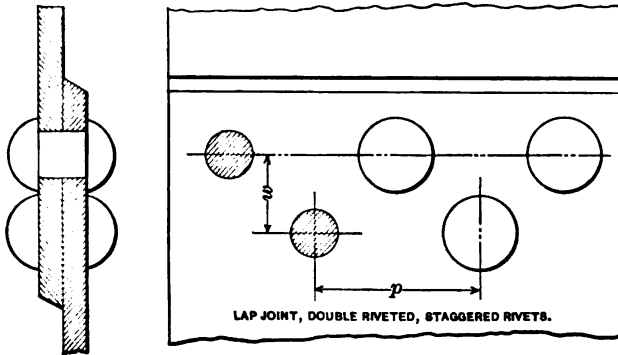
FIG. 143.

the plates are butted against each other, and one or two cover-plates, straps, or welts placed over their junction, the rivets passing

LAP JOINT, DOUBLE CHAIN RIVETED.
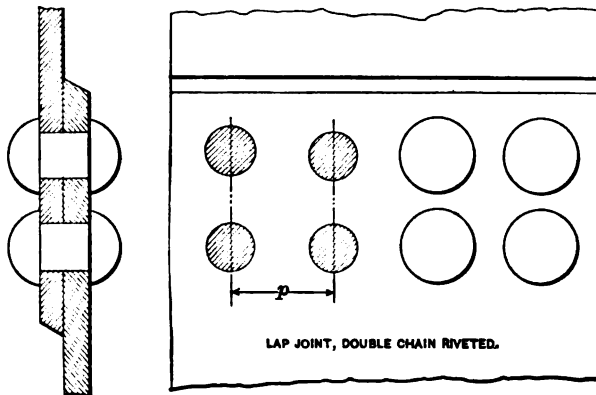
FIG. 144.

through one plate and one side of the strap or straps, as shown in Figs. 146 and 147.   Fig. 145 is a lap-joint with cover-plate.

The seams are further classified according to the number of rows of rivets that are used, and the positions of the rivets of one row

LAP JOINT WITH COVER-PLATE
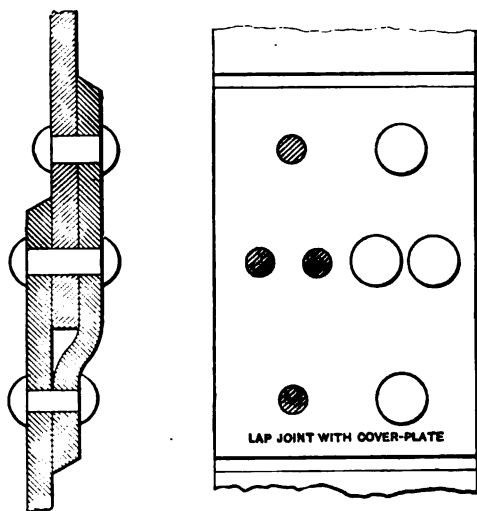
FIG. 145.

relatively to those of the other rows. The rows of rivets run parallel to the length of the seams. · Single-riveted joints are shown in Figs.

BUTT JOINT, SINGLE COVER-PLATE OR STRAP, SINGLE RIVETED.
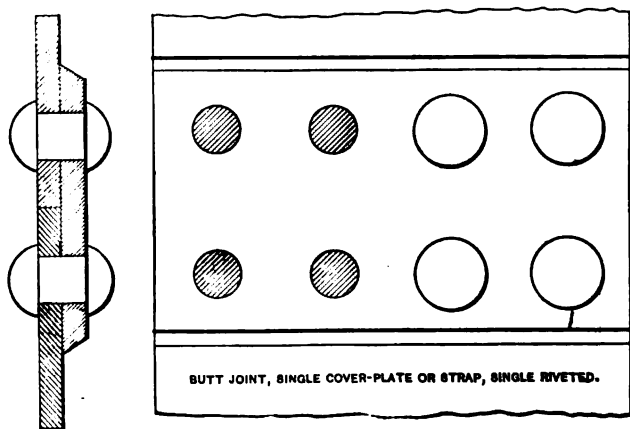
FIG. 146.

142, 146, and 147; in each of these joints the edges of the plates are pierced by but a single row of rivets, although two rows are

BUTT JOINT, TWO COVER-PLATES OR STRAPS.
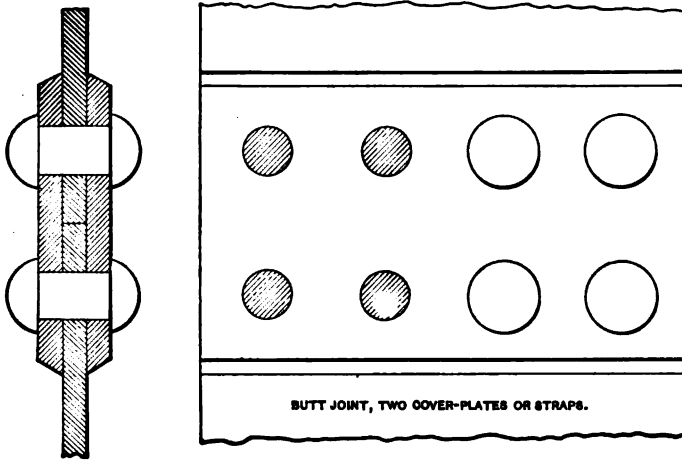
Fig. 147.

embodied in the seam in two of these three cases. Double riveting is shown in Figs. 143 and 144, the edges of the plates being pierced
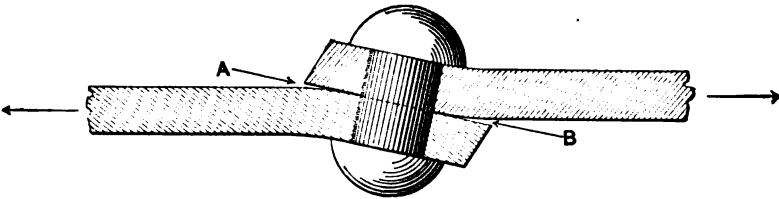
Fig. 148.

with two rows of rivets. The rivets are said to be staggered when those of one row are opposite the spaces of the adjacent row, as in Figs. 143 and 145; when they are opposite each other, as in Figs. 144, 146, and 147, the seam is chain-riveted.

The **pitch** of rivets is the distance between the centres of adjacent rivets in the same row.

The **margin** is the distance from the edge of the plate to the edge of the rivet-hole. (See *m*, Fig. 142.)

The **overlap** is the distance one plate laps over the other.

All the rows of a seam do not need to have the same pitch, as

can be seen by reference to Fig. 145, where the pitch of the outer rows is double that of the middle one.

PROPORTIONS OF RIVETED JOINTS.

139. A single-riveted joint may yield in one of the five following ways when forces are applied as indicated by the arrows in Fig. 142:

1st. Shearing the rivets in the plane of the plate surfaces that are in contact.

2d. Crushing the rivets, or the plate in front of them, by the "bearing-pressure" of the rivets against the plate.

3d. Tearing the plate between the rivets.

4th. Splitting or tearing the plate between the rivet and the edge of the plate.

5th. Shearing out the plate in front of the rivet, the piece sheared out having a width approximately equal to the diameter of the rivet.

Double-riveted joints may fail by either the first or third method, or by crushing the rivets. Any other manner of failure would be a combination of two or more of the five methods given above. The failure of joints with several rows of rivets may be still more complicated.

The frictional resistance to the slipping of the plates over each other, due to their being clamped together by the end contraction of the rivets, is, in a carefully made hydraulic machine-riveted joint, generally from one third to one half of the total strength of the joint. A hand-riveted seam generally offers less frictional resistance to the slipping of the plates over each other than one which is machine-riveted. In designing riveted seams it is not customary to take this frictional resistance into consideration.

The notation for riveted joints is as follows:

A = sectional area of rivet on a plane perpendicular to its axis, square inches. (It is assumed that the rivet fills the hole, and therefore its diameter is equal to that of the hole where the plates touch each other);

$c =$ ultimate bearing- or crushing-strength of plate or rivet for single shear, pounds per square inch;

$c' =$ ultimate crushing-strength of plate or rivets for double shear, pounds per square inch;

$d =$ diameter of rivet-hole, inches;

$f =$ ultimate tensile strength of plate between rivet-holes, pounds per square inch;

$n =$ number of rivets passing through one plate along a length of edge equal to p;

$p =$ pitch of rivets, inches;

$s =$ ultimate shearing-strength of rivet for single shear, pounds per square inch;

$s' =$ ultimate shearing-strength of rivet for double shear, pounds per square inch;

$t =$ thickness of plate, inches;

$w =$ distance between rows of rivets.   (See Fig. 143.)

A rivet is in single shear when the shearing force acts only in one plane, as in a lap-joint, or a butt-joint with one cover-plate. It is in double shear when the tendency is to shear it off in two planes, as in the double-welt butt-joint, Fig. 147.

Table XLVI gives ultimate values that can be safely used in practice with good material. A suitable factor of safety must be introduced. The common practice in this country is to use steel plates with iron rivets for boiler construction. Steel plates with mild steel rivets are used to a considerable extent for other classes of work.

140. Rivets.—Since a rivet may fail either by crushing or shearing, it is first necessary, for a given thickness of plate, to find the diameter of rivet having equal crushing and shearing strengths by equating the strengths as follows:

For single shear,

$$cdt = \frac{\pi d^2}{4}s;$$

whence

$$d = \frac{ct}{.7854s} = 1.27\frac{ct}{s}. \quad \ldots \quad \ldots \quad \textbf{(169)}$$

<div align="center">

TABLE XLVI.

ULTIMATE STRENGTH OF RIVETS AND PLATES.

</div>

	Pounds perSquare Inch.	
	Iron.	Steel.
Rivets, shearing-strength in single shear, s	40,000	50,000
Rivets, shearing-strength in double shear, s'	35,500	45.000
Bearing-pressure in single shear,* c	65,000	90,000
Bearing-pressure in double shear, c'	80,000	110,000
Plates, tensile strength between rivet-holes, t	40,000	55,000

<div align="center">* Bearing area taken as the projected area of the bearing surface.</div>

For double shear,

$$c'dt = \frac{2\pi d^2}{4}s';$$

whence

$$d = \frac{c't}{1.5708s'} = .635\frac{c't}{s'} \ . \ . \ . \ . \ . \ . \ (170)$$

Substituting in equation (169) the values given in Table XLVI gives for single shear:

Iron rivets,

$$d = 1.27\tfrac{40000}{40000}t = 2.06t. \ . \ . \ . \ . \ (171)$$

Steel rivets,

$$d = 1.27\tfrac{90000}{50000}t = 2.28t. \ . \ . \ . \ . \ (172)$$

Substituting in equation (170) gives for double shear:

Iron rivets,

$$d = .635\tfrac{80000}{35500}t = 1.43t. \ . \ . \ . \ . \ (173)$$

Steel rivets,

$$d = .635\tfrac{110000}{45000}t = 1.55t. \ . \ . \ . \ . \ (174)$$

The diameters given by equation (171) hold good in practice for plates from $\frac{3}{16}$ to $\frac{3}{8}$ of an inch thick, but beyond this point the diameter of the hole becomes smaller in proportion to the plate thickness. The difficulty of driving the larger sizes of rivets is largely accountable for this fact. Since the sectional area of a rivet varies as the square of its diameter, it is clear that any rivet smaller than the size giving equality of bearing and shearing strengths will fail by shearing; therefore in designing a joint whose rivet is not larger than the size having equal bearing and shearing strengths, it is not necessary to use the bearing-strength of the rivet and plate, the shearing-strength of the rivet being all that needs consideration; but if the rivet is larger than the size for equality of these two values the bearing-strength must be used.

Table XLVII gives average diameters of holes for plates as commonly used in practice.

TABLE XLVII.

DIAMETERS OF RIVET-HOLES COMMONLY FOUND IN PRACTICE FOR LAP-JOINTS AND SINGLE-STRAP BUTT-JOINTS.

Thickness of plate, inches......	3/16	¼	5/16	⅜	½	⅝	¾	⅞	1	1⅛
Diameter of rivet, inches........	⅝	½	⅝	¾	⅞	⅞	1	1	1⅛	1¼

The form and proportions of some of the rivet-heads most commonly used are shown in Figs. 149, 150, 151, and Table XLVIII.

141. Pitch of rivets.—For a given thickness of plate and diameter of rivet, the pitch required for equal strength of rivets and plates may be found by the following equations, in which it is assumed that the margin is large enough to prevent the rivets from breaking out toward the edge of the plate, and that rupture must occur either by shearing the rivets, tearing the plate along the line of rivet-holes, or crushing the rivets or plate.

When the diameter of the rivet equals or is smaller than the value for equality of shearing- and crushing-strength, then—

For single-riveted joints having the rivets in single shear,

$$\frac{\pi d^2}{4}s = (p - d)tf;$$

whence

$$p = \frac{\pi d^2}{4}\frac{s}{tf} + d = A\frac{s}{tf} + d. \quad . \quad . \quad . \quad (175)$$

TABLE XLVIII.

DIMENSIONS OF COUNTERSUNK RIVET-HEADS.*

(See Fig. 151.)

Dimensions in Inches.

Diam. of rivet, D	¼	5/16	⅜	7/16	½	9/16	⅝	11/16	¾	13/16	⅞	1	1⅛	1¼
Diam. of head, A	½	19/32	11/16	25/32	⅞	31/32	1₁/₁₆	1₅/₃₂	1¼	1₁₁/₃₂	1₇/₁₆	1⅝	1₁₃/₁₆	2

FIG. 149.   FIG. 150.   FIG. 151.

For double shear,

$$\frac{2\pi d^2}{4}s' = (p - d)tf;$$

whence

$$p = \frac{\pi d^2}{2}\frac{s'}{tf} + d = 2A\frac{s'}{tf} + d. \quad . \quad . \quad . \quad (176)$$

And in double-riveted joints, both rows of rivets being of the same pitch:

For single shear,

$$\frac{2\pi d^2}{4}s = (p - d)tf,$$

* Taken from drawings kindly furnished by J. H. Sternbergh & Co.

whence

$$p = \frac{\pi d^2}{2} \frac{s}{tf} + d = 2A\frac{s}{tf} + d. \quad . \quad . \quad . \quad (177)$$

For double shear,

$$\frac{4\pi d^2}{4} s' = (p - d)tf;$$

whence

$$p = \pi d^2 \frac{s'}{tf} + d = 4A\frac{s'}{tf} + d. \quad . \quad . \quad . \quad (178)$$

If the rivet is larger than the theoretical value for equality of shearing and crushing strengths, failure will occur by crushing. In accordance with this—

For single-riveted joints having the rivets in single shear:

$$cdt = (p - d)tf;$$

whence

$$p = \frac{c + f}{f} d. \quad . \quad . \quad . \quad . \quad . \quad (179)$$

For double shear,

$$p = \frac{c' + f}{f} d. \quad . \quad . \quad . \quad . \quad . \quad (180)$$

For double-riveted joints whose rivets have the same pitch in both rows and are in single shear:

$$2cdt = (p - d)tf;$$

whence

$$p = \frac{2c + f}{f} d. \quad . \quad . \quad . \quad . \quad . \quad (181)$$

For double shear,

$$p = \frac{2c' + f}{f} d. \quad . \quad . \quad . \quad . \quad . \quad (182)$$

In general, in any joint of ordinary form having two or more rows of rivets passing through the edge of one plate, the pitch of

the rivets can be determined, when p is taken equal to the pitch of the row farthest from the edge of the plate, by equation (183) or (184) when the rivets are small enough to fail by shearing, or by (185) or (186) when large enough for the joint to fail at the bearing surfaces.

For single shear,

$$p = \frac{n\pi d^2}{4}\frac{s}{tf} + d = nA\frac{s}{tf} + d. \quad \cdots \quad (183)$$

For double shear,

$$p = \frac{2n\pi d^2}{4}\frac{s'}{tf} + d = 2nA\frac{s'}{tf} + d. \quad \cdots \quad (184)$$

For failure by crushing when the rivets are in single shear,

$$p = \frac{nc + f}{f}d. \quad \cdots \cdots \cdots \cdots (185)$$

For double shear,

$$p = \frac{nc' + f}{f}d. \quad \cdots \cdots \cdots \cdots (186)$$

142. The efficiency of a riveted joint is the ratio of its strength to that of the solid plate.   The theoretical efficiency can readily be calculated if it is assumed that the portion of the plate between the holes is of the same strength per unit area as at other places. Thus, for a single-riveted lap-joint of $\frac{3}{8}$-inch steel plates and $\frac{3}{4}$-inch iron rivets, the pitch, by equation (175), is

$$p = .442\frac{40000}{.375 \times 55000} + .75 = 1.6 \text{ inches (about).}$$

Taking $p = 1\frac{11}{16}$ inches,

$$\text{Efficiency} = \frac{p - d}{p} = (1\frac{11}{16} - \frac{3}{4}) \div 1\frac{11}{16} = .55 +.$$

The actual efficiency of a joint may differ considerably from the theoretical on account of various influencing causes, the principal

ones being the effect of punching upon the strength, hardness, and ductility of the plate, and the pressure of the plates against each other due to the contraction of the rivets. It is probable that, with a well-made joint, working with a factor of safety of . four, no slipping of the plates occurs so as to bring the rivets hard against the holes on the bearing side.

143. Shearing, punching, and drilling plates preparatory to riveting.—The plate metal, coming from the rolling-mill in irregular shapes, must be sheared to form and size suitable for its purpose. After this the rivet-holes are made in it, there being three methods of making them, as follows: 1st. Punching to the required size with a power-punch by a single stroke of the punch for each hole; 2d. Punching a hole from $\frac{1}{16}$ to $\frac{1}{8}$ of an inch smaller than required, and then enlarging it to full size with a reamer or similar cutting tool; 3d. Drilling. Of these three methods punching is by far the quickest and cheapest; the punching and reaming process is considerably more expensive; and drilling is the most costly of all.

When a hole is punched in a plate of metal, it is found that the " wad " or " plug " forced out is much smaller in volume than the hole formed by its removal (unless the plate is hard and brittle, and hence unsuitable for riveting up to withstand pressure). It is about the same diameter as the hole, but sometimes not more than half as long as the thickness of the plate. Since the metal in the wad is found to be little, if any, more dense than the plate, it is evident that there must be a flow of the metal as the punch presses upon it, of such a nature that part of the metal, instead of being punched out of the hole with the wad, is forced into the walls of the hole. The thickening of the plate around the hole is proof of this.

This flow of metal into the walls of the hole, which is really a cold working of the material, changes its physical qualities to a depth varying from $\frac{1}{16}$ to $\frac{1}{2}$ of an inch, according to the size of the hole and the quality of the metal, the result being that a thin ring or sleeve, harder and less ductile than the original plate, is formed around the hole. As to the effect of this hardened sleeve upon the strength of the plate along the line of holes, and consequently upon the efficiency of the joint, there has been much discussion with

regard to whether it increases, decreases, or has any effect upon these qualities. The numerous experiments made upon riveted joints seem to indicate, however, that punching is injurious to hard plates and those made from a poor quality of iron or steel, but that soft plates may actually be benefited by the same treatment. Whether any benefit is ever derived or not, it is certainly true that when the operation is injurious its deleterious effects are more marked the poorer the quality and the greater the hardness of the plates.

By reaming out the holes after punching, the effects of punching are nearly or quite removed, and the joint shows practically the same efficiency as when the holes are drilled. The effects of punching can also be partly, sometimes completely, removed by annealing the plate after punching; this process is rather inconvenient and expensive for large plates, however.

The effects of shearing the plate to approximate size are removed by planing off a strip of metal along the edge, which at the same time reduces it to size and makes a smooth edge. The sheared edge always presents a rough surface when it comes from the shears, which in itself would not be acceptable for good work on heavy plates, especially where tight joints are to be made.

144. Faulty construction and grooving of riveted joints.— When the holes in the two plates do not coincide, it is evident that the rivet will be hard to drive into place, and an offset will be formed on it where the plates press together when it is headed; and if the heads are directly opposite each other they cannot both be concentric with the body of the rivet. The whole result is that the rivet is improperly formed and is not as efficient as a perfect one. By careful work very good seams with punched holes can be made.

By punching the holes small, clamping the plates together in the relative positions they will occupy when riveted, and reaming through both plates at the same time, a practically perfect rivet-hole can be obtained; the reaming can be done by hand or power.

Drilled holes can be more accurately located than punched ones, consequently better coincidence of the holes can be obtained when the plates are drilled separately than when punched; and drilling

has the further advantage that the plates can first be clamped together and then the holes drilled perfectly concentric.

In machine-riveting it is necessary for the dies to be held rigidly so that no side motion can occur, for if it does the head will be formed eccentrically with the body and the rivet weakened in consequence. In practice this fault is sometimes so great that the edge of the head is tangent to the body of the rivet. Fortunately such great faults are not found in boiler-work turned out by makers of any reliability.

The stress upon a lap-riveted seam, causing a tendency of the plates to bend, as in Fig. 148, localizes the tension to some extent at A and B. The localization does not generally need serious consideration, however, unless the plates are grooved by calking or corrosion along the edges at A and B. If deeply grooved, the tendency to bend becomes greater, and in badly grooved plates rupture may occur along the groove at the edge of the seam.

The slight bending of the plates at a seam also tends to localize their pressure against the rivets. This localization is at or near the plane of contact of the plates. It has the effect of causing the rivets to shear more readily than if there were no bending of the plates.

When lap-joints are tested to destruction, the head of the rivet is sometimes pulled off on account of the bending of the plate, as indicated in Fig. 148.

145. Examples of riveted joints taken from practice.—Figs. 152 to 163 represent the practice of the Baldwin Locomotive Works with regard to riveted joints.*

Figs. 164 to 171 illustrate the riveted seams adopted by the Continental Iron Works for cylindrical-shell marine-boilers.†

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* Taken from blue-prints kindly furnished by Burnham, Williams & Co., Baldwin Locomotive Works.

†'Taken from printed designs kindly furnished by the Continental Iron Works.
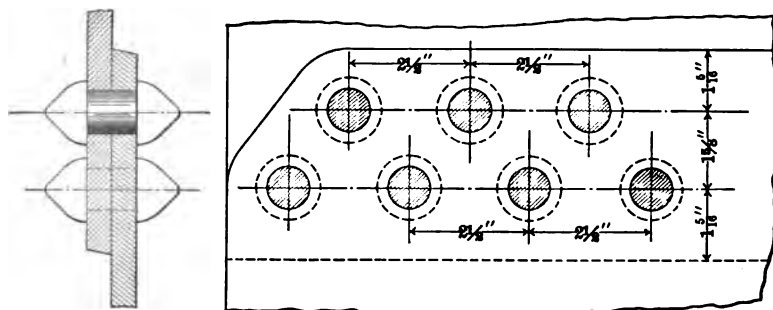
FIG. 152.

Double-riveted lap-joint.   1/2″ plate, 13/16′ rivets, 7/8″ holes.
Baldwin Locomotive Works.
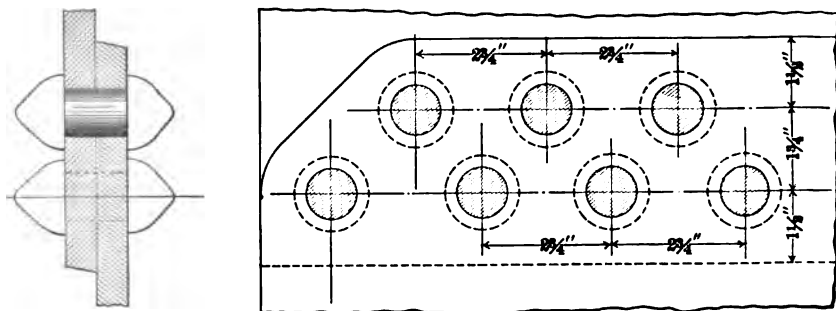
FIG. 153.

Double-riveted lap-joint.   5/8″ plate, 15/16″ rivets, 1″ holes.
Baldwin Locomotive Works.
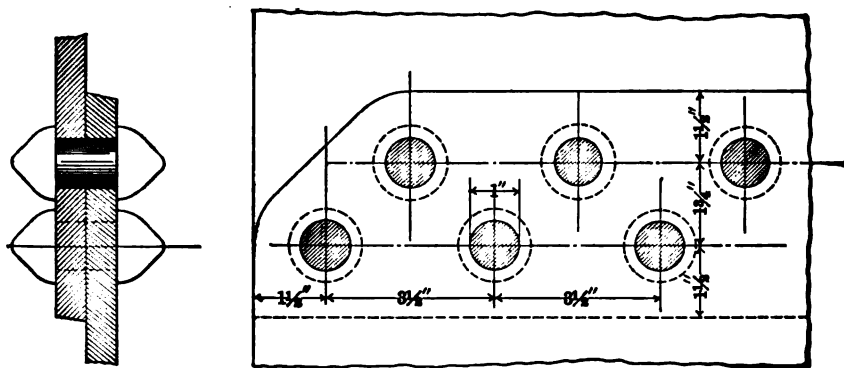
FIG. 154.
Double-riveted lap-joint. 5/8″ plate, 15/16″ rivets, 1″ holes.
Baldwin Locomotive Works.
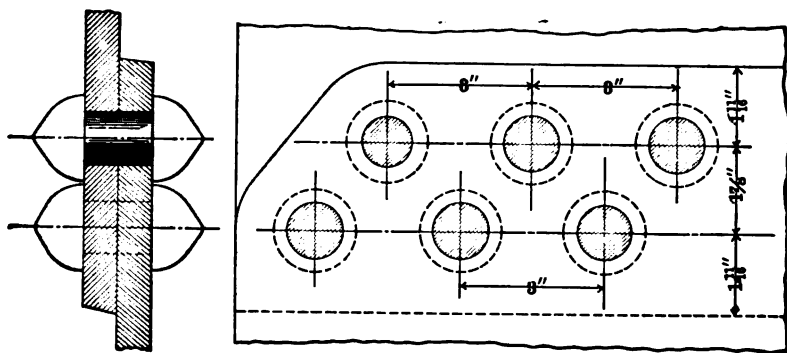
FIG. 155.
Double-riveted lap-joint. 11/16″ plate, 1 1/16″ rivets, 1 1/8″ holes.
Baldwin Locomotive Works.
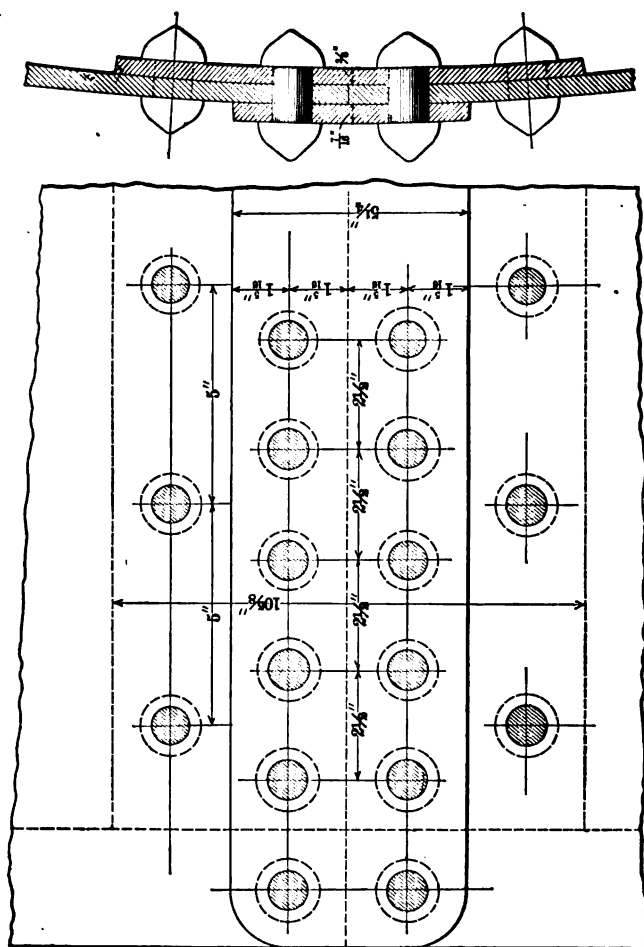
Fig. 156.

Butt-joint with double covering-strips. 7/16″ steel plate, 7/8″ rivets. Baldwin Locomotive Works.
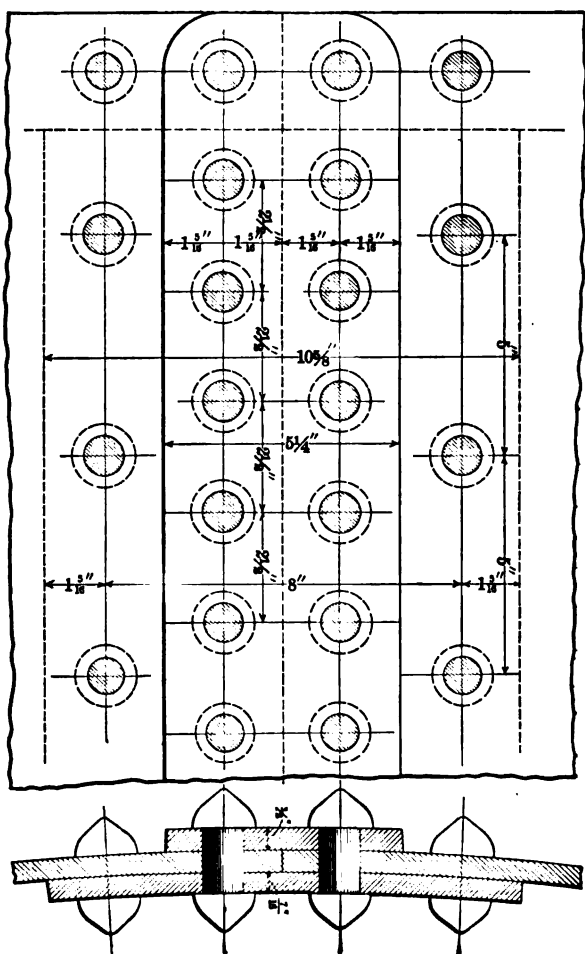
Fig. 157.

Butt-joint with double covering-strips. 1/2" steel plate, 7/8" rivets. Baldwin Locomotive Works.
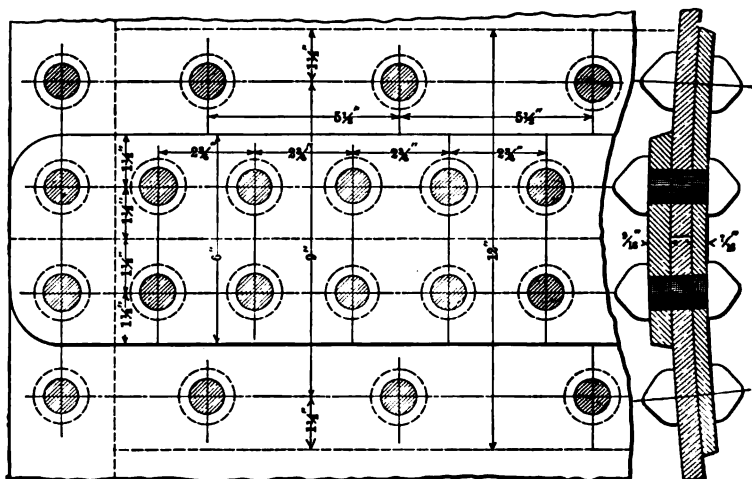
FIG. 158.

Butt-joint with double covering-strips.   9/16″ steel plate, 1″ rivets.
Baldwin Locomotive Works.
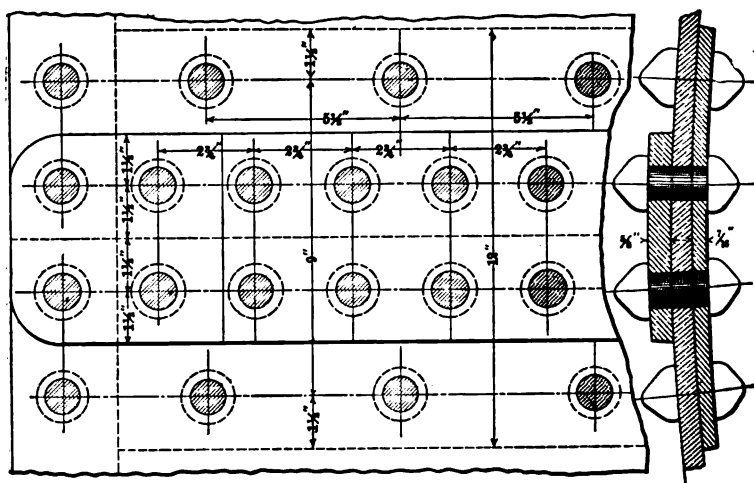
FIG. 159.

Butt-joint with double covering-strips.   5/8″ steel plate, 1″ rivets.
Baldwin Locomotive Works.
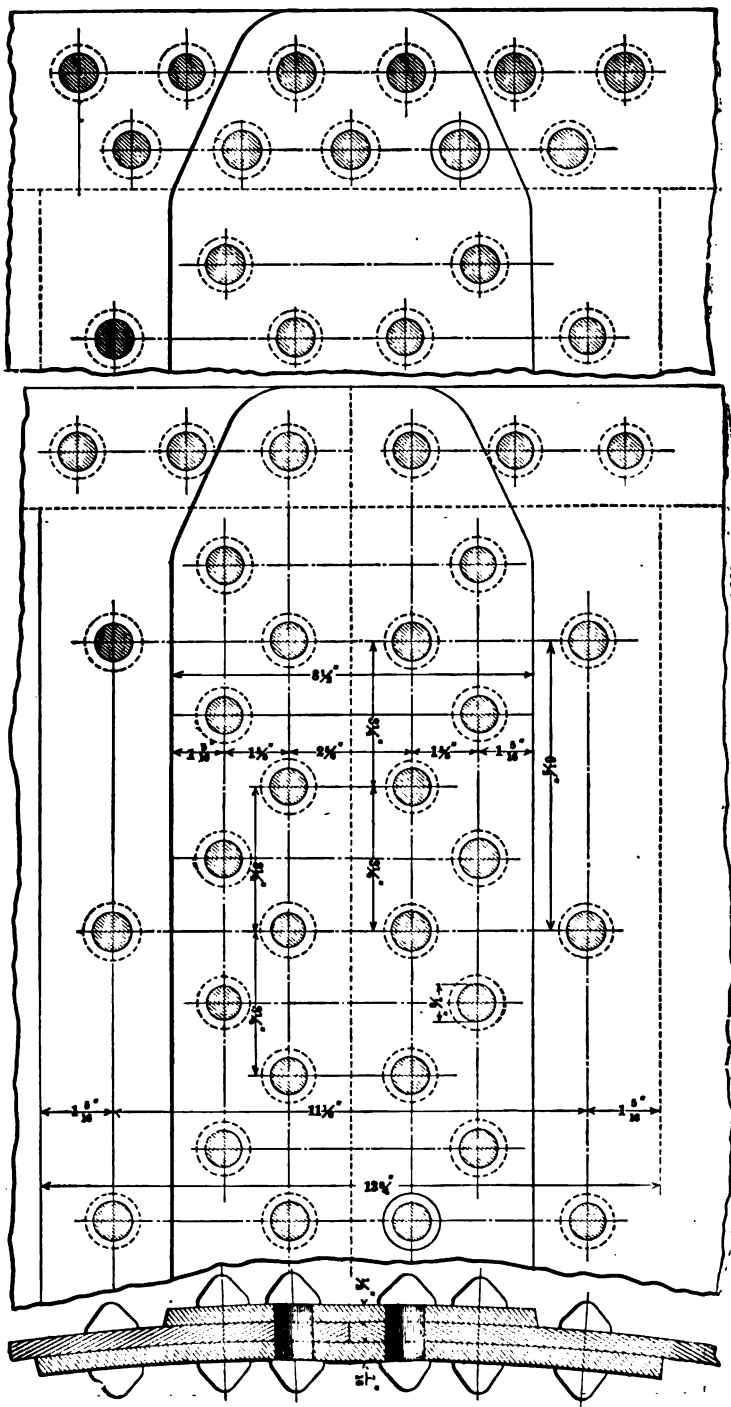
FIG. 160.

Double-riveted butt-joint with double covering-strips. 7/16" steel plate, 7/8" rivets. Standard for all pressures when double-riveted butt-seams are specified. Baldwin Locomotive Works.
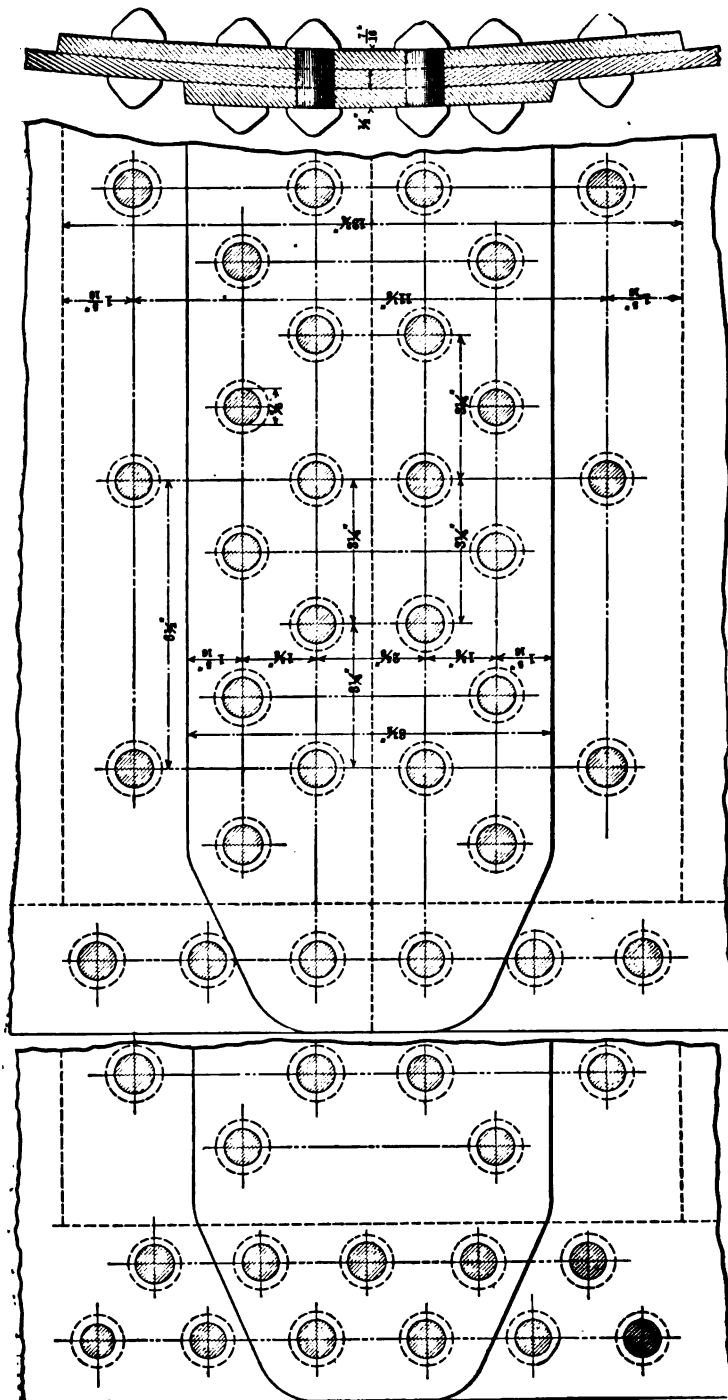
FIG. 161.

Double-riveted butt-joint with double covering-strips. 1/2" steel plate, 7/8" rivets. Standard for all pressures when double-riveted butt-seams are specified. Baldwin Locomotive Works.
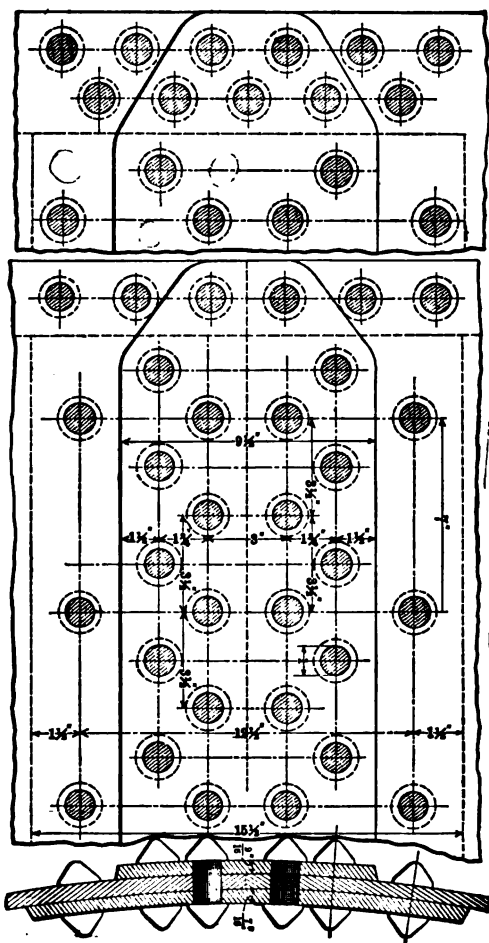
Fig. 162.

Double-riveted butt-joint with double covering-strips. 9/16″ steel plate, 1″ rivets. Standard for all pressures when double-riveted butt-seams are specified. Baldwin Locomotive Works.
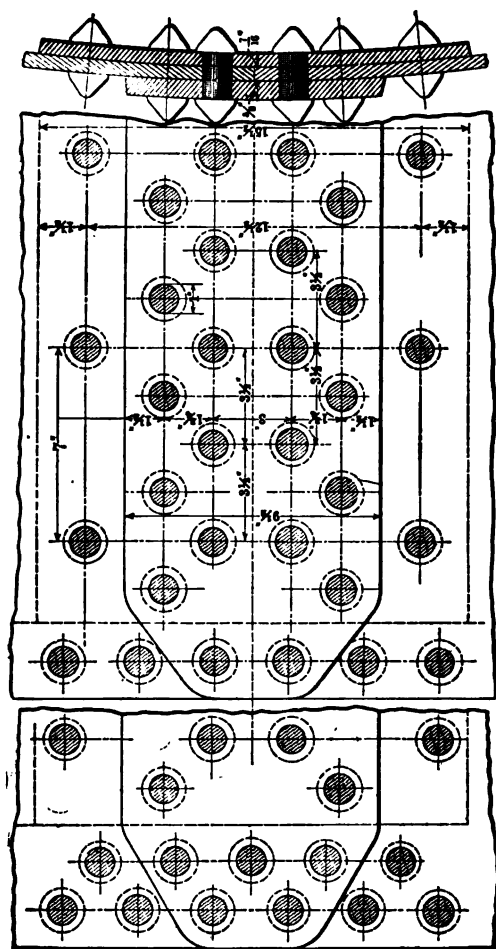
FIG. 168.

Double-riveted butt-joint with double covering-strips. 5/8" steel plate, 1" rivets. Standard for all pressures when double-riveted butt-seams are specified. Baldwin Locomotive Works.
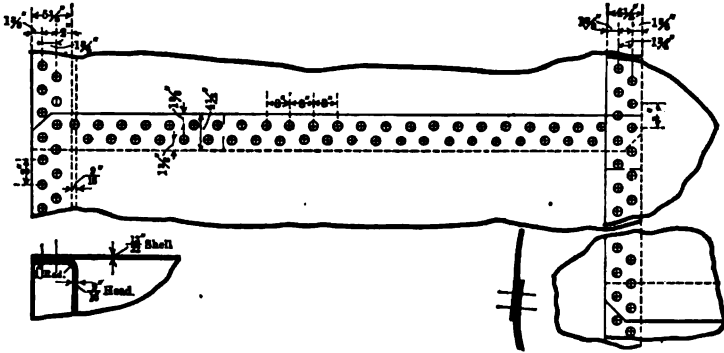
FIG. 164.

Steam-pressure, lbs. per sq. in., 100. Inside diameter of shell 6' 6".
7/8" rivets, 15/16" holes. The Continental Iron Works.
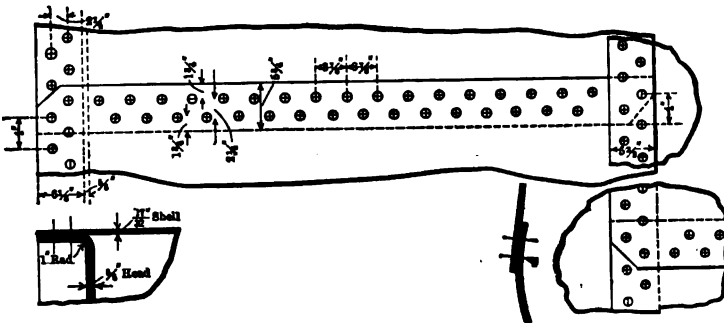
FIG. 165.

Steam-pressure, lbs. per sq. in., 130. Inside diameter of shell 6' 6".
1⅛" rivets, 1¹⁄₁₆" holes. The Continental Iron Works.
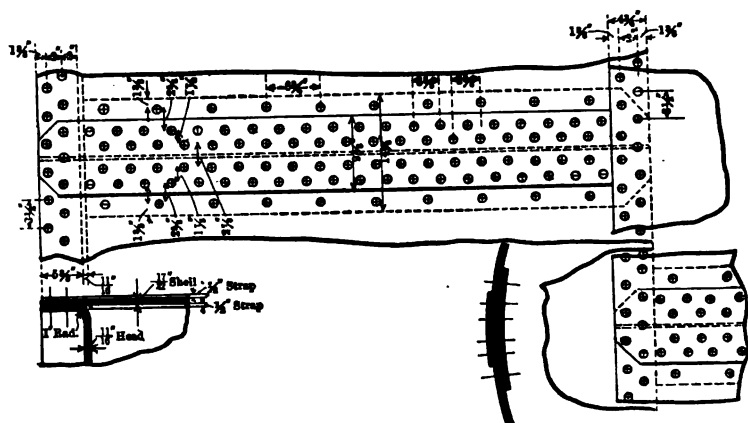
FIG. 166.

Steam-pressure, lbs. per sq. in., 160. Inside diameter of shell 6' 6'.
7/8" rivets, 15/16" holes. The Continental Iron Works.
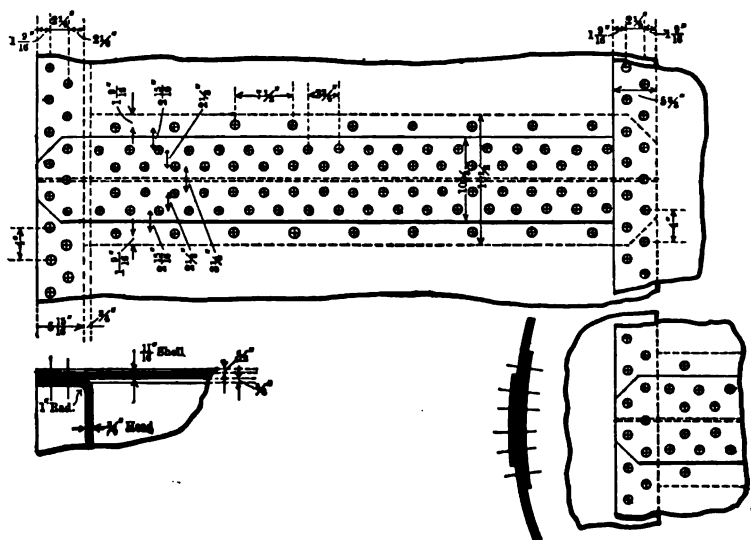
FIG. 167.

Steam-pressure, lbs. per sq. in., 200. Inside diameter of shell 6' 6".
1" rivets, 1 1/16" holes. The Continental Iron Works.
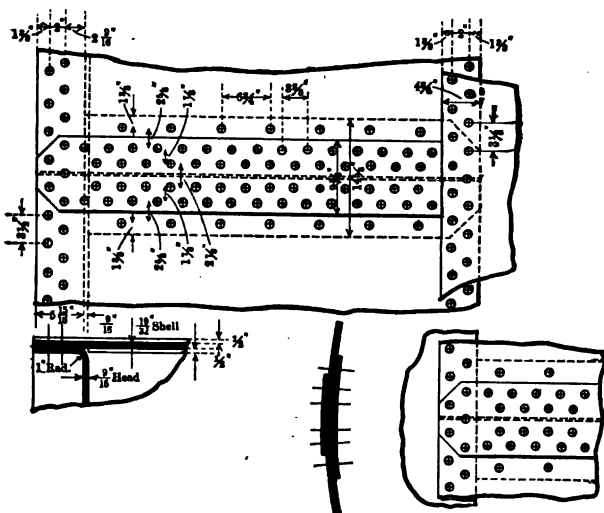
FIG. 168.

Steam-pressure, lbs. per sq. in., 100.   Inside diameter of shell 11' 6".
7/8" rivets, 15/16" holes.   The Continental Iron Works.
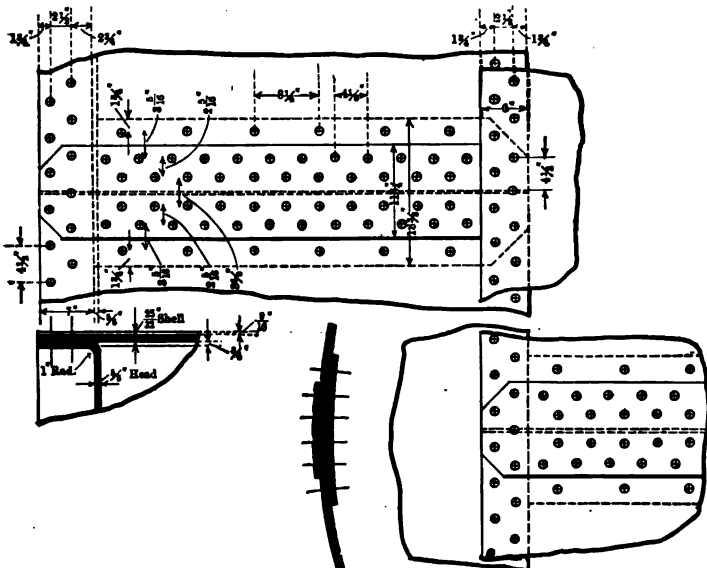
FIG. 169.

Steam-pressure, lbs. per sq. in., 180.   Inside diameter of shell 11' 6".
1⅛" rivets, 1 1/16" holes.   The Continental Iron Works.
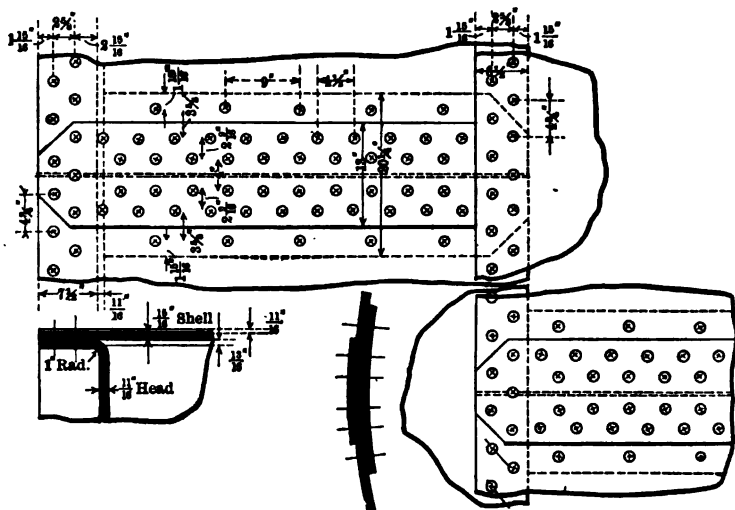
FIG. 170.

Steam-pressure, lbs. per sq. in., 160.   Inside diameter of shell 11' 6".
1¼" rivets, 1⁵⁄₁₆" holes.   The Continental Iron Works.
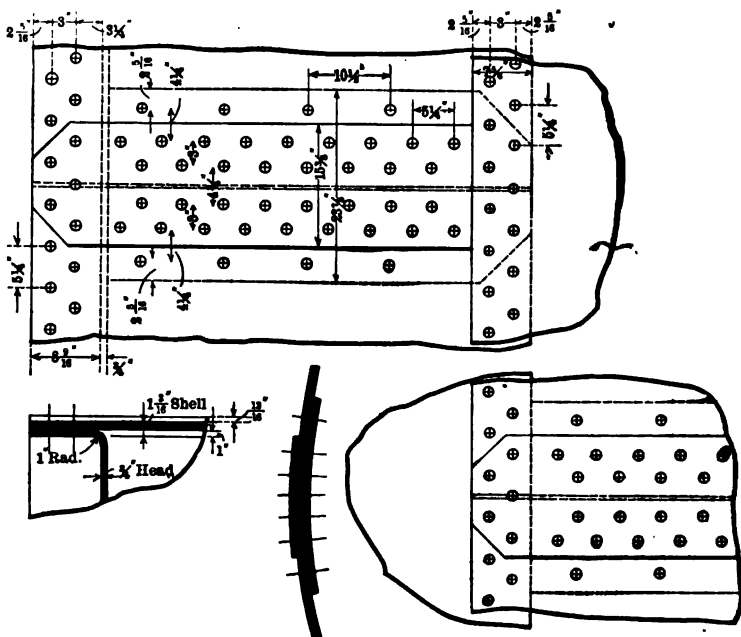
FIG. 171.

Steam-pressure, lbs. per sq. in., 200.   Inside diameter of shell 11' 6".
1⅛" rivets, 1⁹⁄₁₆" holes.   The Continental Iron Works.

CHAPTER XIII.

FRAMES OF PUNCHING, SHEARING, AND RIVETING MACHINES.

PUNCHING AND SHEARING MACHINES.*

146. A C frame of the general form shown in Fig. 172 is very commonly used for punching and shearing machines. The punch
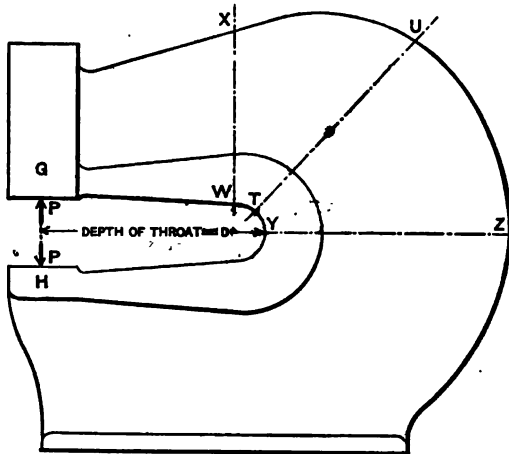
FIG. 172.

is attached to a slide which works in the guides G and operates against a die supported at H. The material to be punched or sheared is placed between the punch and die.

If the frame is for a stationary machine which is to set in the position shown in the figure, it is made with a flat base as shown. When the jaws are to stand vertically instead of horizontally, the base illustrated is not necessary, of course, but some suitable means

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* It is believed that Professor Albert W. Smith was the first to present the method of dealing with the stresses in C frames that is given in this chapter

must be supplied for supporting the frame. Portable punches are generally suspended by a chain or rope, hence no base is necessary.

The frames for heavy stationary punching and shearing machines are generally made of cast iron. Forgings and steel castings are generally used for portable machines.

In designing a punch-frame for strength it is necessary to consider the external forces acting upon it, together with the internal stresses due to the externally applied forces. Sections must be selected at different places along the frame, and each designed so as to give the required working stress in the material, due allowance being made for the localization of shrinkage stresses at re-entrant angles, and for the general appearance of the machine.

In general it will be found that for cast iron the sections near the punch will have to be made heavier than is necessary for the requisite strength in order to obtain walls thick enough to cast well in connection with the remainder of the frame.

The external forces acting on the frame when punching a piece of metal have practically the same effect on the material near the back part of the throat of the frame as if two opposite forces, both coincident with the axis of the punch, were applied to separate the jaws. In ordinary construction the centre line of the punch lies in a plane dividing the frame into two symmetrical halves. This median plane of the frame is parallel to the paper in Fig. 172. When the punch is so located, its pressure against the piece to be punched has no other tendency than to separate the jaws.

When used as a shearing-machine, the cut begins near one end of the shear-blades and travels across them to the other end as they are brought together. Assuming that the blades are set so that they are perpendicular to the median plane, it can be seen that at the beginning of the cut the pressure against the blades will be to one side of the median plane, although approximately parallel to it. On account of being to one side of the median plane, the pressure against the shears at the beginning of the cut has a tendency to open the frame more on one side than the other, and thus cause a torsional or twisting action upon it in the two parts extending forward to form the throat and jaws. When the cut has passed to the opposite end of the blades, a similar twisting action occurs, but in the opposite direction.

The frame should be of such a sectional form as will offer the greatest resistance to the twisting action just mentioned, as well as to the greatest (in ordinary cases) tendency to separate the jaws. This can be done, when the frame is cast, by making it of the hollow or box form.

If the shear-blades are placed parallel to the median plane with their cutting edges in it, and are sharp, there would probably be no great force acting on the frame when shearing other than one tending to separate the jaws in the same manner as when punching. As the blades become dull, however, there is a tendency for them to move sidewise relatively to each other, thus inducing a side bending action on the parts of the frame forming the jaws, and a twisting action on sections back of the throat. The box section is the best to resist this action, and is generally used for cast-iron frames.

The following notation may be used for punch-frames :

$A =$ total area of section of frame;

$I =$ moment of inertia of a section about its gravity axis which is normal to the median plane of the frame;

$I_g =$ moment of inertia, about its gravity axis normal to the median plane, of any part of the section;

$I_a =$ moment of inertia, about the gravity axis of the entire section, of any part of the section;

$P =$ force necessary to drive punch through the material to be perforated;

$S_e = I \div e_o =$ section modulus of the section with regard to compressive fibre-stress;

$S_t = I \div e_t =$ section modulus of the section with regard to tensile fibre-stress;

$a =$ area of any selected part of the section;

$b =$ greatest dimension of any partial section, measured parallel to the gravity axis about which the moment of inertia is required;

$d =$ depth of throat of frame;

$l =$ distance from line of action of P to gravity axis of section under consideration;

$e_o =$ distance from gravity axis of section to outermost fibre in compression;

$e_t =$ distance from gravity axis of section to outermost fibre in tension;

$f_e =$ greatest compressive fibre-stress in section;

$f_t =$ greatest tensile fibre-stress in section;

$h =$ greatest dimension of any partial area measured perpendicular to the gravity axis about which the moment of inertia is required;

$s =$ shearing-stress per unit area uniformly distributed over entire section;

$t =$ tensile stress per unit area uniformly distributed over the entire section;

$z =$ distance between gravity axis of any selected part of the area and the gravity axis of entire section.

147. Stresses in a section perpendicular to the motion of the punch.—The stress in the material on the section YZ, Fig. 172, taken normal to the pressure P against the punch and die, can be determined by considering the portion of the frame above YZ as a free body, as shown in Fig. 173.
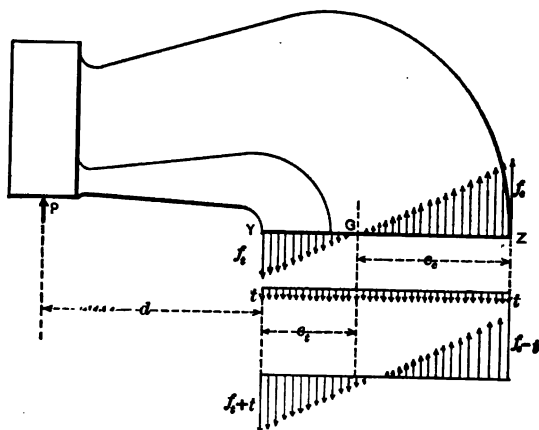
FIG. 173.

The forces acting on this part of the frame may be taken as those due to punching a plate. They are:

1st. The pressure P against the punch as it is forced through

the piece to be perforated. The line of action of P coincides with the centre line of the punch.

2d, The fibre-stress in the material due to the bending action of P. P acts with a lever-arm whose length is the distance $d + e_t$ from its line of action to the centre of gravity of the section. The tensile fibre-stress f_t at Y, and the compressive fibre-stress f_o at Z, are:

$$f_t = \frac{P(d + e_t)}{S_t}; \quad . \quad . \quad . \quad . \quad . \quad . \quad (187)$$

$$f_o = \frac{P(d + e_t)}{S_o}. \quad . \quad . \quad . \quad . \quad . \quad . \quad (188)$$

3d. A uniformly distributed tensile stress of a value

$$t = P \div A$$

over the entire section.

By combining the stresses acting on the section YZ it can be seen that the maximum total tensile stress per unit area is at Y, and is equal to $f_t + t$.

The equation for obtaining the total stress is

$$\left.\begin{array}{c}\text{Maximum tensile stress} \\ \text{on section } YZ\end{array}\right\} = f_t + t = \frac{P(d + e_t)}{S_t} + \frac{P}{A}. \quad (189)$$

Similarly, the maximum total compressive stress per unit area on the section YZ is at Z, and is equal to the numerical difference of f_o and t. Therefore

$$\left.\begin{array}{c}\text{Maximum compressive} \\ \text{stress on section } YZ\end{array}\right\} = f_o - t = \frac{P(d+e_t)}{S_o} - \frac{P}{A}. \quad . \quad (190)$$

If the centre of gravity of the section lies midway between Y and Z, then $f_t = f_o$. Equations (189) and (190) show that if the centre of gravity is thus located the maximum tensile stress at Y will be greater than the maximum compressive stress at Z by an amount equal to $2t = 2P \div A$. This assumes that the modulus of elasticity of the material is the same for tension as for compression,

which is practically true for the materials common to such construction.

If a material having equal strength in both tension and compression is used for the frame, the cross-section should be made heavier on the side next the punch or shear-blades in order that $(f_t + t)$ shall equal $(f_o - t)$, this being the condition for equal maximum tensile and compressive stresses.

For such a material as cast iron, which is much stronger in compression than tension, it becomes necessary for economy of material to make the section much heavier on the side next the punch than at the back. This is done to make $(f_t + t)$ smaller than $(f_c - t)$.

148. Numerical solution for a section perpendicular to the motion of the punch.—This corresponds to the section on YZ in Fig. 172.

Let the work required of the machine be to punch a 1-inch hole 19 inches from the edge of a plate $\frac{3}{4}$ of an inch thick, having a shearing-strength of 55000 pounds per square inch; also to be operated with shear-blades for shearing the same plate. The frame to be cast iron. The throat to be 20 inches deep, so as to allow the edge of the plate to clear it 1 inch, and to be stressed to practically 2000 pounds per square inch tension when punching the hole.

The force P necessary to drive the punch through the plate is found by multiplying the shearing-strength of the material by the area to be sheared by the punch. The sheared area is that of the wall of the hole made by the punch. Whence

$$P = 55000\pi \times 1 \times \tfrac{3}{4} = 129600 \text{ lbs.}$$

It can safely be assumed that the force exerted to separate the jaws of the frame will not be so great when shearing as when punching the size of hole specified. If the frame is made strong enough to operate as a punch, it is therefore only necessary to give it such a form of cross-section as will secure sufficient rigidity against twisting sidewise when shearing.

A drawing of any convenient size, and of a form considered suitable for its purpose, may be made for the required section. Fig. 174 may be taken as the sectional form selected for this problem. The moment of inertia I of this section, and thence its

section modulus, may now be found, the scale of the drawing being taken as unity (i.e., scale 1 inch = 1 inch).   The section thus represented in Fig. 174 is not by any means large enough for the
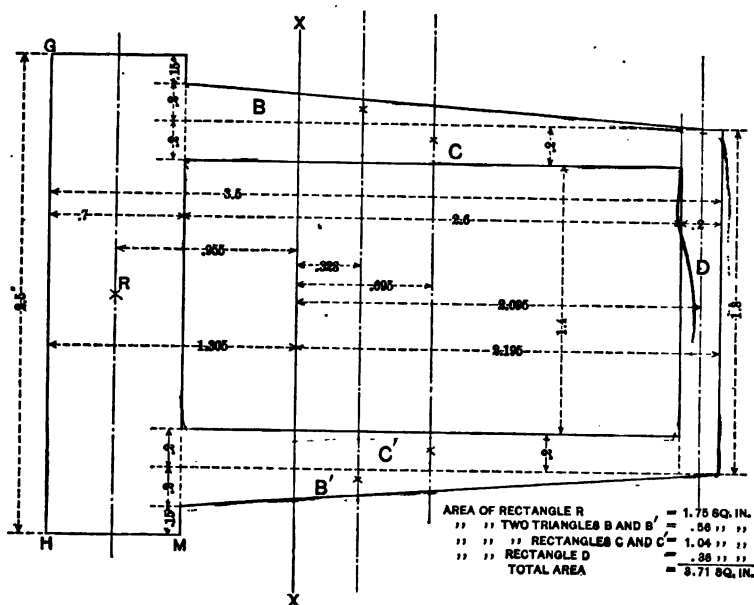
FIG. 174.

required strength but it is not necessary that it shall be, for the scale that must be applied to it to obtain the dimensions for the requisite strength can be determined by trial, as shown later.

In Fig. 174 the section has been taken of a simple form in order. that the calculations for I might be readily made.   The actual working form should have rounded corners and filleted re-entrant angles.

For convenience in determining I, the section is divided into six parts, namely:

1st. A rectangle R;
2d. Two similar and equal triangles B and B';
3d. Two similar and equal rectangles C and C';
4th. A rectangle D.

The divisions through the body of the section are indicated by broken lines. The centre of gravity of each partial area of the section is indicated by $\times$.

In order to determine I, the gravity axis XX of the section must first be located. This can be done conveniently by taking moments about the line GH, considering the areas as forces proportional to the areas, whence

$$e_t = 1.305 \text{ in.} \quad \text{and} \quad e_o = 2.195 \text{ in.}$$

For the rectangle R:

$$I_g = \tfrac{1}{12}bh^3 = \tfrac{1}{12} \times 2.5(.7)^3 = .0714;$$

$$az^2 = 1.75 \times (.955)^2 = 1.596;$$

$$I_x = I_g + az^2 = .0714 + 1.596 = 1.6674.$$

For the triangles B and B':

$$I_g = 2 \times \tfrac{1}{36}bh^3 = \tfrac{1}{18} \times .2(2.8)^3 = .2439;$$

$$az^2 = .56(.328)^2 = .0602;$$

$$I_x = .2439 + .0602 = .3041.$$

For the rectangles C and C':

$$I_g = 2 \times \frac{1}{12}bh^3 = \frac{.2(2.6)^3}{6} = .5856;$$

$$az^2 = 1.04(.695)^2 = .5023;$$

$$I_x = .5856 + .5023 = 1.0879.$$

For the rectangle D:

$$I_g = \tfrac{1}{12}bh^3 = \tfrac{1}{12} \times 1.8(.2)^3 = .0012;$$

$$I_h = az^2 = .36(2.095)^2 = 1.5800;$$

$$I_x = .0012 + 1.58 = 1.5812.$$

For the entire section:

$$I = 1.6674 + .3041 + 1.0879 + 1.5812 = 4.6406.$$

The section modulus for tension is

$$S_t = \frac{I}{e_t} = \frac{4.6406}{1.305} = 3.556.$$

This is the section modulus for the dimensions given in Fig. 174. It is clearly too small for the required service. A scale of drawing must therefore be assumed which will give a larger section.

Assume that the drawing Fig. 174 is one ninth size. The value of e_t for the enlarged section will therefore be 9×1.305, and S_t for the same will be $(9)' \times 3.556$, since the section moduli of similar sections are proportional to the cubes of their linear dimensions. The area will be $(9)' \times 3.71$.

The maximum tensile stress in the enlarged section will therefore be, by equation (189),

$$f_t + t = \frac{129600(20 + 9 \times 1.305)}{(9)' \times 3.556} + \frac{129600}{(9)' \times 3.71}$$

$$= 1590 + 430 = 2020 \text{ lbs. per sq. in.}$$

This value is practically satisfactory. Therefore by multiplying the linear dimensions of Fig. 174 by 9 a section of suitable size may be obtained. The dimensions obtained in this manner should be modified to agree with shop methods of measurement, of course. The corners should be rounded and re-entrant angles filleted. This need not be done to such an extent as to seriously affect the section modulus, however. It is often necessary to apply a fractional scale to the drawing, such, for instance, as 9.1, 8.7, etc.

If the section adopted is of such a form as to make it impossible to divide it into geometrical portions, the moment of inertia may be found graphically, the gravity axis being located during the process.* The section modulus can then be readily determined.

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* For finding moment of inertia graphically see appendix, § P.

149. Section parallel to the motion of the punch.—This corresponds to a section on WX, Fig. 172. The portion of the frame to the left of WX may be considered a free body, as in Fig. 175. The
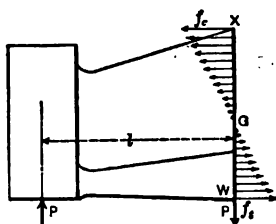
FIG. 175.

forces and stresses acting on it are indicated in the figure. They are:

1st. The force P, coincident with the centre line of the punch.

2d. The fibre-stress in the material due to the bending action of P. P acts on a lever-arm of length l, measured from the line of action of P to the plane of the section. The tensile fibre-stress is at W, and equals

$$f_t = \frac{Pl}{S_t}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (191)$$

The compressive fibre-stress is at X, and equals

$$f_o = \frac{Pl}{S_o}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (192)$$

3d. A shearing-stress uniformly distributed over the section. The value of this shear per unit area is

$$s = \frac{P}{A}. \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad (193)$$

The maximum tension, compression, and shear are found by combining the stresses obtained by the last three equations. The formulas for maximum tension and compression, as given in works on the strength of materials, are:

$$\text{Maximum tension} \quad = \frac{f_t}{2} + \sqrt{s^2 + \left(\frac{f_t}{2}\right)^2};$$

$$\text{Maximum compression} = \frac{f_c}{2} + \sqrt{s^2 + \left(\frac{f_c}{2}\right)^2}.$$

Since the value of s is the same for all parts of the section, the maximum shear will occur where the fibre-stress due to bending is greatest. This will be at X if the gravity axis of the section is farther from X than from W. The equation is

$$\text{Maximum shear} = \sqrt{s^2 + \left(\frac{f_c}{2}\right)^2}. \quad \ldots \quad (194)$$

The section must, of course, be designed strong enough to resist both tension and shear. In general, it is not possible to make it equally strong to resist both when such a material as cast iron is used. At a section near the punch the shearing-stress may be the one to fix the proportions, but when the section is taken near the back of the throat the tensile stress is more apt to be the one requiring consideration.

It should be noted that for the section WX the lever-arm l remains the same length whatever scale is applied to the drawing of the section.

The section WX must have such dimensions normal to the median plane of the frame as will allow it and the section on YZ to be joined together to form the frame.

In order to give a symmetrical appearance to the frame, and to obtain a form that will cast well, the sections near the guides at the working end of the frame are generally made much stronger than calculations show to be necessary.

150. Angular section of a punch-frame.—A section taken at an angle, as the one on TU, Fig. 172, has the following forces and stresses acting upon it. They may be seen by the aid of Fig. 176.

1st. The force P due to the pressure of the punch against the plate.

2d. The fibre-stress due to the bending action of P. The lever-

arm of P is l, measured from the line of action of P to the gravity axis of the section which is projected as a point at G. The tensile fibre-stress at T, due to the bending action of P, is

$$f_t = \frac{Pl}{S_t}; \quad \ldots \ldots \ldots \quad (195)$$
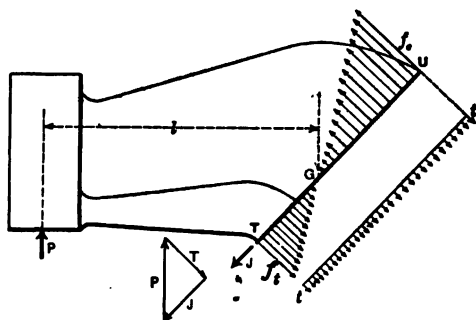
FIG. 176.

and the compressive fibre-stress at U, due to the same cause, is

$$f_o = \frac{Pl}{S_o}. \quad \ldots \ldots \ldots \quad (196)$$

3d. A shearing-stress J acting as shown in Fig. 176. The value of J is found by resolving P into two components, J and T, one parallel and the other perpendicular to the plane of the section. The shear per unit area due to J is

$$s = J \div A.$$

4th. A tensile stress T acting normal to the plane of the section. The tension per unit area due to T is

$$t = T \div A.$$

The formulas for maximum stresses are:

$$\text{Maximum tension} = \frac{f_t + t}{2} + \sqrt{s^2 + \left(\frac{f_t + t}{2}\right)^2};$$

Max. compression $= \dfrac{f_c - t}{2} + \sqrt{s^2 + \left(\dfrac{f_c - t}{2}\right)^2}$;

Maximum shear $= \sqrt{s^2 + \left(\dfrac{f_t + t}{2}\right)^2}$ or $= \sqrt{s^2 + \left(\dfrac{f_c - t}{2}\right)^2}$.

For the maximum shear the greater of the two quantities under the radicals is to be used. In many cases a numerical solution is necessary to determine which of these two quantities is the greater.

This section must have dimensions, normal to the median plane of the frame, intermediate between those of the sections on WX and YZ.

151. General form of a punch-frame.—In designing the frame onough sections should be tested to secure a nearly uniform strength throughout the frame, due allowance being made for the fact that the sections near the punch-guides may have to be heavier than is actually necessary for strength in order that the frame may have a symmetrial appearance. The walls of the frame should not be made so thin at any part that they will not cast well in connection with the heavier parts.

On account of the tendency of the shrinkage stresses in a casting to be greatest about re-entrant angles, it is generally advisable to make the sections bordering on the curve at the back of the throat somewhat stronger than those along the straight sides of the jaws. When the distance between the jaws is not greater than shown in Fig. 172, the back of the throat can be made an arc of a circle, or approximately so, as shown in the figure. When the jaws are far apart, however, this cannot generally be done well, but the re-entrant angles should be well filleted in order to prevent excessive shrinkage stresses in the material about them.

If the frame is made with an open flat bottom, suitable for resting on a foundation, the sections along the lower horizontal part, referring to Fig. 172, should be tested in the same way as those above, unless it is certain that each is at least as strong as the one immediately above it.

In most designs the frame tapers, somewhat in the manner of a pyramid, from the guides toward the back of the throat. The

thickness *HM*, Fig. 174, of the rib generally decreases from the throat toward the punch, as indicated in Fig. 172; less frequently it is kept of the same thickness all the way around the throat and jaws.

Having obtained the general form of the frame with regard to strength and symmetry, the auxiliary parts, such as bearings, etc., can be added to complete the frame.

DIRECT-ACTING HYDRAULIC RIVETER.

152. A hydraulic riveter is required for driving rivets in boiler-plate up to 60 inches wide and $\frac{5}{16}$ of an inch thick, the largest rivet to be $\frac{5}{8}$ of an inch in diameter, corresponding to 0.307 of a square inch area.

Experiments show that a pressure of 160000 pounds per square inch on a hot rivet of this diameter will set the rivet and form the head satisfactorily. The pressure required for this case will therefore be

$$0.307 \times 160000 = 49120 = 50000 \text{ pounds, in round numbers.}$$

By calling this value 50000 there is no greater deviation from the correct value than is shown in the experiments on riveting.

Fig. 177 is a side elevation of the outlines of the most common form of such machines as they are found in practice. The cylinder *Q* contains a piston attached to a piston-rod, one end of which projects at *R*, where one of the dies for compressing the rivet and forming the head is attached. The piston-rod serves as a ram for driving the die. The stationary die is at *S*.

In order to rivet cylindrical shells of small diameters, such as flues, the stake should be made as slender as is possible for the required strength, so that the shell can be passed over it for driving the rivets in the longitudinal seams when riveting up a single section, and for the circular seams when riveting two sections together. It can be made of either a steel forging of rectangular cross-section or a steel casting having an I section.

The frame may be cast iron or a steel casting. Since the over-

reach of the riveter, which is the distance it will reach over a plate from its edge to drive a rivet, is small, the frame can be so made of cast iron as to have sufficient strength without excessive weight.
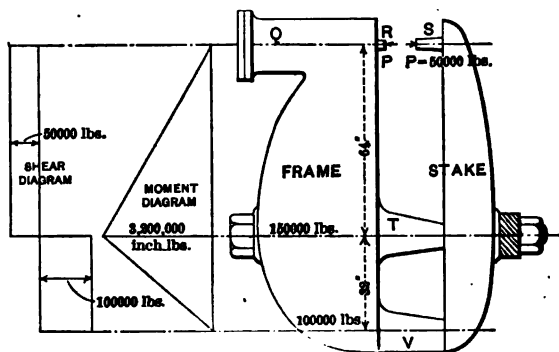
Fig. 177.

The stake and frame are held together by two bolts, one passing on each side of the stake and through a bar passing over the top. This bar, section-lined in the figure, clamps against the top or back of the stake.

The lugs T and V are a part of the frame, their office being to hold the stake in position.

In proportioning for strength the stake and frame can be dealt with separately, each as a beam supported at the ends and loaded eccentrically with a single load. The moment and shear diagrams, identical for both, are shown in the figure.

It will be noticed that the distance from the centre of the dies to the plane of the bolt centres is 64 inches, which is 4 inches in excess of the distance the centre of the rivet is to be from the edge of the widest plate. An examination of the figure shows that about this much must be allowed for the lug and bolts.

By taking moments about a point 64 inches from the centre line of the punch the pressure on the lug V is found. Its value is

$$V = (64 \div 32)50000 = 100000 \text{ pounds.}$$

That on T is

$$T = 50000 + 100000 = 150000 \text{ pounds.}$$

The bending moment is a maximum at the plane of the centre lines of the bolts, and equals

$$M = 50000 \times 64 = 3200000 \text{ inch-pounds.}$$

If the stake has a cross-section fairly thick, measured perpendicular to the plane of the paper, in proportion to its width, and is made of good steel, it will, on account of the length being large in proportion to its thickness and width, have fibre-stresses due to bending which are large in proportion to the shearing-stress uniformly distributed over the section. Under such conditions the shear can be neglected, and the stake designed to resist the stresses due to bending only. This is in accordance with the common method of dealing with beams whose height is small in proportion to their length. In general, the stake should taper from its maximum sectional dimensions at the bolts, growing smaller in both dimensions toward both ends. The thickness is generally reduced uniformly, but the height changes more rapidly as the distance from the largest part increases.

There should always be sufficient thickness to prevent the upper or stake die from springing sidewise to such an extent as to make the head of the rivet objectionably eccentric with the body.

The frame may be of either a solid I section or hollow. In either case the bending moment and shear can both be dealt with in the same manner as for the section WX of the punching and shearing machine.

The bolts must be tightened with a combined initial tension somewhat greater than that which is thrown upon them when a rivet is driven, in order to prevent the springing apart of the stake and lug. The lug T, therefore, must resist in this case a compression somewhat greater than 150000 pounds. The lug V must, of course, resist a compression of 100000 pounds.

The frame may either stand as in the figure, suitable flanges being provided for resting upon a foundation, or it may be placed with the axis of the cylinder vertical. The nature of the service required of the machine is the determining factor for the position in which it shall be placed.

CHAPTER XIV.

SELECTION OF MATERIALS.

153. In selecting materials for machine parts, the properties that must most commonly be considered with regard to their adaptability are as follows: strength, resilience, stiffness, coefficient of friction, durability, convenience and cost of working into the required form, and cost of the material as found on the market in the merchantable form. It is seldom that all of these properties need to be taken into account when deciding upon the material for any given machine-member.

A few examples may serve to show more clearly than any other means how the qualities have to be taken into account.

In an ordinary engine-lathe the bed is of a somewhat complicated form, having ribs, sometimes hollow, extending from side to side, bosses and other raised places for attaching parts, and the sides are ordinarily flanged or ribbed at both top and bottom in order to give rigidity. This complication at once sets forth the necessity of making the bed of some material that can be cast in moulds. There are only two materials, whose cost is such as to allow them to be used for such purposes, that can be formed by casting, namely, cast iron and steel. Cast iron is much the cheaper of the two, and can be planed and otherwise machined much more readily and, consequently, more cheaply than steel. It is a much weaker material than steel and is not so stiff. These two qualities apparently make it objectionable, but this is really not the case on account of the following reasons: In order for a lathe tool to take a broad cut when removing only a thin turning from the surface of the piece worked upon, it is necessary for the lathe-bed to be both rigid and heavy, so as to prevent the tool from springing away from its work and "chattering" against it; the mass of metal in the bed has much to do in the prevention of the small vibrations that

cause the chattering. Therefore the mass of metal necessary to hold the tool steady for wide cuts will make the lathe strong and stiff enough for the heaviest cuts that the driving mechanism of the lathe can carry. Cast iron, therefore, in comparison with steel, has for the lathe-bed the advantage in cost and ease of machining, while its lower strength and moduli of elasticity are of small consideration, on account of the mass of metal to be used; clearly, cast iron is the best material for the purpose.

Even if the vibration of the tool had not been an item in the conditions above, the cast iron would have still been the best material on account of its softness making it easy to machine as well as the low cost of the raw material, both together making the finished product much less costly in cast iron than in steel. This is also true of numerous other machine-tool parts and machines.

The spindle or arbor of an ordinary engine-lathe rests in two bearings supported by the head-stock, and is driven by a gear attached to it near the bearing next to the face-plate or live-centre, as the case may be. A stepped pulley occupies all or most of the remaining length of the part of the spindle which lies between the bearings. When the lathe is working, all the power transmitted to overcome the resistance of the cutting edge of the tool must pass through the part of the spindle lying in the bearing just back of the face-plate, thus causing a torsional moment in the spindle. In order to withstand this twisting moment, and at the same time the weight of the face-plate and the work upon it or the centres, the spindle, unless large in diameter, must be made of some strong material, which should also be resilient in order to withstand any of the shocks that are apt to occur accidentally. By keeping the spindle small the frictional loss in the head-stock bearings is kept down, and there is probably less liability to cutting and seizing of the journal by the bearings when running at high speeds. There is also less frictional loss between the bearing surfaces of the stepped pulley and spindle when the latter is not larger than is necessary to give ample bearing surface. The best material for the spindle is, therefore, a moderately hard machine steel, used without hardening. In case great accuracy and very little wear of the bearing surfaces are desired, wrought iron or very low carbon steel can be used by case-hardening and then grinding to form in an

emery grinding machine. This is the method frequently used in making milling-machine arbors.

In very heavy lathes, such as are used for turning locomotive drive-wheels, shafts for ocean vessels, or heavy ordnance, many designs have the driving-cone and back-gears placed on shafts serving to support them only, and the spindle is driven by a pinion meshing in a gear attached directly to the face-plate. By this means the spindle is relieved of all duty except supporting the face-plate and the load upon it or the live-centre. Since there is no torsional force to be resisted by the spindle, and furthermore because such lathes run at moderate speeds, the spindle can be enlarged and made of a weaker material than steel. Cast iron is found to give good service in such cases when made much larger than would be necessary for steel, lightness and rigidity being secured by making the spindle hollow. The large diameter gives a low pressure per unit area upon the bearing surfaces and, with proper care during the early life of the machine, they become glazed and wear well, even when the bearing as well as the spindle is of cast iron. The cost is much less for such a spindle of cast iron than for steel, even when the diameter is kept down with the latter.

In general, a steel lathe-spindle does not run satisfactorily upon cast iron, and since the head-stock of a lathe is almost invariably made of this material, it is necessary to provide some other material for the bearing surface upon which the journal of a steel spindle runs. In this case the material of the boxes is selected with regard to its coefficient of friction, durability in resisting frictional abrasion and wear, and possibly cost. The last item is generally not a very important one, however, since there is a comparatively small amount of material in them; brass, bronze, Babbitt metal, or some of the alloys resembling some one of them are the materials commonly chosen. Brass and bronze are more expensive, but they give a better and more finished appearance to the machine than Babbitt's or similar alloys which are used as linings for a cast-iron shell. The cost is far less for these metal linings with cast-iron casings than for brass or bronze.

In many cases the conditions to be fulfilled indicate clearly the necessary material; thus a spring that must have considerable strength and resilience must be made of steel containing enough

carbon to cause it to harden and temper under proper heat treatment, or it must be made of brass or some similar alloy; in the general case, therefore, there are narrow limits to choose between. As soon as the purpose of the spring is defined, as for car- or wagon-springs, the cost immediately limits the selection to a single choice, steel being the only material admissible.

Hydraulic presses when made of cast iron allow the water to percolate through the walls of the cylinder when subjected to high pressures. This was formerly a source of great trouble, and various devices were invented to overcome it, one being to line the cylinder with copper, brass, or bronze. When steel castings came to be successfully made, it was found that they were water-tight even under pressures much higher than had been attempted with the cast iron. Naturally the material to be selected for a high-pressure hydraulic press is steel, the selection in this case being based mainly upon the density and consequent water-tightness of the metal, although the greater strength is a considerable factor, these two more than overbalancing the increased cost due to the higher price of steel castings and the more expensive work of machining.

TABLE XLIX.

GENERAL PROPERTIES AND METHODS OF WORKING THE MATERIALS MOST COMMON TO ENGINEERING CONSTRUCTION.

Material.	Tensile Strength.	Compressive Strength.	Shock Resisting Power.	Methods of Shaping for Use.
Cast iron..................................	Low.	Very high.	Low.	Casting.
Wrought iron, also called malleable iron........	Medium.	Medium.	High.	Rolling, forging, and wire-drawing.
Bessemer and open-hearth steel, also called machinery steel, soft steel, mild steel, and ingot iron	High.	High.	High.	Rolling, forging, and wire-drawing.
Crucible steel, also called cast steel and tool steel....................................	Very high.	Very high.	Medium.	Rolling, forging, and wire-drawing.
Steel castings................................	High.	High.	High.	Casting.
Malleableized castings, also called malleable iron castings................................	Medium.	High.	High.	Casting.
Brass	Medium to low.	....	Medium.	Casting, rolling, forging, and wire-drawing.
Bronze.....................................	Medium to low.	....	Medium.	Casting, rolling, forging, and wire-drawing.
Babbitt metal and other grades of white metal used for lining journal boxes..............	Very low.	....	....	Casting.

TABLE L.

STRENGTHS AND MODULI OF ELASTICITY OF THE MATERIALS MOST COMMON TO ENGINEERING CONSTRUCTION.

Material.	Per Cent. Carbon.	Tensile Strength.		Compressive Strength.		Shearing Strength.	Flexure.	Modulus of Elasticity.	
		Elastic Limit.	Ultimate.	Elastic Limit.	Ultimate.	Ultimate.	Stress in Outer Fibre.	Tension.	Shear.
Bessemer and open-hearth steel	0.15	42,000	63,000	39,000	......	48,000	......	30,000,000	9,000,000
	0.20	47,000	66,000	43,000	......	53,000	......		
	0.50	48,000	80,000	46,000	......	57,000	......		
	0.70	53,000	89,000	53,000	......	60,000	......		
	0.80	57,000	108,000	63,000	......	66,000	......		
	0.95	69,000	118,000	71,000	......	88,000	......		
High-grade wrought iron	......	28,000	56,000	28,000	......	40,000	{ Elas. lim. 52,000	28,000,000	9,000,000
Common wrought iron	......	22,000	40,000	28,000	......	32,000	......	28,000,000	9,000,000
Crucible or tool steel	......	58,000	116,000	58,000	......	......	......	31,000,000	12,400,000
Malleable cast iron	......	......	85,000	......	......	......	{ Ultimate 70,000	......	......
Steel castings	......	35,000	65,000	29,000	......	......	......	19,000,000 to 31,000,000 Average 25,000,000	......
Cast iron	......	......	10,000 to 33,000 Average 20,000	......	56,000 to 145,000 Average 90,000	18,000 to 30,000	Ultimate 30,000 to 54,000 Average 42,000	18,000,000	7,000,000

APPENDIX.

A. Development of equations for an angular-thread screw.— Fig. 178 shows a portion of an angular-thread screw of which $HH^\blacktriangle$
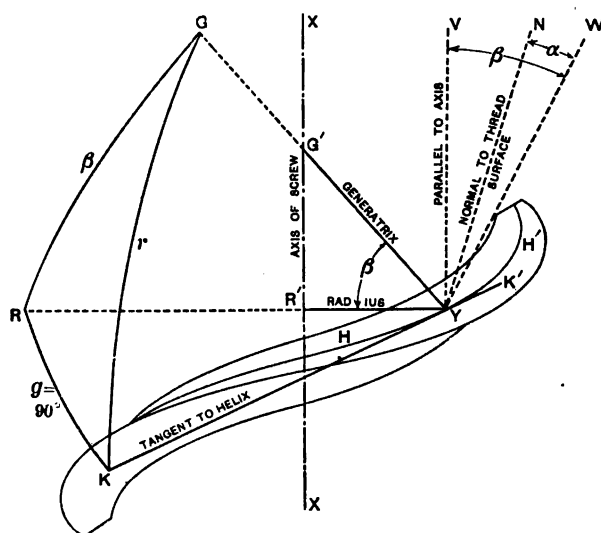
FIG 178.

is the mean helix, and KK' a tangent to it at Y. Y is a point on HH', and VY is parallel to the axis of the screw. VY may be taken as representing an elementary force acting on the thread at Y. $G'Y$ is the position of the generating-line when passing through the point Y; the angle of $G'Y$ with the radial line $R'Y$ is β. $R'Y$ is, of course, normal to the axis XX of the screw.

The elementary force VY may be resolved into two forces, one radial along YR', and the other along the line WY, normal to $G'Y$ and lying in the axial plane containing $G'Y$ and $R'Y$. The radial

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component is annulled, so far as forces external to the screw are concerned, by the radial component of the elementary force corresponding to VY, and lying diametrically opposite it; therefore the radial component does not need further consideration.

The component acting along WY is held in equilibrium by two forces, one (NY) normal to the surface of the thread, and the other (not shown in the figure) perpendicular to the axial plane through Y (i.e., perpendicular to the plane containing $G'Y$ and $R'Y$). This latter force is the one which, acting with a lever-arm equal to the radius of the mean helix, tends to produce rotation of the screw.

In order to determine the relative values of this rotative force and that along WY it is necessary to know the value of the angle α = angle NYW in terms of the thread-angle β and the helix-angle θ. (θ is the angle between KK' and a normal to the axial plane through Y.)

To facilitate the determination of the value of α, the spherical triangle GRT may be formed by extending YG' and YR'. In this triangle the side $g = 90°$ and β are known, as well as the solid angle R along RY, which equals $90° + \theta$. The angles G and K are each less than $90°$.

The line NY is normal to the plane KYG, and since GY is the intersection of this plane with the axial plane RYG, α is the complement of the solid angle G, or $\alpha = (90° - G)$.

The determination of the value of α in terms of θ and β is as follows:

$$\begin{aligned}
\cos r &= \cos g \cos \beta + \sin g \sin \beta \cos R \\
&= \cos 90° \cos \beta + \sin 90° \sin \beta \cos (90° + \theta) \\
&= 0 + \sin \beta \sin \theta \quad . \quad . \quad . \quad . \quad . \quad . \quad . \quad (197)
\end{aligned}$$

and

$$\sin G = \sin g \, \frac{\sin R}{\sin r} = \sin 90° \frac{\sin (90° + \theta)}{\sqrt{1 - \cos^2 r}}.$$

Substituting the value of $\cos r$ given in equation (197) gives

$$\sin G = \frac{\cos \theta}{\sqrt{1 - \sin^2 \beta \sin^2 \theta}}.$$

By trigonometrical relations

$$\cot G = \frac{\sqrt{1 - \sin^2 G}}{\sin G}$$

$$= \sqrt{1 - \frac{\cos^2 \theta}{1 - \sin^2 \beta \sin^2 \theta}} \times \frac{\sqrt{1 - \sin^2 \beta \sin^2 \theta}}{\cos \theta}$$

$$= \frac{\sqrt{1 - \sin^2 \beta \sin^2 \theta - \cos^2 \theta}}{\cos \theta}.$$

This reduces to

$$\cot G = \tan \theta \cos \beta = \tan \alpha. \quad . \quad . \quad . \quad . \quad (198)$$

The relation between the force VY and, the turning force. through Y and normal to the axial plane RYG is as follows:

The force $WY = VY \sec \beta$, and the turning force $= WY \tan (\alpha + \phi)$, in which ϕ is the friction-angle. Therefore

$$\text{Turning force} = VY \sec \beta \tan (\alpha + \phi)$$

$$= VY \sec \beta \frac{\tan \alpha + \tan \phi}{1 - \tan \alpha \tan \phi}.$$

And by substituting the value of α given in equation (198)

$$\text{Turning force} = VY \sec \beta \frac{\tan \theta \cos \beta + \tan \phi}{1 - \tan \theta \cos \beta \tan \phi}$$

$$= VY \frac{\tan \theta + \sec \beta \tan \phi}{1 - \tan \theta \cos \beta \tan \phi}.$$

By adding together all the elementary forces VY and calling. their sum $T =$ tension in bolt, the total turning force F becomes

$$F = T \frac{\tan \theta + \sec \beta \tan \phi}{1 - \tan \theta \cos \beta \tan \phi}.$$

B. **Graphical determination of the moment of inertia of a plane area. Approximate method.**—In the numerical solution given in § 148 it can be seen that the I_x of the rectangle R (referring to Fig. 174) is made up of the sum of two quantities, one of which is the moment of inertia I_g of the rectangle about its own gravity axis, and the other the product az^2 of the area of the rectangle R by the square of the distance of its centre of gravity from the gravity axis XX of the entire plane area. If the rectangle R were very narrow, as measured perpendicular to the gravity axis XX, its I_g would become so small as to be negligible for practical purposes. Its I_x would therefore practically be equal to az^2. The graphical solution which follows is a method of finding the sum of all the az^2's for the small areas into which a given plane area may be divided.

In Fig. 179 A is the plane section whose approximate moment of inertia about its gravity axis normal to the centre line of the area is required. This approximate moment of inertia will be given the same symbol I_d as the accurate moment of inertia about the gravity axis.

Divide the section A into a number of thin strips, 1, 2, 3, 4, . . . , 13, 14, parallel to the gravity axis about which the moment of inertia is to be determined. The dotted lines indicate the divisions. In Fig. 179 these strips are made somewhat wide in order to secun sufficient space for lettering and to make the drawing clear. For very accurate work they should be made narrower. These strips may be of the same or different widths. The position of the centre of gravity of each strip may now be estimated, and a line drawn through it parallel to the gravity axis about which I_x is to be found. The same can also be done for all the other strips. The area of strip 1 may now be represented by a line AB parallel to the given direction of the gravity axis. The line through the centre of gravity of strip 1, whose area is represented by the vector AB, is indicated by the letters ab written on opposite sides of the line. In the same manner bc is the line through the centre of gravity of strip 2, and BC the vector representing the area of 2. BC is a continuation of AB. By proceeding in this manner for all the strips, the line AO, which is the vector representing the entire area of the section A, is obtained.

Select a point P at a distance $\frac{1}{4}AO$ from the line AO, and from

it draw rays to the points A, B, C, ..., N, O. One position of P may be readily located by drawing lines from A and O, each

FIG. 179.

making an angle of 45° with AO; their intersection determines one position of P.

From any point a' on ab draw $a'X$ parallel to AP, and of indefinite length; from a' also draw $a'b'$ parallel to BP, intersecting bc at b'; through b' draw $b'c'$ parallel to CP, intersecting cd at c'; through c' draw $c'd'$ parallel to DP, intersecting de at d'. Continuing this operation, $n'X$ is finally drawn through n' parallel to OP, intersecting $a'X$ at X. The point X lies on the gravity axis of the entire section which is parallel to the strips into which the area was first divided. The gravity axis is therefore determined by drawing XX' perpendicular to the centre line of the area. (The proof that X determines the gravity axis is similar to that for finding a point through which the resultant of a number of parallel forces passes. It does not seem necessary to give it here.)

Extend $a'b'$ to intersect XX' at q. The triangles $a'Xq$ and PAB are similar, since their sides are parallel by construction. The altitude of $a'Xq$ is z_1; that of PAB is $\frac{1}{2}AO$.

Therefore

$$(AB) : (Xq) = (\tfrac{1}{2}AO) : z_1;$$

whence

$$(AB)z_1 = (\tfrac{1}{2}AO)(Xq).$$

By multiplying both sides of this equation by z_1 it becomes

$$(AB)z_1^{\,2} = (AO)(\tfrac{1}{2}Xq \times z_1).$$

In this last equation the first member $(AB)z_1^{\,2}$ corresponds to the required product represented by the general expression az^2 for the strip under consideration; AO represents the entire area of the given section; and $\frac{1}{2}Xq \times z_1$ equals the area of the triangle $a'Xq$. Therefore for strip 1

Approximate I_x = (area of given section) × (area of triangle $a'Xq$).

In the same manner it may be shown for the strip 2 that its

approximate I_x = (area of total section) × (area of triangle $b'qr$),

and so on for all the strips into which the given section is divided.

The sum of the areas of all the triangles of a nature similar to

$a'Xq$ and $b'qr$ equals the area of the funicular polygon B, shaded in the figure. Therefore for the entire section A,

Approx. $I =$ (area of given section) $\times$ (area of funicular polygon)
$$= (\text{area } A) \times (\text{area } B).$$

Both of these areas can be conveniently measured with a planimeter.

In a manner similar to the above it may be shown that the moment of inertia of the section A about any axis YY' parallel to XX' is

Approximate $I_v = (\text{area } A) \times (\text{area } a'b'c' \ldots m'n'vsa')$.

The points s, t, and u are obtained by prolonging $a'z$, $a'q$, and $a'r$ to intersect YY'.
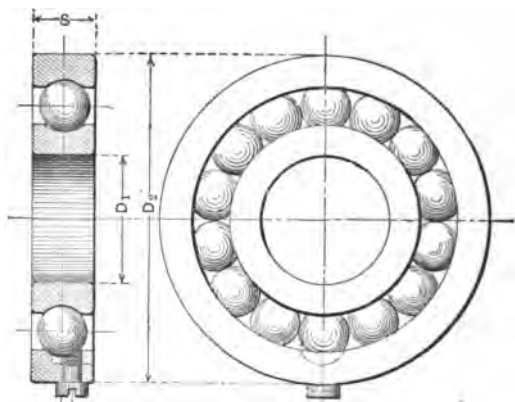
FIG. 180.
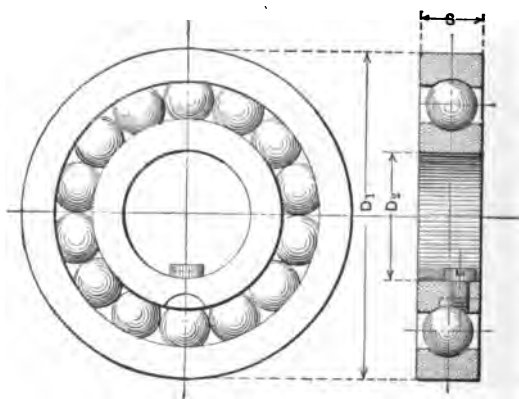
FIG. 181.

TABLE LII.

BALL BEARINGS. DIMENSIONS AND LOADS.

Refers to Figs. 182 and 183.

(From Catalogue of the Deutsche Waffen- und Munitions-Fabriken, Berlin, with slight modifications.)

25.4 millimetres = 1 inch; .4536 kilograms = 1 pound.

Number Corresponding to		Diameter of Rings, Millimetres.		Width of Rings. *S.* Millimetres.	Revolutions per Minute.	Safe Load Normal to Axis. in Kilograms.	Weight of Bearing, in Kilograms.
Fig. 182.	Fig. 183.	Inside, D_1.	Outside, D_2.				
102	....	10	32	9	3500	95	0.035
103	103	15	37	9		115	0.050
104	104	20	42	9		135	0.060
105	105	25	52	9		250	0.087
106	106	30	62	10	Decreases uniformly to	340	0.137
107	107	35	70	10		375	0.175
108	108	40	80	11		420	0.250
109	109	45	85	11		450	0.275
110	110	50	90	11		500	0.320
111	111	55	100	12		650	0.410
112	112	60	105	12		700	0.450
113	113	65	115	14		900	0.620
114	114	70	120	14		950	0.650
115	115	75	130	16		1200	0.765
116	116	80	135	16		1250	0.950
117	117	85	145	18		· 1520	1.160
118	118	90	150	18		1590	1.235
119	119	95	160	20		1875	1.530
120	120	100	165	20		1950	1.585
121	121	105	180	22		2300	2.235
122	122	110	185	22	1500	2350	2.275

Thrust bearings for 1500 to 4000 R.P.M. are made in numbers 102 to 122 similar to Fig. 182, except that one less ball is used. The loads for thrust bearings of this type are one-third of the values in the "safe load" column of the above table.
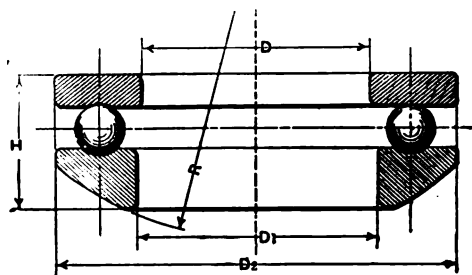
TOP RING HAS FLAT BEARING SURFACE.

FIG. 184.
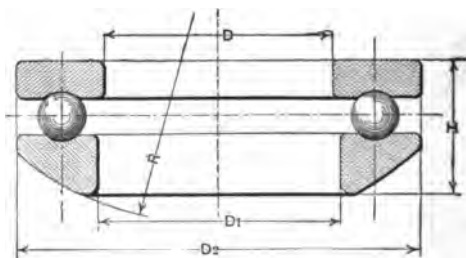
FIG. 185.
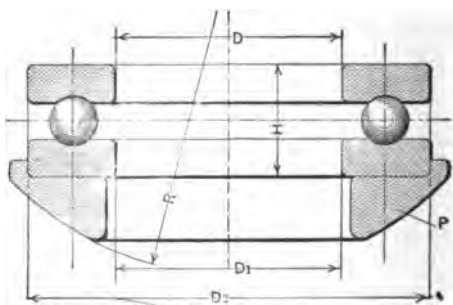
FIG. 186.

TABLE LIII.

BALL THRUST BEARINGS. DIMENSIONS AND LOADS.

Refers to Figs. 184, 185, and 186.

(From Catalogue of the Deutsche Waffen- und Munitions-Fabriken, Berlin)

The bearings are made either with both rings grooved, as in Fig. 185. or with one ring flat and the other grooved, as in Fig. 184 Numbers 21, 23, 25, and 28 have a separate spherical-surface plate as shown in Fig. 186.

25.4 millimetres = 1 inch; .4536 kilograms = 1 pound

Number of Bearing.	Dimensions in Millimetres.					Safe Loads in Kilograms						Weight of Bearing. Kilograms.
						Fig. 184.			Fig 185			
	Inside Diameter.	Inside Diameter.	Outside Diameter.	Height.	Radius of Spherical Surface.	Up to 1500 Revolutions per Minute	Up to 1000 Revolutions per Minute.	Up to 500 Revolutions per Minute.	Up to 300 Revolutions per Minute	Up to 150 Revolutions per Minute.	Up to 10 Revolutions per Minute	
	D	D_1	D_2	H	R							
6	30	32	53	18	40	150	200	250	300	350	1100	0.15
7	35	37	62	21	50	200	250	300	400	450	1500	0.21
8	40	42	64	21	50	250	300	350	450	550	1600	0.24
9	45	47	73	25	60	300	350	400	550	700	2100	0.36
10	50	52	78	25	65	350	400	500	650	800	2300	0.40
11	55	57	88	28	70	400	500	600	750	1000	2900	0.57
12	60	62	90	28	75	450	550	700	850	1100	3100	0.80
13	65	67	100	32	80	550	650	800	1000	1200	3800	0.88
14	70	72	103	32	85	600	700	900	1100	1400	4000	0.91
16	80	82	115	35	95	700	800	1100	1200	1700	5000	1.10
17	85	88	125	38	105	850	950	1300	1500	2000	6000	1.44
19	95	98	140	41	115	1000	1150	1600	1900	2400	7000	1 90
21	105	108	155	40	130	1200	1400	1800	2200	2700	8000	2.42
23	115	118	165	43	140	1300	1600	2200	2500	3200	10000	2.91
25	125	128	175	46	150	1400	1900	2400	2900	3700	11000	3.57
28	140	143	200	52	170	1700	2200	3000	3700	4800	13000	4.99

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